Banking Competition, Capital Accumulation, and Monetary Policy*

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Abstract

In recent years, the increased concentration of activity in the banking system has received much attention. In addition, numerous central banks have expanded their range of policy tools to include paying interest on reserves. The objective of this paper is to study the implications of concentration in the banking sector and the effects of various reserve policies. Changes in the competitive structure affect investment, risk-sharing, and social welfare. A key aspect of our analysis is that banks in more concentrated systems allocate a lot of resources towards cash reserves rather than investing in physical capital. However, in general, perfect competition should not be a regulatory goal for the banking system. The model also demonstrates that the efficacy of changes in interest rates on reserves to real variables such as the capital stock varies with the degree of concentration. Such observations are important as regulators and monetary policy authorities confront the challenges of an evolving competitive landscape out of the recent financial crisis.

1 Introduction

There have been considerable changes in the concentration of the banking sector across the globe. In particular, during the recent financial crisis, a number of institutions were so large that they were deemed "too big to fail." Yet, such trends have been part of a long-term pattern of consolidation activity. Notably, the Bank for International Settlements (2001) provides a thorough review of consolidation in the banking sector across countries. In the United States, for example, there were nearly 19,000 financial institutions in 1989. Just a decade later, only 10,000 were active in the sector. Moreover, Janicki and Prescott (2006) provide evidence indicating that the largest banks in the United States

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(which consist of less than 1% of active institutions) held over 75% of assets in the banking system prior to the crisis. With this large of a role in economic activity, it is hard to believe that these institutions would not take advantage of their market power in the financial system.

Furthermore, it is widely acknowledged that such institutions generally became even larger after the crisis as the number of banks declined.[1] To begin, McCord and Prescott (2014) document that the number of commercial banks in the United States decreased by around 14% from 2007 through 2013 – both McCord and Prescott and Adams and Gramlich (2014) stress that part of the reason for the decrease in the number of banks is due to incredibly low levels of bank entry. In particular, Adams and Gramlich point out that more than 100 new banks were formed on average each year from 1990 - 2008. By comparison, only seven new banks were established from 2009 to 2013. Given average levels of exit over time, the long-term pattern of consolidation will certainly continue.

McCord and Prescott suggest increased regulatory and entry costs since the end of the financial crisis may be responsible.[2] In addition, Barth et al. (2013) argue that barriers to entry are one of the most important factors in determining the number of firms and the degree of competition in the banking sector. Beck et al. (2006) also provide a discussion of the different roles of barriers to entry and other regulatory factors for concentration and the number of firms in the banking system across countries. Finally, Barth et al. observe that there has been an increase in banking requirements across 136 different countries from 1999-2011. Consequently, entry barriers and regulatory costs have not only played a role in activity in the banking system in the United States – they have also been important for many other countries. As a result, the trend towards increasing concentration is also relevant for other banking systems than the U.S.

On top of the changes in banking concentration in recent years, the tools of central banks used to conduct monetary policy have expanded. Notably, the Federal Reserve began paying interest on bank reserves in 2008 and it continues to do so. Around the same time, the Bank of England adopted a similar policy. By comparison, the European Central Bank (ECB) has had this authority since the ECB was established, but lowered the rate down to zero in 2012. In fact, the ECB has been charging banks to hold excess reserves since 2014.

Such dramatic observations raise some very important questions for regulatory and monetary policy authorities to address. What is the optimal competitive structure of the financial system? Should perfect competition in the banking system be a regulatory goal? How does the degree of concentration affect investment and capital accumulation? Do the effects of interest on reserves depend on the degree of concentration? What is the optimal interest rate policy? How does it depend on the competitive structure of the banking

[1] See Gongloff et al. (2013) and Gandel (2013).

[2] McCord and Prescott discuss how the Dodd-Frank Act of 2010 created increased regulatory costs for banks in the United States. They also contend that entry costs have increased due to additional regulations since the end of the financial crisis. As an example, Adams and Gramlich describe how the regulatory period to apply for participation in deposit insurance in the United States increased from three to seven years in 2009.

system?

