The Size of the Central Bank's Balance Sheet: Implications for Capital Formation and the Yield Curve*

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Abstract

The tools used by central banks have evolved considerably in recent years. One important aspect of unconventional policy has been the move of central banks to aggressively purchase longer-term government securities along with expanding the size of its balance sheet. In this paper, we construct a rigorous modeling framework to investigate the impact of such policies. The inclusion of long-term assets and an independent central bank allows us to focus on a Treasury bond purchase program as a monetary policy tool. Outright open market operations through purchases of short-term government securities unambiguously lower yields and promote economic activity. By comparison, long-term asset purchases are only effective if short-term yields are sufficiently low. Should central bankers in countries with high debt loads and high short-term interest rates implement a long-term asset purchase program, it will be ineffective as it would only encourage fiscal authorities to issue more debt. Moreover, we also explain how the transmission channels of monetary policy between unconventional and conventional policies have important consequences for risk-sharing in the economy.

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1 Introduction

In recent years, central banks around the world resorted to balance sheet expansion policies that reached unprecedented levels in order to promote economic activity. In the United States, for example, the Federal Reserve increased the size of its balance sheet more than fourfold: total assets increased from 0.92 trillion dollars in August 2008 to 4.3 trillion dollars in May 2014.[1] The impact of balance sheet policies in the U.S. had important effects on longer-term interest rates. Notably, a number of studies point out that the cumulative effects of quantitative easing as well as Operation Twist led to a reduction between 80 and 120 basis points on the yield of 10 year Treasury-securities.[2] In a similar manner, Bauer and Neely (2014) argue that the purchases also affected international bond yields. In addition, other major central banks such as the Bank of England have also pursued extensive balance sheet policies around the same time as the Federal Reserve.

Moreover, aside from recent years, the possibility of such interventions was raised earlier by Bernanke (2003) and Bernanke and Reinhart (2004). They posit that expansion of the balance sheet can be effective when overnight interest rates are close to zero. Through purchases of government bonds along with shifting the maturity structure to longer-dated securities, the size of the central bank's balance sheet can reduce their yields to stimulate economic performance.

By comparison to other central banks, the Japanese experience with balance sheet policies began in the Spring of 2001 – a much longer timeframe than other countries. Similar to recent observations, Ueda (2012) and Takeda and Yajima (2014) provide evidence that such policies led to a decline in long-term yields of Japanese government bonds as well as in the returns of other privately-issued assets.

Given the extensive use of balance sheet policies in recent years and the possibility that they may be pursued in the future, the objective of this paper is

[1] `www.federalreserve.gov/monetarypolicy/bst_recenttrends.htm`.

[2] See Pandl (2012), Meyer and Bomfim (2012), and Li and Wei (2012) for studies on the cumulative effects of asset purchase programs.

to develop a framework to study the size of the balance sheet as well as other policies designed to affect long-term interest rates.[3] In so doing, we seek to study the transmission channels of conventional monetary policies such as the money growth rate in comparison to "unconventional" balance sheet policies. We are particularly interested in the effects of the various policy tools on investment, capital formation, and the yield curve. In our framework, the fiscal authority issues riskless short and long-term bonds. In the absence of purchases from the monetary authority, financial intermediaries buy the bonds from the fiscal authority on behalf of their depositors. By comparison, the central bank prints money over time and can also acquire bonds.

Central bank actions – whether conventional monetary policies or unconventional balance sheet policies – affect economic activity through the incentives of intermediaries in the banking system. Therefore, one cannot understand the transmission channels of policy in the absence of well-defined intermediaries who perform services that depositors cannot achieve individually. In particular, Diamond and Dybvig (1983) demonstrate that intermediaries perform important risk-pooling functions. Further, Bencivenga and Smith (1991) show that risk-pooling institutions economize on holdings of unproductive liquid assets. Thus, it naturally follows that unconventional policies - with an emphasis on purchases of long-term securities - can be best understood in a framework where intermediaries favor long-term investment opportunities. Finally, in order to study the effects of conventional policies such as money growth, it is also important that a model feature a well-defined transactions role for money as a medium of exchange.

In order to study the effects of conventional versus "unconventional" balance sheet policies for economic outcomes, we consider an overlapping generations version of Diamond and Dybvig (1983) with production as articulated by Bencivenga and Smith (1991). In addition, following Schreft and Smith (1997, 1998), limited communication and restrictions on asset portability lead to a well-defined transactions role for money. In our framework, a fraction of individuals in the economy will be randomly relocated to another geographic location. Due to the friction of limited communication across locations, these individuals must hold fiat money in order to realize consumption. In this manner, the liquidity risk is analogous to a liquidity preference shock as in Diamond and Dybvig. Financial intermediaries exist to provide insurance against such shocks by providing risk pooling services.

[3]This is especially true for the case of the European Central Bank.

There are two significant departures in our framework from the previous literature. First, in order to study asset purchases of both short and long-term bonds, we extend our model to a three period overlapping generations setup. Second, the fiscal authority and the central bank are separate institutions. Only a fraction of central bank net revenues will be redistributed back to the fiscal authority in each period. Consequently, asset purchases by the central bank can have an effect on yields of government debt. In turn, the reduction in yields has the potential to promote income and capital accumulation.

Our framework clearly articulates the following transmission channels of conventional policy versus balance sheet policy.[4] First, money growth affects economic activity by generating seigniorage revenue. It also interrupts risk-sharing. By comparison, a second conventional policy - open market operations targeted towards short-term government bonds - attempts to discourage institutions from providing risk-sharing to depositors. As a result, it promotes capital formation as more resources are devoted to investment in productive assets. Finally, balance sheet policy targeted towards long-term government bonds reflects the desire to lower long-term yields to push institutions away from unproductive government debt.

Interestingly, the framework demonstrates that outright open market operations through purchases of short-term government securities unambiguously lower yields and promote economic activity. By comparison, long-term asset purchases are only effective if short-term yields are sufficiently low. Should central bankers in countries with high debt loads and high short-term interest rates implement a long-term asset purchase program, it will be ineffective as it would only encourage fiscal authorities to issue more debt. In this manner, our work contributes to a number of recent papers that show that the effects of monetary policy are likely to vary across countries. For example, Ghossoub and Reed (2010) develop a framework that demonstrates that the effects of money growth are qualitatively different between rich and poor countries. In particular, monetary policy generates a Tobin effect in advanced countries but reduces economic activity in poor countries. Moreover, Bullard and Keating (1995) observe that money growth positively affects output in low inflation countries. In contrast to previous work studying the effects of conventional policy across countries, we also find that the effects of unconventional policy vary.

[4]Bhattacharya and Singh (2008, 2010) compare the effects of a money growth rule to an interest rate target.

From a methodological perspective, there are two papers that are closely related to our work. First, Schreft and Smith (1997) develop a monetary growth framework with multiple steady-states in which the amount of government debt varies. As a result of the different levels of government debt which crowd out capital formation, the effects of monetary policy vary across countries. In comparison to their work, the fiscal and monetary authorities are "independent" institutions in our model. In particular, the central bank acquires government bonds which are issued by the fiscal authority. In their model, bonds are issued by a hybrid fiscal-monetary authority and only private intermediaries hold government debt. While Schreft and Smith (1997) focus on the crowding-out effects of government debt, purchases of securities by the central bank may relieve the crowding-out problem in our framework.

By comparison, Schreft and Smith (2002) study how the stock of government debt affects the ability of the monetary authority to conduct open market operations. Finally, both Schreft and Smith (1997) and Schreft and Smith (2002) construct two-period OLG models while we consider a three-period framework so that we can study yields of government debt at different maturities.

The remainder of the paper is as follows. As a simple starting point, Section 2 establishes a model of two periods in order to focus exclusively on the size of the balance sheet in comparison to a standard money growth rule. Section 3 extends the model to three periods. Section 4 studies steady-states in the three period framework. Section 5 compares the effects of different monetary policy instruments. Section 6 concludes. The proofs of major results are presented in the Appendix.

2 Benchmark Model of Treasury Purchases

We begin by establishing a benchmark model that will allow us to analyze the effects of a conventional policy tool - changes in the money growth rate. By comparison, we also analyze asset purchases of the central bank. The relatively simple two period structure highlights the distinction between activities of the fiscal authority from the central bank (CB). Notably, the fiscal authority and the central bank are separate institutions. Only a fraction of central bank net revenues will be redistributed back to the fiscal authority in each period. It also allows us to focus on the implications of large-scale open market operations in a setting where all bonds have the same maturity. In comparison to Schreft and

Smith (2002), we consider that the monetary authority retains a proportion of its revenues.

Moreover, we allow for the CB to use two different policy tools. As in Schreft and Smith (2002), the CB can purchase government securities. In addition, in our framework, the monetary authority prints money over time following a fixed money growth rule. In this manner, there are two different ways in which the monetary authority influences the ability of private sector intermediaries to achieve risk-sharing.

Finally, the transmission of policy actions of the CB in Schreft and Smith (2002) are limited as they only study an endowment economy. Instead, we investigate the impact of CB activities in a production economy with capital accumulation.

2.1 The Environment

Time in the economy is discrete, indexed by $t = 1, 2, ...\infty$. Two geographically-separate locations coexist. Each is populated by a unit-mass continuum of two-period lived agents that exert labor effort when "young" and value consumption of a homogeneous good when "old". Given an initial population of old, the economy is populated by an infinite stream of individuals within each location such that an overlapping generations structure prevails. Agents' preferences are $u(c_t) = \log c_t$ and they are endowed with one unit of labor when young. In every period, each young agent has a probability $\pi \in (0, 1)$ of moving to the other location.

The banking sector is represented by a market for deposits in which private banks accept agents' labor income in exchange for a return at a future date. Moreover, the market for deposits is perfectly competitive. Consequently, each intermediary will offer rates of return to maximize depositors' expected utility.

Following Schreft & Smith (1997), there is limited communication between banks at different locations. The resulting anonymity friction eliminates the possibility of trading privately issued liabilities. Only fiat money can be transported between locations. Thus, movers must liquidate their savings and carry money balances to their new site despite that money is dominated in rate of return by all other assets in the economy.

Due to the potential for geographic displacement, individuals are subject to relocation risk. Since relocation shocks force individuals to liquidate their asset

holdings early, relocation shocks are analogous to liquidity preference shocks in Diamond and Dybvig (1983). Intermediaries provide insurance against such risk by pooling income and offering returns on deposits contingent on relocation status. As the banking sector is perfectly competitive, institutions will offer a level of risk-sharing that maximizes depositors' expected utility.

There are separate fiscal and monetary authorities. The fiscal agent generates its stream of revenue from issuing risk-free bonds B_t in financial markets as well as from supplementary transfers by the monetary authority. Letting the real value of bonds be $b_t \equiv B_t/P_t$ where P_t is the unit price of the consumption good, a holder of a unit of government debt at $t-1$ has a sure claim to R_t units of consumption in period t. Hence, debt payouts constitute the liability side of the fiscal authority.