The objective of this paper is to study the implications of concentration in the banking sector for economic activity in a monetary economy where the central bank uses interest rates on reserves as its primary policy tool. In particular, we demonstrate that changes in the competitive structure affect the level of investment, risk-sharing, and social welfare. A key aspect of our analysis is that banks in more concentrated systems allocate a lot of resources towards money balances. That is, they have a tendency to hoard cash reserves. The implications of the competitive structure in our model lie at the core of the role of financial intermediation for economic activity. Notably, Bencivenga and Smith (1991) stress that an active intermediary sector promotes risk-pooling services and "eliminates excessive investment in unproductive liquid assets...[the absence] leads towards unfavorable levels of capital accumulation." Yet, in stark contrast, we demonstrate that more concentrated banking systems considerably deviate from this function.

In order to study the role of financial sector competition for investment and capital accumulation, we follow Bencivenga and Smith (1991) by studying an overlapping generations version of Diamond and Dybvig (1983) with production. As in Schreft and Smith (1997, 1998), limited communication and restrictions on asset portability generate a transactions role for fiat money. Physical capital and money balances are the only two assets in the economy. In contrast to Schreft and Smith, the banking sector is imperfectly competitive with fixed entry. In particular, intermediaries engage in Cournot competition in the market for capital. As a result, differing degrees of concentration in the banking sector affect the provision of risk-pooling and investment. In addition, interest rates on bank reserves serve as the primary policy tool of the monetary authority rather than the rate of money growth.

Notably, banks engage in strategic behavior and exploit their market power in the capital sector. That is, in contrast to perfectly competitive banks lacking market power, banks take into account that they face a downward-sloping demand for capital by firms. As a result, the industrial organization of the banking sector has serious consequences for real activity. In particular, intermediaries exploit their market power by holding back resources available to firms and devote more funding to money balances. Consequently, highly concentrated sectors provide a large amount of insurance against liquidity risk – this tendency may affect the transmission channels of interest rate policy. Thus, the model presents a trade-off between the provision of productive resources to firms and risk-pooling to depositors as the competitive structure of the banking system varies.

We turn to the questions posed above. For example, how do the effects of interest on reserves depend on the degree of concentration? In particular, changes in the degree of concentration can render policy less effective in affecting investment and capital accumulation. Moreover, as concentration increases (and banks become very large in size), monetary policy has a weaker impact on the capital stock because banks would respond more to the desire to exploit their market power and would acquire even more money balances. In this manner,

increased concentration "clogs" the transmission channels of monetary policy – changes in payments on reserves have stronger economic effects if the banking sector is less concentrated.

Next, should policymakers unambiguously strive for increased competition in the banking sector? Should perfect competition be a regulatory goal? In general, no. Moreover, the competitive structure that maximizes aggregate welfare depends on the amount of liquidity risk that individuals face. If the degree of liquidity risk is high and reserve requirements are weak, the amount of capital accumulation is unfavorably low and banks hold a high amount of excess reserves that pay a relatively low interest rate relative to required reserves. In addition, the distortions from market power in the capital market are relatively high. Thus, an increase in competition would lead to a welfare gain as the capital stock would be significantly higher. However, there are limits to the gains from promoting competition. It is possible that the degree of competition can be too high because excessive amounts of competition would lead to minimal gains in income derived from the capital sector at the expense of less risk sharing as institutions less aggressively exploit their market power. Therefore, policymakers should carefully consider recent policy initiatives in many countries to promote competition. For example, in the U.K. there have been changes in banking requirements to lower barriers to entry in order to encourage credit funding.[3,4] Yet, in the same scenario with stronger reserve requirements, perfect competition can be optimal because banks would be offering an excessive amount of insurance against liquidity risk as they would be holding a large amount of reserves.

By comparison, if liquidity risk is low and there are minimal reserve requirements, a lot of resources are devoted to investment. As a result, the returns from additional capital investment are too low and the banking sector would benefit from greater concentration than under perfect competition. In fact, in some environments, a highly concentrated sector would produce the best social outcome.

Finally, we address the implications of the competitive structure for optimal monetary policy. Interestingly, irrespective of agents' degree of exposure to liquidity risk, the welfare-maximizing interest rate on excess reserves is (weakly) increasing in the degree of banking competition. In other words, in response to the recent trend towards concentration of firms in the banking sector, the model would call for lower rates on excess reserves. Simply put, concentrated banks with market power tend to hold excessively large amounts of money balances – the optimal policy would discourage such behavior.

The remainder of the paper is as follows. Section 2 provides a brief overview of the related literature. Section 3 outlines the physical environment of the

[3] Financial Services Authority (2013) describes the changes in banking requirements. One of them is a reduction in capital requirements for new entrants.

[4] Both Hannan (1991) and Corvoisier and Gropp (2002) contend that interest rates on loans are higher in markets with higher concentration ratios. Furthermore, Beck, Demirguc-Kunt and Maksimovic (2004) document that credit rationing occurs more often in concentrated banking systems.

model. Section 4 studies a monetary steady-state equilibrium. Section 5 addresses the welfare implications of the model, including the optimal competitive structure and the optimal monetary policy rule under different degrees of concentration of the banking sector. Next, in Section 6, we endogenize participation in the banking system. Finally, Section 7 concludes.

2 Related Literature

Our paper contributes to recent work which examines the implications of the industrial organization of the financial system for economic activity. In particular, Paal, Smith, and Wang (2005) construct an endogenous growth framework to study the effects of different competitive structures of the banking system for economic growth. Their model demonstrates that a monopolistic banking system may produce a higher rate of growth than a competitive banking system.

In a model of credit market activity, Ghossoub, Laosuthi, and Reed (2012) investigate the effects of monetary policy and find that there are important qualitative differences in the effects of policy across competitive structures. Under perfect competition, the transmission of monetary policy is straightforward in which higher rates of money growth lead to an increase in credit market activity. However, in a distorted monopolistic setting, policy produces the opposite effect. Building on the structure of Ghossoub, Laosuthi, and Reed (2012), Matsuoka (2011) examines optimal policy across competitive structures. In addition, Ghossoub (2012) studies the implications of the competitive structure for prices in capital markets.

In related work, Monnet and Sanches (2015) study the role of the competitive structure of the banking system for private money creation. Due to a low return on bank loans under perfect competition, the supply of money is inefficiently low. By comparison, an optimum quantity of money is incentive feasible in a monopolistic system. In contrast to their work, our paper emphasizes the role of financial intermediaries as private providers of *insurance* against liquidity risk as in Diamond and Dybvig (1983) – Monnet and Sanches focus on the role of banks as private providers of *liquidity* which promotes trade in a framework that builds on Lagos and Wright (2005) and Cavalcanti and Wallace (1999a, 1999b).

All of these papers focus exclusively on financial markets under price competition. Consequently, they are unable to address the implications of changes in the degree of concentration. By comparison, Ghossoub and Reed (2015) develop a model of a monetary economy where banks differ in size and engage in Cournot competition to consider the optimal size distribution of the banking system and the strength of the impact of monetary policy. However, in contrast to our framework, they study an endowment economy and focus on the role of credit markets to promote consumption smoothing.[5] In addition, Ghossoub

[5] In a similar vein, Williamson (1986) studies a model of Cournot competition in credit markets. However, financial intermediaries do not perform risk-pooling services as in our setup.

and Reed (2017) develop a model of imperfect competition in the capital sector to study the effects of concentration in economies with production externalities. However, the rate of money growth is the primary policy tool used by the central bank in both papers. Notably, the current structure is the *first* to examine the implications of *interest on reserves* in a model with an imperfectly competitive banking system where depositors are subject to liquidity risk.

Further, Gerali et al. (2010) looks at an economy with a monopolistically competitive banking sector in an infinitely-lived heterogeneous agent discrete stochastic general equilibrium model to study the implications of shocks from the banking sector for the business cycle. Moreover, Mandelman (2011) develops a framework with imperfect competition in the banking system to study the behavior of mark-ups over the business cycle.