The CB implements policy based on two instruments. First, it controls the supply of money in the economy according to the rule $M_t = \sigma M_{t-1}$ where M_t denotes the period t nominal money supply per agent and $\sigma > 1$ the gross rate of money growth between adjacent periods. Real money balances, $m_t \equiv M_t/P_t$, offer a rate of return that depends on the economy's relative price level $\frac{P_{t-1}}{P_t}$.

The second instrument derives from the CB's ability to engage in an active bond purchase program. Consequently, total demand for government debt in the economy will be split between the CB and private banks. A constant fraction $\lambda \in [0,1]$ of net revenue that is generated by the CB will be transferred to the fiscal entity in every period. We consider this value as an "institutional" parameter which reflects the degree of resource-independence of the CB.

2.2 Factor Markets

Labor (L_t) and capital (K_t) are the two factors of production in the economy. Labor effort is exclusively provided by young agents. Capital earns a rate of return of r_t and completely depreciates at the end of each period. As in the neoclassical growth model, output of a homogeneous consumption good is realized through a constant returns to scale production technology $F(K_t, L_t)$. In standard capital-intensive form it is $f(k_t)$, where $k_t \equiv K_t/L_t$, $f'(k_t) > 0$ and $f''(k_t) < 0$.

Due to perfect competition in each factor market, each input earns their marginal product:

$$w_t = w(k_t) = f(k_t) - k_t f'(k_t) \tag{1}$$

$$r_t = f'(k_t), \tag{2}$$

and we assume $w(0) > 0$.

2.3 Timing of Actions

The timing of events that take place within an agent's two-period life span is illustrated in Figure 1. Young individuals provide labor effort and production takes place. Afterwards, factors of production get paid according to (1) and (2) from which young workers deposit all of their labor income in private banks. Because the fraction of young individuals that are going to relocate is publicly known, banks determine the optimal allocation of deposits into currency and other assets during this phase. At the end of the initial period when the relocation shock π is observed, every young depositor who needs to move withdraws their deposits. However, due to limited communication and restrictions on asset portability, movers will only demand cash. At the end of this first period relocation occurs. When period $t+1$ arrives, old nonmovers withdraw deposits whereas their moving counterparts exchange cash for the consumption good with banks in their new location. Finally, all old agents consume and exit the economy at the end of the period.

2.4 Financial Intermediation

All banks are identical so we focus on a single representative bank that offers returns on deposits. A bank can participate in the market for deposits by offering rates of return to its clients as dictated by a perfectly competitive financial sector. Together with the assumption of zero transaction costs, the profit maximization program for this institution translates into expected utility maximization on behalf of its depositors.

Consequently, banks compete for deposits w_t by announcing a rate of return r_t^m for the π movers and r_t^n for the remaining $(1-\pi)$ non-movers. Therefore, the consumption available to each agent is:

$$c_t^m = w_t r_t^m \tag{3}$$

if a mover, and

$$c_t^n = w_t r_t^n, \tag{4}$$

for a nonmover.

Banks allocate deposits into the three different assets in the economy: real money balances in the amount m_t, capital per depositor i_t and government issued bonds b_t^P such that:

$$m_t + i_t + b_t^P \leq w_t. \tag{5}$$

Moreover, banks face additional constraints in order to guarantee that the actual returns are in line with those offered to the moving and nonmoving populations. That is, for those who relocate, it must be that

$$\pi w_t r_t^m \leq m_t \frac{p_t}{p_{t+1}} \tag{6}$$

and

$$(1-\pi) w_t r_t^n \leq r_t i_t + R_t b_t^P \tag{7}$$

otherwise. Under this scenario, the representative bank's optimization program is

$$\max_{r_t^m, r_t^n} \{\pi \ln [w_t r_t^m] + (1-\pi) \ln [w_t r_t^n]\}$$

subject to (5) - (7).

The solution to this maximization problem yields the following first order conditions:

$$R_t - r_t = 0. \tag{8}$$

and

$$m_t / w_t - \pi = 0. \tag{9}$$

Equation (8) is a no-arbitrage condition that makes bank's investment allocations indifferent between bonds and capital. The efficient risk-sharing condition in (9) allows banks to insure agents against liquidity risk by allocating a relative amount of cash balances equal to the fraction of moving individuals. In this manner, banks contribute to the agent's welfare by risk-pooling.

2.5 The Central Bank

Monetary policy is the sole responsibility of the monetary authority. As previously mentioned, it has two types of instruments: the growth rate of money as its conventional policy tool and the acquisition of government debt obligations. In any given period, the CB follows a constant money growth rule so that inflation generates seigniorage revenue equal to $\left(\frac{\sigma-1}{\sigma}\right) m_t$. The CB also participates in financial markets by purchasing government bonds in the amount b_t^{CB}. In addition, the CB transfers a fraction λ of its net revenue to the fiscal authority in every period. Through this resource injection, the CB can have an indirect influence on the impact of fiscal debt in the economy. Recalling R_t as the return on government securities, the CB's resource constraint is

$$R_{t-1} b_{t-1}^{CB} + \left(\frac{\sigma - 1}{\sigma}\right) m_t - b_t^{CB} \geq 0. \tag{10}$$

The first two terms in (10) represent the revenues from the return to bonds and seigniorage revenue. The third term depicts expenses incurred in the purchase of government debt during the same time period. As noted in (10), we do not allow the central bank to impose net losses upon the fiscal authority.

2.6 The Fiscal Authority

The fiscal authority in this economy issues riskless government bonds b_t in financial markets. As pointed out above, demand for these assets comes from private banks and from the CB so that the bond market clearing condition becomes:

$$b_t = b_t^P + b_t^{CB}, \quad \text{for all} \quad t > 0. \tag{11}$$

The government obtains its income from the sale of debt obligations as well as from CB transfers. Specifically, the latter represents an injection of a fraction (λ) of the CB's net receipts as depicted in (10). Similarly, the expense side of the fiscal authority's budget constraint consists of payments disbursed to honor its debt obligations at maturity. In equilibrium, government revenue equals expenses so that:

$$R_{t-1} b_{t-1} = b_t + \lambda[R_{t-1} b_{t-1}^{CB} + \left(\frac{\sigma - 1}{\sigma}\right) m_t - b_t^{CB}], \quad \text{for all} \quad t > 0. \tag{12}$$

2.7 Steady State Equilibrium

We now study economic activity in steady-state. First, currency as well as bond markets clear by equations (9) and (11), respectively. In addition, we also have $i_t = k_{t+1}$. Consequently, using the private bank's balance sheet it must be that

$$k_{t+1} = w_t - m_t - b_t^P \quad \text{for} \quad t \geq 0, \tag{13}$$

which highlights that in private bank's portfolios, investment in capital declines as demand for liquidity or government debt increases. Next, using the fiscal authority's budgetary constraint (12) along with (9) and (11), we can solve for the amount of government bonds purchased by private banks, b_t^P which yields

$$b_t^P - (1-\lambda) b_t^{CB} = \lambda \left(\frac{\sigma - 1}{I_t - \sigma} \right) m_t, \tag{14}$$

where $I_t \equiv \frac{P_{t+1}}{P_t} R_t$.

A closer look at the results allows us to see two important aspects related to fiscal debt. First, the CB can absorb government debt that otherwise would be acquired by private banks. This implies that the monetary authority has a tool that can act as a buffer for government debt distortions. Notably, the amount of bonds available for the private sector to hold depends on the degree of resource-independence of the monetary authority. For example, if $\lambda = 1$, then effectively all of the bonds issued by fiscal authority would be held by the private sector. In this case, though the central bank may hold bonds, it redistributes everything back to the fiscal authority. So, in net terms, the fiscal authority would not have any debt obligations to the CB. Moreover, all of the debt would be financed by seigniorage. As the CB retains more of its revenues (λ smaller), there would be less bonds available for intermediaries to purchase.

Second, the relationship between seigniorage and bond holdings also depends on the degree of independence. The relationship between seigniorage and bonds held by the private sector is increasing in λ. That is, the extent of inflation-financing of debt is higher if the CB transfers more of its revenues to the fiscal authority.

Next, (14) into (13) yields:

$$\Omega(k) = \frac{(1-\pi)(I_1 - \sigma) - \lambda\pi(\sigma - 1)}{I - \sigma} + (1-\lambda)\frac{b^{CB}}{w(k)}, \tag{15}$$

where $\Omega(k) \equiv k/w(k)$ and we assume $w_0 \equiv w(0) > 0$, so that $\Omega'(k) > 0$ for $k \geq 0$.

If $\lambda = 1$, the government budget constraint would be the same as Schreft and Smith (1997).

This analysis also differs from their study in another important aspect. Focusing on the share of capital to deposits, CB asset purchases have two effects. On one hand, they contribute to capital formation by reducing the crowding out effect. On the other, as economic activity increases, intermediaries now have a higher level of deposits. Hence, the effects of this policy on the share of capital in private banks' balance sheets must be controlled for the size of the deposit base. Thus, the behavior of (15) in relation to any policy that stimulates capital formation must take into account this additional effect.

Equation (15) shows the steady state level of capital in the economy depending explicitly on the rate of money growth and bond purchases by the CB. Additionally, the stock of capital is negatively affected by the nominal return on government debt. As increasing debt levels imply higher borrowing costs for the fiscal authority, a potential pathway for a crowding out effect remains.

We also have a no-arbitrage condition as described by (8) in steady-state. Money market equilibrium along with the money growth rule implies $\sigma = P_{t+1}/P_t$. From the capital market condition, (2), and (8), the no-arbitrage condition is:

$$I = \sigma f'(k) \tag{16}$$

Together, (15) and (16) capture the complete steady-state equilibrium behavior of the economy.

The following proposition provides the conditions for multiple steady states to exist:

Proposition 1. (*Existence of Multiple Steady-States*). *Suppose that the condition in (10) holds. In addition, let* $\hat{k}$ *be such that* $\Omega(\hat{k}) = 1 - \pi(1-\lambda) + (1-\lambda)b^{CB}/w(\hat{k})$ *and* $\hat{\sigma} = 1/f(\hat{k})$. *Further, suppose that* $\sigma > \hat{\sigma}$. *Under these conditions, multiply steady-states exist in which the financial position of the fiscal authority varies. In the low capital steady-state, the fiscal authority is a net debtor. In the other steady-state, the fiscal authority transfers income to the private sector.*

Figure 2 illustrates the existence of multiple steady state equilibria in the model. Interestingly, each steady-state has distinct features. In equilibrium A

for instance, a highly distorted economy prevails as characterized by a low level of capital and high nominal interest rates. As explained in Schreft & Smith (1997), this steady state resembles a developing economy with a high amount of outstanding government debt. Particularly, looking at (14) in steady state and because $I > \sigma$ in this regime, there is a positive amount of bonds outstanding that makes the government hold a net-debtor position in financial markets.

In contrast, the economy represented by the equilibrium shown in B exists in a more efficient financial sector with a low interest rate and high capital stock. If the prevailing interest rate I has an upper bound $I < \sigma$, the fiscal authority holds a net lending position as in Schreft and Smith (1997).

2.8 Comparative Statics

2.8.1 Money Growth Rule

We begin our analysis by illustrating how the effects of a money growth rule policy transmit to the real sector. Proposition 2 summarizes the implications of a higher rate of money growth for activity across steady-states:

Proposition 2.

i. In economies where the government is a net debtor, $\frac{dk}{d\sigma} < 0$, $\frac{dR}{d\sigma} > 0$, *and* $\frac{dI}{d\sigma} > 0$.

ii. In economies where the government is a net creditor, $\frac{dk}{d\sigma} > 0$, $\frac{dR}{d\sigma} < 0$, *and* $\frac{dI}{d\sigma} > 0$.

Figure 3 shows the two differing outcomes depending on the steady-state. In the economy where the government is a net debtor, an increase in the rate of money growth enhances the government's ability to issue debt. However, due to (11), the bond market clears so that an increased bond supply gives way to higher nominal yields which ultimately crowd out capital accumulation. In contrast to Schreft and Smith (1997), the increase in nominal interest rates and the reduction in the capital stock will be higher if the CB is less "independent." That is, the crowding-out effect will be larger if the CB transfers a greater proportion of its revenues to the fiscal authority.

In the economy where the government is a net lender, a low-interest rate steady state regime, an increase in σ contributes to capital formation. Since

nominal interest rates are sufficiently low, as the government is subsidizing economic activity through its net lending status, an increase in the rate of money growth erodes the real return on bonds which promotes the ability of intermediaries to invest in capital. Again, the effects are stronger as λ is larger.

2.8.2 Treasury Purchase Program

When the CB engages in a bond purchase program, there are nontrivial effects on capital formation. Proposition 3 summarizes the implications of an increase in bond purchases:

Proposition 3. *Regardless of the fiscal position of the government,* $\frac{dk}{db^{CB}} > 0$, $\frac{dR}{db^{CB}} < 0$, *and* $\frac{dI}{db^{CB}} < 0$.

Unlike a money growth instrument, purchases by the CB unambiguously increase capital formation in both steady states. In the low capital economy, the policy action reduces the supply of government debt available to private investors. Therefore, it alleviates the crowding out problem of inflation-financed government debt. This allows a lower rate of marginal productivity to drive down nominal interest rates throughout the economy in the absence of arbitrage opportunities.

Within the high capital economy where the cost of borrowing is lower than the inflation rate, an expansion in the balance sheet acts as an increase in loans to the fiscal authority at a low interest rate. The "low cost" borrowing translates into additional resources available for private investment. In turn, the actions of the CB promote capital formation. Figure 4 illustrates the findings. In comparison to the effects of money growth, the effects of open market operations are stronger when there is a greater degree of independence of the monetary authority from the fiscal authority.

Notably, we find that conventional money growth policy has distinct implications for risk-sharing relative to unconventional policy. First, an increase in the rate of money growth lowers the return on money which distorts the level of risk-sharing in the economy regardless of the steady-state. However, as the real return to government debt is lower in response to higher money growth in the advanced steady-state, the implications for risk-sharing are less severe in advanced economies. However, unconventional balance sheet policy promotes welfare without the negative implications for the return to money. From this perspective,

open market operations may be a preferable tool relative to money growth since it promotes activity without disturbing risk-sharing in the financial system.

3 A 3-Period Model

Having shown the reach of the benchmark model, we now proceed to enrich its structure with the aim of analyzing unconventional monetary policy measures. As these actions constitute an expansion of the CB's balance sheet through the purchase of longer-term government securities, we allow for the existence of such assets by including an additional period in the life cycle of agents. Specifically, we use a three-period overlapping generations structure that allows the fiscal authority to issue short-term (one-period) and long-term (two-period) bonds.

3.1 The Environment

The unit-mass continuum of individuals experience three stages. Now, after a young stage, an intermediate "middle-age" follows which then leads to their last existing period or old phase. There is an initial population of each type for which only the old agents hold M_0 units of currency. Labor effort is still provided when young and consumption valued when old. In this three-period setup, relocation shocks are experienced by middle-aged depositors.

While the fiscal authority continues to issue one-period or short-term real bonds which we now denote as $b_{1,t}$ with rate of return $R_{1,t}$, it also sells long-term, two-period maturity bonds in amount $B_{2,t}$ with real value is $b_{2,t} \equiv B_{2,t}/P_t$. A holder of a unit of this security at period t is entitled to $R_{2,t+2}$ units of the consumption good in period $t+2$.

Therefore, the CB has a third instrument that belongs to its set of balance sheet tools, namely, purchases of long-term government securities in amount b_2^{CB}. Together with outright open market operations (described now by b_1^{CB}) and control of the money growth rate, the CB implements monetary policy.

3.2 Factor Markets

We preserve the production technology studied in the benchmark model. However, investment in capital takes two periods to materialize. This implies that returns to inputs follow:

$$r_t(k_t) = f'(k_t) \tag{17}$$

and

$$w_t \equiv w_t(k_t) = f(k_t) - k_t f'(k_t). \tag{18}$$

3.3 Timing of Actions

Following the timeline described in Figure 5, the supply of labor, return to factors, bank deposits and their corresponding investment allocations take place in an individual's first period. As the middle aged period arrives, agents learn their relocation status. Then movers liquidate their savings from banks and exchange them for cash with the previous generation's old movers and relocation takes place. This action marks a difference relative to our benchmark in that the bank does not explicitly hold cash balances. Finally, when old, agents realize consumption based on returns to assets and exit the economy.

3.4 Financial Intermediation

Private banks create portfolios of capital i_t and bonds (both short-term and long-term maturities $b^P_{1,t}$ and $b^P_{2,t}$, respectively). In turn, the nominal return to government debt is $I_{1,t} \equiv R_{1,t}(P_{t+1}/P_t)$ if short-term and $I_{2,t} \equiv R_{2,t}(P_{t+2}/P_t)$, otherwise.

The distribution of asset holdings among depositors depends upon their liquidity preference. Notably, in contrast to standard Schreft-Smith (1997, 1998) models there is a short-term, interest-bearing asset that intermediaries can acquire in order to meet demands among those withdraw early. The amount acquired on behalf of movers is equal to $b^m_{1,t}$. However, there is still an information friction that such individuals will eventually need to confront. That is, upon receiving their payments from bond holdings in the short-term, middle-aged movers will buy cash from old individuals who are carrying cash from the other location. Thus, in contrast to Schreft-Smith (1997, 1998), money is primarily a medium of exchange since there is an alternative short-term interest bearing asset available.

The returns of nonmovers come from both short and long-term bonds ($b^n_{1,t}$ and $b^n_{2,t}$) along with capital i_t. In this manner, a representative bank offers a rate of return on deposits of $r_{m,t}$ if relocating and $r_{n,t}$ otherwise. Therefore, consumption for movers is:

$$c_t^m = r_t^m w_t \tag{19}$$

and

$$c_t^n = r_t^n w_t \tag{20}$$

for nonmovers.

The private bank's balance sheet constraint is:

$$w_t = b_{1,t}^m + b_{1,t}^n + b_{2,t}^n + i_t. \tag{21}$$

Similarly, payments to movers are constrained by:

$$\pi w_t r_{m,t} = \left(R_{1,t+1} b_{1,t}^m\right) \frac{P_t}{P_{t+1}}, \tag{22}$$

whereas for the nonmoving depositors:

$$(1-\pi) w_t r_t^n = \left(R_{1,t+1} b_{1,t}^n\right) R_{1,t+2} + R_{2,t+2} b_{2,t}^n + r_{t+2} i_t, \tag{23}$$

Based on these conditions along with (19) and (20), banks offer deposit rates that solve

$$\max_{r_t^m, r_t^n} \left\{\pi \ln\left[w_t r_t^m\right] + (1-\pi) \ln\left[w_t r_t^n\right]\right\} \tag{24}$$

subject to (19) - (23). In turn, the solution to the bank's problem yields the following first order conditions:

$$R_{2,t+2} - r_{t+2} = 0 \tag{25}$$

$$R_{1,t+1} R_{1,t+2} - r_{t+2} = 0 \tag{26}$$

where (25) establishes that optimal bank allocations in long term bonds and capital eliminate arbitrage opportunities. Likewise, (26) shows that compounded short term bond returns must be the same as capital. Combining these two conditions provides a no-arbitrage condition among bonds with different maturities:

$$R_{1,t+1} R_{1,t+2} = R_{2,t+2}. \tag{27}$$

Equation (27) represents the basis for a term structure of interest rates that underlies the yield curve in the economy.

By solving (24), the private bank's optimal allocation of short term bonds for movers is:

$$b_t^m = \pi w_t, \tag{28}$$

which yields both the private bank's short-term bond demand and the optimal risk sharing rule. Real returns from debt are used to purchase real money balances from the current old movers:

$$m_{t+1} = R_{1,t+1} b_{1,t}^m. \tag{29}$$

3.5 The Central Bank

The CB controls the rate of money growth which allows for seigniorage revenue as well as the implementation of open market operations. In this economy however, the monetary authority is able to access a third policy instrument. In particular, it can also acquire long-term bonds equal to $b_{2,t}^{CB}$. Therefore, the CB's resource constraint is:

$$R_{1,t-1} b_{1,t-1}^{CB} + R_{2,t-2} b_{2,t-2}^{CB} + \left(\frac{\sigma - 1}{\sigma}\right) m_t - b_{1,t}^{CB} - b_{2,t}^{CB} \geq 0. \tag{30}$$

3.6 The Fiscal Authority

Fiscal debt obligations can be redeemed within one or two periods depending on their maturity structure. The supply of short-term bonds issued by the fiscal authority is $b_{1,t}$. Short-term bond demand on behalf of movers and non-movers is $b_{1,t}^m + b_{1,t}^n$. In addition, holdings by central bank are equal to $b_{1,t}^{CB}$. Thus, the clearing condition for short-term bonds implies:

$$b_{1,t} = b_{1,t}^m + b_{1,t}^n + b_{1,t}^{CB} \tag{31}$$

On the other hand, the demand for long-term bonds only comes from two sources: the demand by intermediaries on behalf of the non-movers ($b_{2,t}^n$) and the demand by the central bank ($b_{2,t}^{CB}$):

$$b_{2,t} = b_{2,t}^n + b_{2,t}^{CB}. \tag{32}$$

At any time period, the government budget is balanced so that

$$b_{1,t} + b_{2,t} + \lambda \left[R_{1,t-1} b^{CB}_{1,t-1} + R_{2,t-2} b^{CB}_{2,t-2} + \left(\frac{\sigma - 1}{\sigma} \right) m_t - b^{CB}_{1,t} - b^{CB}_{2,t} \right] = R_{1,t-1} b_{1,t-1} + R_{2,t-2} b_{2,t-2}. \tag{33}$$

3.7 Steady-State Analysis

From the bank's demand for short-term bonds (28) and since $i_t = k_{t+2}$ we rewrite its balance sheet constraint (21) in the steady-state as:

$$k = w(1 - \pi) - b^n_1 - b^n_2. \tag{34}$$

Again, there is potential displacement of capital in bank's portfolios from government debt. However, bond market equilibrium conditions (31) and (32) show that the CB can affect the magnitude of such displacement.

To see how acquisitions by the monetary authority impact private bond demand, we focus on the fiscal authority's constraint (33). The no-arbitrage condition on real interest rates (27) along with $\sigma = P_{t+1}/P_t$ in the steady state, yields the no arbitrage condition between nominal interest rates:

$$(I_1)^2 = I_2. \tag{35}$$

It follows that equation (33) can be arranged as:

$$b^n_1 + b^m_1 + \left(\frac{I_1 + \sigma}{\sigma} \right) b^n_2 + (1 - \lambda) \left[b^{CB}_1 + \left(\frac{I_1 + \sigma}{\sigma} \right) b^{CB}_2 \right] = \lambda \left(\frac{\sigma - 1}{I_1 - \sigma} \right) m, \tag{36}$$

so that the total amount of government obligations is equal to current revenues.

In this manner, the LHS of (36) shows government outstanding liabilities - net of CB transfers - in the form of short and long-term bonds. These securities are held by private banks and the monetary authority. As denoted by the RHS of this constraint, bonds are financed via seigniorage revenue transfers relative to the net interest obligations of the fiscal authority. It is evident that the degree of redistribution of income from the central bank (implied by λ) and central bank bond purchases play a significant role in the fiscal authority's actions. In what follows, we analyze the interaction between the fiscal and the monetary authority.

First, following the risk-sharing rule from (28), private banks allocate movers' deposits into government debt $b^m_1 = \pi w$. Upon receiving returns from their bond

holdings, movers acquire money balances in the amount $m = R_1(\pi w)$. Also, using (NA locus) we can substitute these values into (36) and rearrange it so that:

$$b_1^n + \left(\frac{I_1+\sigma}{\sigma}\right) b_2^n + (1-\lambda)\left[b_1^M + \left(\frac{I_1+\sigma}{\sigma}\right) b_2^{CB}\right] = \pi w\left[\lambda\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{I_1}{I_1-\sigma}\right) - 1\right]. \tag{37}$$

Now, after taking into account the risk-pooling actions of intermediaries, government expenses involve debt liabilities to private banks as well as to the monetary authority. However, the term on the right hand side now represents net seigniorage revenue.

We begin by considering economies in which $I_1 < \sigma$. In this scenario, the RHS of (37) is negative which implies that the fiscal authority – on net – transfers income to the private sector. This is possible because the fiscal authority earns more income from seigniorage than it pays for its bonds. Should the monetary authority engage in a bond purchase program, this provides the fiscal agent with more resources to redistribute to the private sector.

In Schreft and Smith (1997), the government is a net debtor if $I_1 > \sigma$. However, they do not allow for the government to issue bonds of different maturities as in our model. The additional complications arising from the maturity structure imply that stronger restrictions on short-term interest rates are required in order for the fiscal authority to persistently run as a net debtor. Interestingly, $\sigma < I_1 < \frac{\sigma}{1-\lambda\left(\frac{\sigma-1}{\sigma}\right)}$ must be satisfied.

The second inequality implies that the net revenue raised through payments by movers, $\lambda\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{I_1}{I_1-\sigma}\right) - 1$, must be positive, allowing the government to run a deficit. Notably, $(I_1 - \sigma)$ represents the net interest obligations of the fiscal authority from issuing bonds that will eventually be held by movers. By comparison, $\lambda\left(\frac{\sigma-1}{\sigma}\right) I_1$, is the seigniorage tax rate generated by issuing new currency to short-term bond holders. In other words, net income generated from the movers is positive as long as $\frac{\lambda\left(\frac{\sigma-1}{\sigma}\right)I_1}{I_1-\sigma} > 1$. Alternatively, the seigniorage revenue exceeds the short-term payments to movers as long as $I_1 < \frac{\sigma}{1-\lambda\left(\frac{\sigma-1}{\sigma}\right)}$. The lower bound, $I_1 > \sigma$, implies that the fiscal authority is a net debtor among the remaining bond holders.

The following corollaries shed insight into the effects of bond purchases on the fiscal authority's budget constraint:

Corollary 1. *Suppose $\lambda = 1$. Under this condition, the CB transfers all of its net revenues to the fiscal authority:*

$$b_n^{1P} + \left(\frac{I_1 + \sigma}{\sigma}\right) b_2^n = \pi w \left[\left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{I_1}{I_1 - \sigma}\right) - 1\right].$$

As a result, all public debt is absorbed by private banks. Central bank purchases do not have any net impact on the resources of the fiscal authority.

Corollary 2. *Suppose $\lambda = 0$. In this case, the fiscal authority does not have any income:*

$$b_1^n + \pi w + \left(\frac{I_1 + \sigma}{\sigma}\right) b_2^n + b_1^{CB} + \left(\frac{I_1 + \sigma}{\sigma}\right) b_2^{CB} = 0.$$

However, it is still possible that the fiscal authority issues some bonds. Zero net debt obligations implies that borrowing by the fiscal authority is offset by loans (i.e., negative bond holdings).

Proposition 4. **(***Existence of Multiple Steady States) Suppose that (30) holds. In addition, let $\hat{k}$ be such that $\Omega(\hat{k}) = (1-\lambda)b_1^{CB}/w(\hat{k}) + [\sigma + \pi\lambda + (\sigma - \lambda(\sigma+1))b_2^{CB}/w(\hat{k}) + b_2/w(\hat{k})]\sigma^{-1}$ and $\hat{\sigma} \equiv 1/f'(\hat{k})$. Further, suppose that $\sigma > \hat{\sigma}$. Under these conditions, multiply steady-states exist in which the financial position of the fiscal authority varies. In the low capital steady-state, the fiscal authority is a net debtor. In the other steady-state, the fiscal authority transfers income to the private sector.*

We now analyze the steady-state equilibrium in the economy. Using (35) and (34) we derive the evolution of the steady state capital stock depending on the actions of the monetary authority:

$$\begin{aligned}\Omega(k) = 1 - \pi\lambda\left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{I_1}{I_1 - \sigma}\right) + (1-\lambda)\frac{b_1^{CB}}{w(k)} + \\ \left[1 - \lambda\left(\frac{I_1 + \sigma}{\sigma}\right)\right]\frac{b_2^{CB}}{w(k)} + \left(\frac{I_1}{\sigma}\right)\frac{b_2}{w(k)}.\end{aligned} \tag{38}$$

We find it useful to compare (38) to the same equation in Schreft and Smith (1997). Notably, the analogue to Schreft and Smith (1997) in our framework would summarize the capital-deposit ratio ($\Omega(k)$) by $\Omega(k) = 1 - \pi\lambda\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{I_1}{I_1-\sigma}\right)$. If $I_1 < \sigma$, the government would be able to inject additional resources to the

banking system so that investment in capital would exceed deposits. In our framework, there are *three* additional mechanisms which would encourage the same outcome. First, the central bank may inject resources to the economy in the form of one-period loans, b_1^{CB}. It may also inject resources through long-term purchases, b_2^{CB}. Finally, the fiscal authority has the ability to borrow against the future and settle its obligations in the next period instead of today.

We next provide some cursory evidence that short-term and long-term bond purchases have different effects on economic activity. Notably, the impact of long-term bond purchases hinges on the level of interest rates economy through the term $\left[1-\lambda\left(\frac{I_1+\sigma}{\sigma}\right)\right]$. However, the coefficient on short-term purchases is independent of interest rates.

A no-arbitrage condition between short-term bonds and long-term returns in the capital market is obtained through (17), (25), as well as the term structure depicted in (27):

$$I_1 = \sigma\sqrt{f'(k)}. \tag{39}$$

In this manner, we analyze the steady state equilibrium behavior of the economy in (I_1, k) space through (38) and (39).

Figure 3 shows the steady state equilibrium diagram of the economy. The existence of multiple equilibria is illustrated by points A and B. We next proceed with monetary policy analysis.

3.8 Monetary Policy Instruments: Comparative Statics

3.8.1 Money growth rule

We start by considering the effects of money growth for economic activity. Proposition 2 summarizes the impact of the change in the (conventional) policy:

Proposition 5.

i. In economies where the government is a net debtor, $\frac{dk}{d\sigma} < 0$, $\frac{dR_1}{d\sigma} > 0, \frac{dI_1}{d\sigma} > 0$, *and* $\frac{dI_2}{d\sigma} > 0$.

ii. In economies where the government is a net creditor, $\frac{dk}{d\sigma} > 0$, $\frac{dR_1}{d\sigma} < 0$, *and* $\frac{dI_1}{d\sigma} > 0$, *and* $\frac{dI_2}{d\sigma} > 0$.

Figure 4 illustrates the impact of an increase in σ. The capital stock in the developing economy is lower under higher money growth as seen by comparing

equilibrium A relative to A'. Because the government is a net borrower, an increase in σ creates additional seigniorage income transfers. In turn, the fiscal authority can issue more debt. Consequently, the crowding out effect on the capital stock is more severe.

In the case of the high capital economy at initial steady state B, an increase in the supply of money promotes capital accumulation as denoted in B'. The increase in seigniorage promotes the ability of the fiscal authority to transfer income to the private sector.

Regardless of the stage of development, higher money growth leads to higher short-term nominal interest rates. By no-arbitrage across the maturity structure, long-term nominal rates will also increase. Since long-term rates are equal to the compounded short-term rate, an increase in the money growth makes the "yield curve" steeper in nominal terms. However, in the advanced economy, real interest rates are lower. From this perspective, the yield curve in real terms would become flatter.

Consequently, if the monetary authority in a poor country with a large debt service load is concerned about the cost of borrowing by the fiscal authority, an increase in the rate of money growth (a standard monetary policy tool) would be inappropriate. However, an increase in the rate of money growth would be effective in promoting investment activity in advanced countries.

3.8.2 Balance sheet policy: short-term bonds

We now turn to the next conventional monetary policy - outright open market operations. Proposition 3 summarizes the effects:

Proposition 6. *Regardless of the fiscal position of the government,* $\frac{dk}{db_1^{CB}} > 0$, $\frac{dR_1}{db_1^{CB}} < 0$, $\frac{dI_1}{db_1^{CB}} < 0$, *and* $\frac{dI_2}{db_1^{CB}} < 0$.

Figure 8 depicts the steady state response of the capital stock and interest rates to this policy. It is clear that for both economies, open market purchases of short-term securities unambiguously increase capital formation and lower interest rates. By comparison to the effects of money growth, the yield curve will be flatter under the change in policy.

For the low capital economy, CB purchases of short term debt mitigate crowding out by absorbing government debt from private bank's portfolios. With the

additional source of funding to the fiscal authority, nominal short-term interest rates do not need to be as high in order for bond markets to clear. In turn, as investment in the capital stock increases, there is a lower return to capital. This is a second-round effect of an open market operation – lower real returns lead to lower long-term interest rates due to no-arbitrage between long-term bonds and capital investment.

In the developed economy, the government transfers income to the private sector. In this case, short-term government bond purchases transfer even more funds to the fiscal authority. The low-cost resources find their way to the private sector stimulating productive investment. Then, returns to capital decrease and long-term yields decline.

Therefore, regardless of the stage of development, conventional balance sheet policy eases interest rates and promotes capital formation. In addition, the yield curve would not be as steep since long-term rates are equal to compounded short-term interest rates.

3.8.3 Balance sheet policy: long-term bonds and the yield curve

We next analyze the implications of purchases of long-term bonds. As we demonstrate below, long-term asset purchases do not necessarily improve investment and capital accumulation. That is, resources injected to the fiscal entity through long-term purchases may exacerbate fiscal distortions in the economy. For example, we start by looking at the marginal effect of b_2^{CB} on steady state capital from (38). Recalling that $I_1 = \sigma R_1$ we can rewrite this effect as:

$$1 - \lambda(R_1 + 1). \tag{40}$$

Notably, an increase in short-term purchases would be expected to promote capital formation by alleviating the distortions from the crowding-out of government debt *as long as real short-term interest rates are sufficiently low.* In terms of nominal returns:

$$1 - \lambda(\frac{I_1 + \sigma}{\sigma}). \tag{41}$$

As long as the coefficient is positive, long-term purchases can promote economic activity. This holds as long as:

$$I_1 < I_1^* \equiv \sigma \left(\frac{1 - \lambda}{\lambda} \right). \tag{42}$$

A proposition follows based on this result.

Proposition 7. *Whether the government is a net lender or net creditor,* $\frac{dk}{db_2^{CB}} > 0$, $\frac{dR_1}{db_2^{CB}} < 0$, *and* $\frac{dI_1}{db_2^{CB}} < 0$ *as long as short term interest rates are such that* $I_1 < I_1^*$.

This condition posits that long-term asset purchases are only effective in a low short-term interest rate environment. The rationale is straightforward. Asset purchases of short-term government bonds reduce the obligations to the private sector, depending on the proportion of income retained by central bank. By comparison, interest expenses for long-term bonds are *compounded* in the government budget constraint. The higher the short-term interest rate, the greater the extent of long-term interest obligations. Therefore, a long-term asset purchase program would be less effective if interest rates are high and asset purchases would not have a substantial impact on the debt obligations of the fiscal authority.

Given that the effects of long-term asset purchases are dependent on the level of short-term yields, it is unlikely that the effects of such a policy would be invariant across countries. That is, as shown in Figure 9, the effects of long-term asset purchases are likely to depend on the stage of economic development. In a poor economy with high nominal interest rates, the supportive role of unconventional policy portrayed in Proposition 4 does not apply.

The government budget constraint involves two sources of expenditures: payments for one-period (short-term) bonds or two-period bonds. Because of compounding, long-term bonds can be a significant drain on the obligations of the fiscal authority. As a result, long-term asset purchases of the central bank simply withdraw the less significant obligations from circulation. In this manner, long-term asset purchases aggravate the budget constraint of the fiscal authority so that the fiscal authority merely substitutes long-term debt with even greater short-term debt. The increase in short-term debt financing exacerbates the crowding-out problem.

By comparison, in an advanced country with sufficiently low short-term interest rates, the fiscal authority provides net transfers to the private sector. Should the monetary authority increase its long-term asset purchases, it is effectively lending more income to the fiscal authority. The increase in lending promotes the ability of the fiscal authority to transfer resources if short-term interest rates are low. Moreover, if short-term interest rates are low, then long-term interest

rates are also low. In this manner, the increase in interest expenses to the central bank won't be enough to deter the fiscal authority from promoting capital formation. However, Figure 10 shows that a long-term asset program would be ineffective if short-term rates are too high.

In terms of the implications for interest rates along the yield curve, both short and long-term asset purchases would cause yields to decline as long as short-term interest rates are sufficiently low. Yet, the impact of short-term asset purchases would be more effective across the maturity structure than long-term asset purchases. From this perspective, outright open market operations promote risk-sharing in the economy more than a long-term asset purchase program. As we discussed in the simple two period benchmark, asset purchases at either part of the maturity structure are likely to be a more effective policy tool than a simple money growth rule as higher rates of money growth disturb risk-sharing.

4 The Information Content Behind the Yield Curve

Given the amount of attention to studying yields in recent years, we turn to the model to provide insights into the role of yields for relaying information about the state of the economy. For example, do interest rates respond more to changes in real economic activity or inflation? Does the relationship depend on the level of economic development?

As our framework is built upon inflation-financed government debt, we principally focus on the impact of productivity growth on nominal interest rates through the government budget constraint (38):

$$\frac{dI_1}{dA} = \frac{\frac{w_A}{w}\left[\frac{(1-\lambda)(b_1^{CB}+b_2^{CB})}{w} + \frac{I_1(b_2-\lambda b_2^{CB})}{\sigma w}\right]}{\frac{\lambda\pi(\sigma-1)}{(I_1-\sigma)^2} + \frac{b_2-\lambda b_2^{CB}}{\sigma w}} \tag{43}$$

By comparison, an increase in the rate of money growth would also affect interest rates:

$$\frac{dI_1}{d\sigma} = \frac{(\sigma)^{-1}\left[\frac{\lambda\pi[\sigma^2(I_1-\sigma)^2+(I_1-1)]}{\sigma(I_1-\sigma)^2} + \frac{I_1(b_2-\lambda b_2^{CB})}{\sigma w}\right]}{\frac{\lambda\pi(\sigma-1)}{(I_1-\sigma)^2} + \frac{b_2-\lambda b_2^{CB}}{\sigma w}} \tag{44}$$

One can easily compare the effects of productivity growth to inflation by focusing on the numerator in (43) and (44).

In terms of looking at the impact of productivity on nominal interest rates, the key factor is that an increase in productivity causes wages to increase. In turn, the increase in wages fuels deposit growth in the banking system. In order for the capital stock to remain constant, nominal interest rates must sufficiently increase in order to limit the increase in government debt. The increase is more significant if the growth rate of deposits (as reflected by $\frac{w_A}{w}$) is higher.

Moreover, the response of interest rates depends on the amount of debt outstanding and who holds the debt, the private sector or the central bank. If $(1-\lambda)$ is higher, then the net interest obligations to the central bank would also be higher and the interest rate would need to adjust further to accommodate the increase in productivity. Second, due to similar reasoning, the adjustment is also higher if the monetary authority holds more of the debt issued by the fiscal authority. Finally, the nominal interest also increases more if the stock of long-term debt held by the private sector $(b_2 - \lambda b_2^{CB})$ is higher.

Clearly, the numerator in (44) depends on the impact of money growth on seigniorage revenue. If $\lambda\pi$ is higher, then the amount of seigniorage which is transferred to the fiscal authority will also be higher. The increase in revenues would fuel the ability of the fiscal authority to issue more short-term debt. Therefore, in order for the capital stock to remain constant, the nominal interest rate must rise.

Which effect is more significant? Productivity growth or money growth? The following Proposition summarizes our insights:

Proposition 8. *Let $A = \sigma$. In addition, suppose that $\sigma > \frac{w_A}{w}$ and $I_1 \neq \sigma$ is such that $1 < I_1 < \tilde{I}_1$ where $\tilde{I}_1$ is defined in the Appendix. Let $\gamma \equiv \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{\sigma^2 w_A}{\pi w^2}\right)(b_1^M + b_2^M)$ and $\gamma > 1$. Under these conditions, $\frac{dI_1}{dA} > \frac{dI_1}{d\sigma}$.*

According to the Proposition, the effects of productivity growth on nominal interest rates are stronger than inflation if nominal interest rates are low. From an alternative perspective, the Proposition indicates that the effects of productivity growth are stronger if the government has a large stock of bonds in circulation

(which is possible if nominal costs of debt financing are low). So, the information content of interest rates clearly hinges on the level of interest rates and the fiscal position of the government.

5 Conclusions

The tools used by central banks have evolved considerably in recent years. One important aspect of unconventional policy has been the move of central banks to aggressively purchase longer-term government securities along with expanding the size of its balance sheet. In this paper, we construct a rigorous modeling framework to investigate the impact of such policies. The inclusion of long-term assets and an independent central bank allows us to focus on a Treasury bond purchase program as a monetary policy tool. Outright open market operations through purchases of short-term government securities unambiguously lower yields and promote economic activity. By comparison, long-term asset purchases are only effective if short-term yields are sufficiently low. Should central bankers in countries with high debt loads and high short-term interest rates implement a long-term asset purchase program, it will be ineffective as it would only encourage fiscal authorities to issue more debt. Moreover, we also explain how the transmission channels of monetary policy between unconventional and conventional policies have important consequences for risk-sharing in the economy.

In particular, we believe that the three period framework that we offer opens up an array of interesting issues to be addressed in future research. For example, the model could be used to study the term structure of lending and credit market behavior. In addition, given the additional period in the lifecycle of agents in our model, future work could study how the transmission channels of monetary policy depend on the demographic structure of an economy. Notably, the degree of liquidity risk across the lifecycle could have important implications for asset prices in the economy.

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5.1 Appendix

1. Proof of Proposition 1.

We begin by characterizing the behavior of (15). It can be described by:

$$I = \sigma - \frac{\lambda\pi(\sigma - 1)}{\Omega(k) - [(1-\pi) + (1-\lambda)b^{CB}/w]}. \tag{45}$$

First, suppose that $I > \sigma$. For $k \in (0, k')$,

$$\lim_{k\to 0} I(k) = \sigma + \frac{\lambda\pi(\sigma - 1)}{(1-\pi)w_0 + (1-\lambda)b^{CB}/w_0} \equiv I_0 \tag{46}$$

$$\lim_{k\to k'} I(k) = \sigma + \infty = \infty. \tag{47}$$

If $I < \sigma$, we let $\hat{k} > k'$ be defined as:

$$\Omega(\hat{k}) = [1 - \pi(1-\lambda)] + (1-\lambda)b^{CB}/w(\hat{k}). \tag{48}$$

It follows that for $k \in (\hat{k}, \infty)$:

$$\lim_{k\to\hat{k}} I = \sigma - (\sigma - 1) = 1 \tag{49}$$

$$\lim_{k\to\infty} I = \sigma - 0 = \sigma \tag{50}$$

For the locus representing the no-arbitrage condition in (16), the limit behavior is captured by the following two equations:

$$\lim_{k\to 0} \sigma f'(k) = \infty,$$

$$\lim_{k\to\infty} \sigma f'(k) = 0.$$

Next, if $I > \sigma$, (15) is characterized by $\lim_{k\to 0} I(k) = I_0$. By comparison, the no arbitrage condition (16) is such that $\lim_{k\to 0} \sigma f'(k) = \infty$. Further, $\lim_{k\to\tilde{k}} I(k) = \infty$ and $\lim_{k\to\tilde{k}} \sigma f'(k) = \sigma f'(\tilde{k})$ hold. Since both loci are continuous for any $k \in (0, \tilde{k})$, there exists a k^* such that $0 < k^* < \tilde{k}$ in which a steady-state exists where $I > \sigma$.

When $I < \sigma$, $\lim_{k\to\hat{k}} I(k) = 1$ and $\lim_{k\to\infty} I(k) = \sigma$. In the same manner, for (16),

$\lim_{k\to\hat{k}^+} \sigma f'(k) = \sigma f'(\hat{k})$ and also $\lim_{k\to\infty} \sigma f'(k) = 0$. Hence, there must be a k^{**} such that $k^{**} > \hat{k}$ in which a steady state takes place as long as $\sigma f'(\hat{k}) > 1$.

2. Derivation of (38).

By disaggregating total bonds across holdings, we rewrite (33) as:

$$(b^n_{1,t} + b^m_{1,t} + b^{CB}_{1,t}) + (b^n_{2,t} + b^{CB}_{2,t}) +$$
$$\lambda\left[R_{1,t-1}b^{CB}_{1,t-1} + R_{2,t-2}b^{CB}_{2,t-2} + (\frac{\sigma-1}{\sigma})m_t - b^{CB}_{1,t} - b^{CB}_{2,t}\right] =$$
$$R_{1,t-1}(b^n_{1,t-1} + b^m_{1,t-1} + b^{CB}_{1,t-1}) + R_{2,t-2}(b^n_{2,t-2} + b^{CB}_{2,t-2}).$$

But since optimal risk-sharing implies $b^m_{1,t} = \pi w(k_t)$ and $m_{t+1} = \pi R_{1,t+1} w(k_t)$:

$$b^n_{1,t} + \pi w_t + b^{CB}_{1,t} + b^n_{2,t} + b^{CB}_{2,t} +$$
$$\lambda\left[R_{1,t-1}b^{CB}_{1,t-1} + R_{2,t-2}b^{CB}_{2,t-2} + \left(\frac{\sigma-1}{\sigma}\right)\pi R_{1,t+1}w_t - b^{CB}_{1,t} - b^{CB}_{2,t}\right] = \tag{51}$$
$$R_{1,t-1}b^n_{1,t-1} + R_{1,t-1}\pi w_t + R_{1,t-1}b^{CB}_{1,t-1} + R_{2,t-2}b^n_{2,t-2} + R_{2,t-2}b^{CB}_{2,t-2}.$$

Imposing steady state (SS) on (51) and gathering common terms gives:

$$\pi w\left[\lambda\left(\frac{\sigma-1}{\sigma}\right)R_1 - (R_1-1)\right] + \left[(R_1-1)(1-\lambda)\right]b^{CB}_1 -$$
$$\left[(R_2-1)(1-\lambda)\right]b^{CB}_2 = (R_1-1)b^n_1 + (R_2-1)b^n_2.$$

Dividing the above equation by $(R_1 - 1)$:

$$b^n_1 = -\pi w\left[1 - \lambda\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{R_1}{R_1-1}\right)\right] -$$
$$(1-\lambda)\left[b^{CB}_1 + b^{CB}_2\left(\frac{R_2-1}{R_1-1}\right)\right] - b^n_2\left(\frac{R_2-1}{R_1-1}\right). \tag{52}$$

Similarly, using the private bank's balance sheet constraint (59):

$$w = b^n_1 + b^m_1 + b^n_2 + k \tag{53}$$

We again substitute $b^m_{1,t} = \pi w_t$ as well as the value of b^n_1 according to (52) so that in steady state we have:

$$\begin{aligned} w = \pi w - \pi w \left[1 - \lambda \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{R_1}{R_1 - 1}\right)\right] - \\ (1-\lambda)\left[b^{CB}_1 + b^{CB}_2 \left(\frac{R_2 - 1}{R_1 - 1}\right)\right] - b^n_2 \left(\frac{R_2 - 1}{R_1 - 1}\right) + b^n_2 + k. \end{aligned} \tag{54}$$

Also, since $b^n_2 = b_2 - b^{CB}_2$:

$$\begin{aligned} w = \pi w \lambda \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{R_1}{R_1 - 1}\right) - (1-\lambda) b^{CB}_1 - \\ \left[1 - \lambda \left(\frac{R_2 - 1}{R_1 - 1}\right)\right] b^{CB}_2 + \left[1 - \left(\frac{R_2 - 1}{R_1 - 1}\right)\right] b_2 + k. \end{aligned} \tag{55}$$

From (26), (25) and (39), it must be that $(I_1)^2 = I_2$. Hence, we can write:

$$\begin{aligned} w = \pi w \lambda \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{I_1}{I_1 - \sigma}\right) - (1-\lambda) b^{CB}_1 - \\ \left[1 - \lambda \left(\frac{I_2 - \sigma}{I_1 - \sigma}\right)\right] b^{CB}_2 + \left[1 - \left(\frac{I_2 - \sigma}{I_1 - \sigma}\right)\right] b_2 + k. \end{aligned} \tag{56}$$

Solving for the stock of capital:

$$\begin{aligned} k = w \left[1 - \pi \lambda \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{I_1}{I_1 - \sigma}\right)\right] + (1-\lambda) b^{CB}_1 + \\ \left[1 - \lambda \left(\frac{I_1 + \sigma}{\sigma}\right)\right] b^{CB}_2 + \left(\frac{I_1}{\sigma}\right) b_2 \end{aligned}$$

which equivalently can be written as:

$$\begin{aligned} \Omega(k) = 1 - \pi \lambda \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{I^1}{I^1 - \sigma}\right) + (1-\lambda)\frac{b^{1M}}{w} + \\ \left[1 - \lambda \left(\frac{I^1 + \sigma}{\sigma}\right)\right] \frac{b^{2M}}{w} + \left(\frac{I^1}{\sigma}\right) \frac{b^2}{w} \end{aligned}$$

5.1.1 The Private Bank's Problem

$$\max_{r_t^m; r_t^n} \left[\pi \ln(r_t^m w_t) + (1-\pi)\ln(r_t^n w_t)\right]$$

s.t.

$$\pi r_t^m w_t = \left(R_{1,t+1} b_{1,t}^m\right) R_{1,t+2} \tag{57}$$

$$(1-\pi) r_t^n w_t = \left(R_{1,t+1} b_{1,t}^n\right) R_{1,t+2} + R_{2,t+2} b_{2,t}^n + r_{t+2} i_t \tag{58}$$

$$b_{1,t}^m + b_{1,t}^n + b_{2t}^n + i_t = w_t, \tag{59}$$

where (59) represents the private bank's balance sheet constraint.

We now describe that consumption relative to bank deposits:

$$\frac{c_t^m}{w_t} = R_{1,t+1}\beta_{1,t}^m R_{1,t+2}\left(\frac{1}{\pi}\right)$$

$$\frac{c_t^n}{w_t} = \left(R_{1,t+1}\beta_{1,t}^n R_{1,t+2} + R_{2,t+2}\beta_{2,t+2}^n + r_{t+2}(1-\beta_{1,t}^m - \beta_{1,t}^n - \beta_{2,t+2}^n)\right)\left(\frac{1}{1-\pi}\right)$$

where $\beta_{1,t}^m \equiv b_{1,t}/w_t$, $\beta_{1,t}^n \equiv b_{1,t}^n/w_t$ and $\beta_{2,t}^n \equiv b_{2,t}^n/w_t$. In this manner, we can rewrite the private bank's optimization program as follows:

$$\max_{\beta_{1,t}^m; \beta_{1,t}^n; \beta_{2,t+2}^n} \pi \ln\left[\frac{R_{1,t+1}}{\pi}\frac{P_{t+1}}{P_{t+2}}\beta_{1,t}^m\right] +$$

$$(1-\pi)\ln\left[\left(R_{1,t+1}R_{1,t+2}\beta_{1,n,t}^n + R_{2,t+2}\beta_{2,t+2}^n + r_{t+2}(1-\beta_{1,t}^m - \beta_{1,t}^n - \beta_{2,t+2}^n)\right)\left(\frac{1}{1-\pi}\right)\right]$$

The first-order condition for $\beta_{1,t}^m$:

$$\frac{\pi}{\beta_{1,t}^m} - \frac{(1-\pi)r_{t+2}}{(R_{1,t+1}\beta_{1,t}^n)R_{1,t+2} + R_{2,t+2}\beta_{2,t+2}^n + r_{t+2}(1-\beta_{1,t}^m - \beta_{1,t}^n - \beta_{2,t+2}^n)} = 0$$

The first-order condition for $\beta_{1,t}^n$:

$$(R_{1,t+1}R_{1,t+2} - r_{t+2}) = 0$$

The first-order condition for $\beta_{2,t+2}^n$:

$$(R_{2,t+2} - r_{t+2}) = 0$$

Substituting these two last conditions into the first one allows us to solve for the private bank's optimal holdings of short-term bonds for movers:

$$\beta^m_{1,t} = \pi \implies b^m_{1,t} = \pi w_t.$$

Therefore:

$$m_{t+1} = R_{1,t+1} b^m_{1,t}.$$

Equivalently,

$$m_{t+1} = \pi R_{1,t+1} w_t.$$

3. Proof of Proposition 4.

We begin by describing the behavior of the locus implied by (38). First, find the value of I_1 for which $k = 0$ in (38) when $I_1 > \sigma$. If $k = 0$, the equation becomes:

$$(I_1)^2 + [\sigma(\phi_1 - \phi_2) + \psi w_0]\ I_1 - [\sigma((\phi_1 - \phi_2) + 1) + w_0]\sigma = 0 \tag{60}$$

where

$$\phi_1 \equiv \frac{(1-\lambda)b_1^{CB}}{b_2 - \lambda b_2^{CB}} > 0, \quad \phi_2 \equiv \frac{b_2 - b_2^{CB}}{b_2 - \lambda b_2^{CB}} > 0 \text{ and } \psi \equiv 1 - \pi\lambda\left(\frac{\sigma - 1}{\sigma}\right) \in (0,1).$$

The uniquely-positive real root that solves (60) is:

$$I_1^0 = \frac{-[\sigma(\phi_1 - \phi_2) + \psi w_0] + \sqrt{[\sigma(\phi_1 - \phi_2 + 2) + \theta w_0]^2 + 4(1-\psi)w_0\sigma}}{2}\ . \tag{61}$$

We verify that $I_1^0 > \sigma$. If this holds, then equivalently:

$$\frac{-[\sigma(\phi_1 - \phi_2) + \psi w_0] + \sqrt{[\sigma(\phi_1 - \phi_2 + 2) + \psi w_0]^2 + 4(1-\psi)w_0\sigma}}{2} > \sigma \iff$$

$$4(1-\psi)w_0\sigma > 0 \iff$$

$$4 > 0,$$

Therefore, and we conclude $I_1^0 > \sigma$.
Next, the slope of (38) is determined by:

$$\frac{dI_1}{dk} = \frac{\Omega'(k) + \left[(1-\lambda)b_1^{CB} + \lambda b_2^{CB} + \frac{I_1}{\sigma}(b_2 - \lambda b_2^{CB})\right]\frac{w'(k)}{w^2}}{\frac{\lambda\pi(\sigma-1)}{(I_1-\sigma)^2} + \frac{b_2 - \lambda b_2^{CB}}{\sigma w}}, \tag{62}$$

which is positive for any $k \geq 0$.

For the case when $1 < I_1 < \sigma$, let $\hat{k}$ be such that $I_1(\hat{k}) = 1$ in (38), or equivalently:

$$\Omega(\hat{k}) = (1-\lambda)b_1^{CB}/w(\hat{k}) + \left[\sigma + \pi\lambda + (\sigma - \lambda(\sigma+1))b_2^{CB}/w(\hat{k}) + b^2/w(\hat{k})\right]\sigma^{-1} > 0.$$

We next look at the limiting behavior of (38). We let $k \to \infty$ for any $k \in (\hat{k}, \infty)$ and given that $\frac{dI_1}{dk} > 0$, we posit that $I_1(k) \to \sigma$ must follow. Note that:

$$\lim_{k\to\infty}\left\{\frac{I_1(k)\psi - \sigma}{I_1(k) - \sigma}\right\} = \lim_{k\to\infty}\Omega(k) - \left(\frac{b_2 - \lambda b_2^{CB}}{\sigma}\right)\lim_{k\to\infty}\left\{\frac{I_1(k)}{w(k)}\right\}. \tag{63}$$

If this is the case, then the LHS of (63) goes to ∞ as long as $I_1(k) \to \sigma$.

$$\lim_{k\to\infty}\left\{\frac{I_1(k)\psi - \sigma}{I_1(k) - \sigma}\right\} = \infty,$$

Further, note that since $I_1(k) \to \sigma$, $\lim_{k\to\infty}\left\{\frac{I_1(k)}{w(k)}\right\} = 0$ and the limiting properties from (63) are valid.

The no arbitrage condition represented by (39) slopes downward with the same limiting properties as (16).

Now, consider $I_1(k) \in (\sigma, \infty)$. Further, let $I_1^{gc}(k)$ and $I_1^{na}(k)$ represent (21) and (22), respectively. There is a $\tilde{k} > 0$ so that $I_1^{na}(\tilde{k}) = \sigma$ and $I_1^{gc}(\tilde{k}) > \sigma$. Define $g(k) = I_1^{na}(k) - I_1^{gc}(k)$ for $k > 0$. It follows that $g(0) = \infty$ and $g(\tilde{k}) = -\epsilon(\tilde{k})$ where $\epsilon(\tilde{k}) > 0$. Then by an intermediate value theorem argument, there is a $k^* > 0$ such that $g(k^*) = 0$.

Next, consider $I_1(k) \in [1, \sigma)$. There is a $\hat{k}$ such that $I_1(\hat{k}) = 1$. Further, suppose that $\sigma\sqrt{f'(\hat{k})} > 1$. By the monotonicity properties of $I_1^{gc}(k)$ and $I_1^{na}(k)$, there also exists a $\overline{k}$ where $I_1^{gc}(\overline{k}) > I_1^{na}(\overline{k})$. Consequently, there is $k^{**} > \hat{k}$ for which $I_1^{gc}(k^{**}) = I_1^{na}(k^{**})$. Since $I_1^{na}(k^{**}) < \sigma < I_1^{na}(k^*)$, it must be that $k^{**} > k^*$.

5. Proof of Proposition 5.

Concerning the effects of money growth on capital formation, differentiating

(38) with respect to σ results in

$$\frac{dk}{d\sigma} = \frac{-\frac{w\lambda\pi I_1}{\sigma^2(I_1-\sigma)}}{\Omega'(k) - \frac{(f''(k))^2(\sigma-1)}{I_1(I_1-\sigma)^2} + \frac{(1-\lambda)(b_1^{CB}+b_2^{CB})w'}{w^2} + \frac{[2(I_1)^2w'^2(f''(k))^2-\lambda b_2^{CB})}{2\sigma I_1 w^2}}, \tag{64}$$

which implies $dk/d\sigma > (<)0$ when $I_1 < (>)\sigma$.

The effect of money growth on short term interest rates can be seen in equation (44).

6. Proof of Proposition 6.

We start by showing the impact of short-term government bond purchases by the CB on steady state capital. Note that the NA condition does not shift, so we limit the analysis to (38) just to see the direction of the effect of asset purchases on the stock of capital.

Differentiating (38) with respect to b_1^{CB}:

$$\frac{dk}{db_1^{CB}} = \frac{1-\lambda}{\Omega'(k)w + [(1-\lambda)(b_1^{CB}+b_2^{CB}) + (b_2 - \lambda b_2^{CB})I_1/\sigma]\, w'/w}, \tag{65}$$

which is positive for any positive nominal interest rate.

7. Proof of Proposition 7.

We derive (38) with respect to b_2^{CB} which results in the expression:

$$\frac{dk}{db_2^{CB}} = \frac{1-\lambda(I_1+\sigma)/\sigma}{\Omega'(k)w + [(1-\lambda)(b_1^{CB}+b_2^{CB}) + (b^2 - \lambda b_2^{CB})I_1/\sigma]\, w'/w}. \tag{66}$$

Note that this equation has the same denominator as (65) and we uphold the conditions under which it is positive. Namely, $dk/db_2^{CB} > 0$ when

$$I_1 < \sigma\left(\frac{1-\lambda}{\lambda}\right). \tag{67}$$

8. Proof of Proposition 8.

The capital stock depends on the level of productivity denoted by A such that $y = A^2 f[k(A)]$. It follows that wage income is $w \equiv w[A, k(A)]$. Using this, we rewrite (38) and (39) as

$$k(A) = \left[1 - \pi\lambda\left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{I_1}{I_1 - \sigma}\right)\right] + \frac{(1-\lambda)(b_1^{CB} + b_2^{CB})}{w} + \frac{(b_2 - \lambda b_2^{CB})I^1}{\sigma w} \tag{68}$$

$$I_1 = \sigma A\sqrt{f'[k(A)]}, \tag{69}$$

respectively. Taking the derivative of the above equation with respect to A and σ gives:

$$\frac{dI_1}{dA} = \sigma f'[k(A)] \tag{70}$$

$$\frac{dI_1}{d\sigma} = Af'[k(A)], \tag{71}$$

For tractability we assume $\sigma = A$, so that $\frac{dI_1}{dA} = \frac{dI_1}{d\sigma}$ in the no arbitrage locus depicted by (69). This allows us to compare the effect on short term interest rates of the money growth rule σ versus that of the productivity parameter A by focusing only on (68). According to this equation, the impact of A on I_1 is given by:

$$\frac{dI_1}{dA} = \frac{\frac{w_A}{w}\left[\frac{(1-\lambda)(b_1^{CB} + b_2^{CB})}{w} + \frac{I_1(b_2 - \lambda b_2^{CB})}{\sigma w}\right]}{\frac{\lambda\pi(\sigma - 1)}{(I_1 - \sigma)^2} + \frac{b_2 - \lambda b_2^{CB}}{\sigma w}} \tag{72}$$

where w_A is the direct impact of A on the wage rate $w(A, k(A))$.

$$\frac{dI_1}{d\sigma} = \frac{(\sigma)^{-1}\left[\frac{\lambda\pi[\sigma^2(I_1 - \sigma)^2 + (I_1 - 1)]}{\sigma(I_1 - \sigma)^2} + \frac{I_1(b_2 - \lambda b_2^{CB})}{\sigma w}\right]}{\frac{\lambda\pi(\sigma - 1)}{(I_1 - \sigma)^2} + \frac{b_2 - \lambda b_2^{CB}}{\sigma w}} \tag{73}$$

The objective is to find a range of I_1 for which $\frac{dI_1}{dA} > \frac{dI_1}{d\sigma}$. Note that the denominators of (44) and (43) are the same. Hence we focus on their numerators. The second term in the numerator of (43) is greater that its corresponding term in (44) when the money growth rate is high enough. That is,

$$\frac{w_A I_1(b_2 - \lambda b_2^{CB})}{\sigma w^2} > \frac{I_1(b_2 - \lambda b_2^{CB})}{w} \iff \sigma > \frac{w_A}{w}. \tag{74}$$

Similarly, the first term in the numerator of (43) is greater that the corresponding

term in (44) when

$$(1-\gamma)(I_1-\sigma)^2+\sigma^2(I_1-1)<0 \tag{75}$$

where $\gamma \equiv \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{\sigma^2 w_A}{\pi w^2}\right)(b_1^{CB}+b_2^{CB})$ and we assume $\gamma > 1$. This inequality holds for any $I_1 \neq \sigma$ such that $1 < I_1 < \tilde{I}_1$, where

$$\tilde{I}_1 = \frac{2(1-\gamma)-\sigma+\sqrt{\sigma^2-4(1-\gamma)(\sigma-1)}}{2(1-\gamma)}. \tag{76}$$

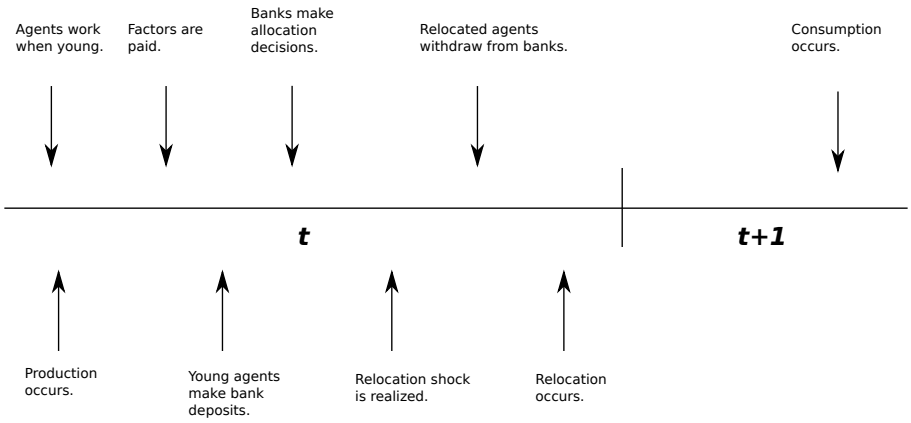

Figure 1: Benchmark model: depositor's timing of actions (Schreft & Smith, 1997).

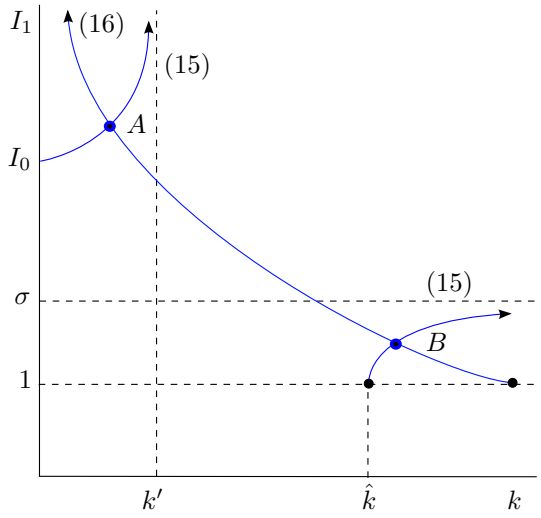

Figure 2: Benchmark model: steady state equilibria.

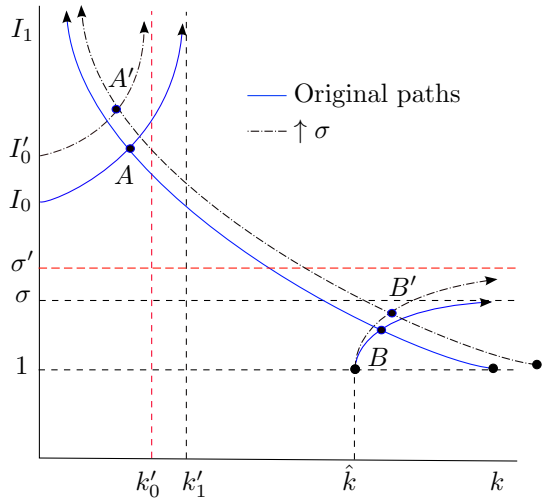

Figure 3: Benchmark model: money growth policy.

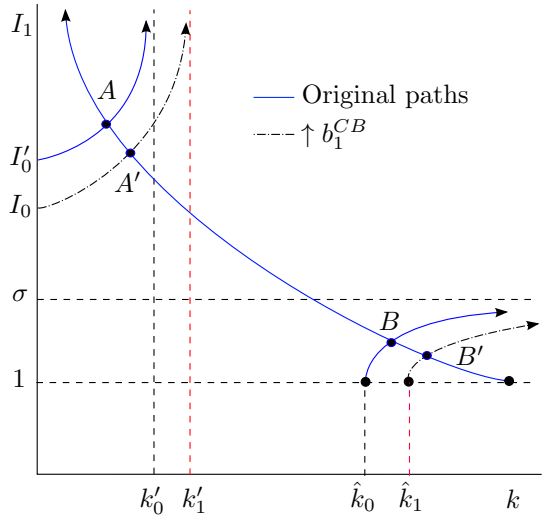

Figure 4: Benchmark model: short-term bond purchases.

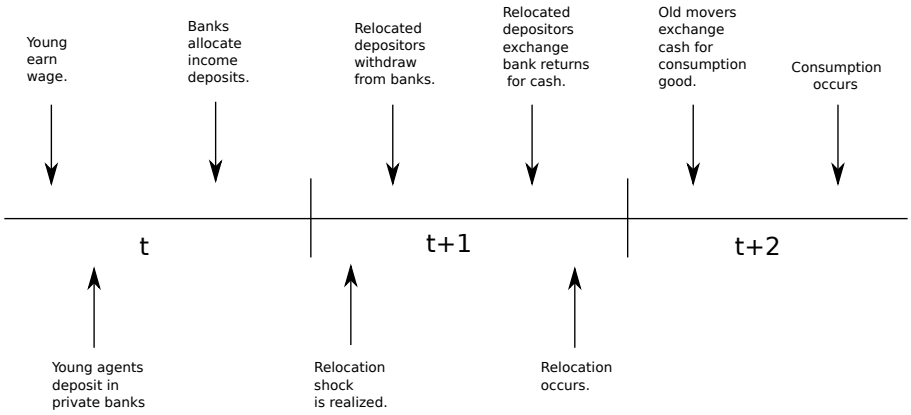

Figure 5: Depositor's timing of actions

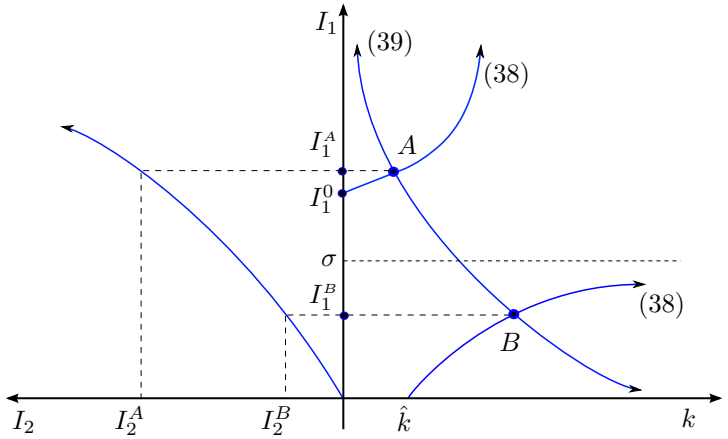

Figure 6: Steady state equilibria.

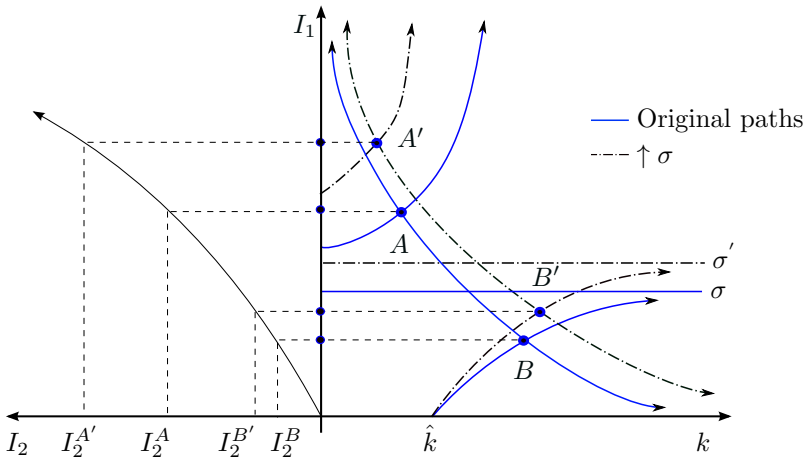

Figure 7: Effects of money growth rule.

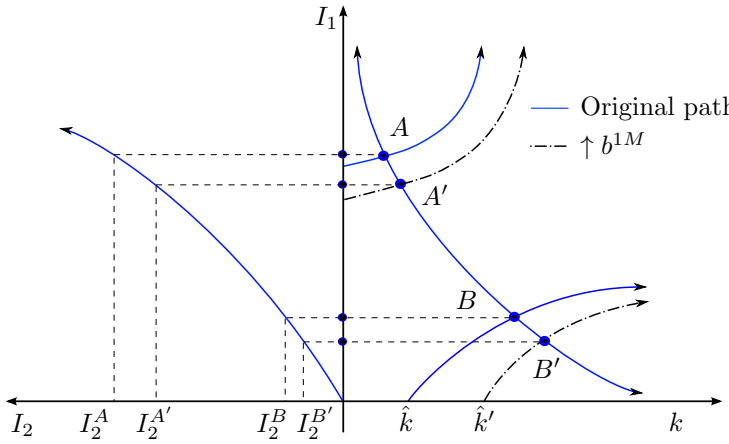

Figure 8: Effects of short-term bond purchases.

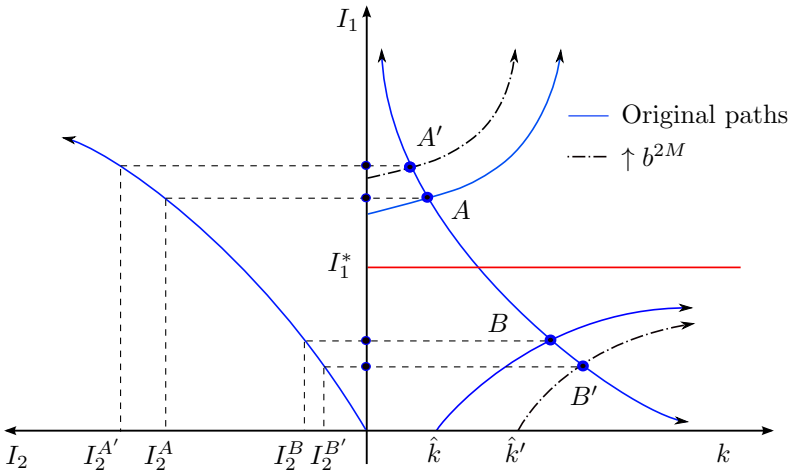

Figure 9: Long-term bond purchases: differential impact.

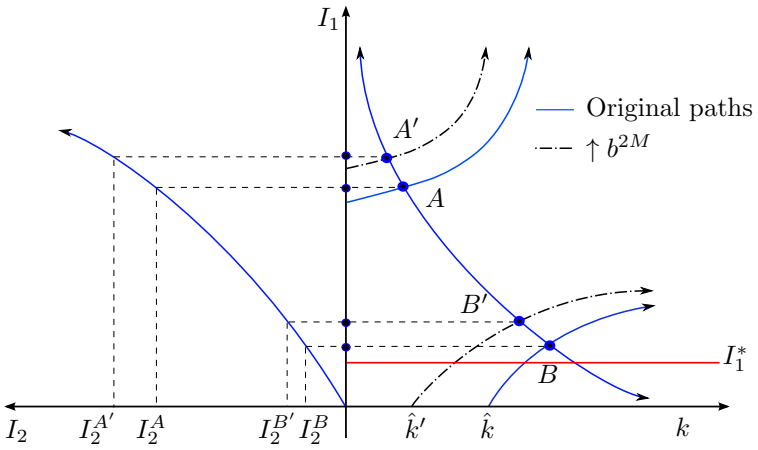

Figure 10: Long-term bond purchases: negative impact.