There are other important papers that also introduce imperfect competition in real economies. For example, Cetorelli and Peretto (2012) construct a general equilibrium model with capital accumulation in which financial intermediaries engage in Cournot competition. However, as money does not circulate in their framework, they do not consider the connections between monetary policy and the competitive structure of the banking system. Similarly, in a structure based on Allen and Gale (chapter 8, 2000), Boyd and De Nicolo (2005) show as we do that more concentrated banks withhold resources from firms. In their analysis, banks provide loans to risky borrowers. By producing higher interest rates for borrowers, this leads to a riskier portfolio of assets on a bank's balance sheet.

Yet, such efforts to study the effects of concentration in the banking system and their implications for monetary policy are not meaningful unless they have empirical relevance. For example, theoretical predictions that the effects of changes in policy on real activity are weaker are not useful for policy discussions unless monetary policy appears to lead to real effects in the data. In this regard, Ghossoub and Reed (2010) summarize important recent empirical contributions that examine the issue. In particular, Ahmed and Rogers (2000), using annual data for the U.S. economy, find evidence which favors a long-run Tobin effect. They argue that models with predictions of super-neutrality or even negative effects from inflation are not consistent with the data for low-inflation economies. While we develop a framework to look at the impact of a different monetary policy tool, the evidence does indicate that monetary policy generally does produce long-run real effects.

Moreover, numerous empirical papers have looked at the implications of the industrial organization of the banking system for economic activity. Much of the existing empirical work suggests that concentration limits activity. To begin, Berger and Hannan (1991) study price rigidities in the deposit sector in the United States following monetary policy shocks. They find that price rigidities in deposit rates are stronger in regional banking markets with greater concentration, providing evidence that concentration leads to less competitive outcomes in the banking sector. In addition, Beck et al. (2004) point out that higher degrees of concentration (as measured by the three-firm concentration ratio) are positively correlated with obstacles to obtain funding for firms across 74 countries.

Further, Cetorelli (2004) observes that efforts to promote competition in the European banking system have contributed to more competitive outcomes in non-financial sectors. Using individual loan applications data for Spain, Jiménez et al. (2012) find that a loan application is more likely to be granted if the local banking market is more competitive. Finally, Ghossoub and Reed (2015) note that banks in countries in the European Monetary Union with higher degrees of concentration tend to hold more liquid assets. Such countries also tend to have lower measures of overall credit market activity where the effects of money growth on credit markets are weaker.

Other research looks at how the degree of concentration in the banking sector affects the transmission of monetary policy to output. For example, Cechetti (1999) and Kashyap and Stein (1997) find that the effects of money growth on output are stronger if the banking system is more competitive but models that formalize why this is true have not been extensively developed. Peersman's (2004) evidence focusing on countries in Europe is also consistent with the work of Cechetti and Kashyap and Stein.

3 Environment

Consider a discrete-time economy with two geographically separated locations or islands. Let $t = 1, 2, .., \infty$, index the time period. On each location, there are two types of agents that live for two periods: workers (potential depositors) and bankers. At the beginning of each time period, a unit mass of ex-ante identical workers and N financial intermediaries (or bankers) are born on each island. Each bank is indexed by j, where $j = 1, 2, ..., N$.

Workers are born with one unit of labor effort which they supply inelastically when young and are retired when old. In comparison to workers, bankers have no endowments. Furthermore, all agents derive utility from consuming the economy's single consumption good when old (c_{t+1}). As in Schreft and Smith (1997) and Antinolfi et al. (2007), the preferences of a typical worker are expressed by $u(c_{t+1}) = \ln(c_{t+1})$.[6] On the other hand, bankers are risk-neutral

[6] As agents only value consumption in old-age, the model abstracts away from intertemporal consumption. There are two related reasons for doing so. First, our approach follows previous work by Diamond-Dybvig (1983), Schreft-Smith (1997, 1998), and Champ, Smith, and Williamson (1996) who study settings in which agents only value consumption in old-age. Thus, such models focus explicitly on the role of banking institutions to insure individuals against liquidity risk which raises aggregate income by holding a portfolio of more productive assets than in the absence of an active banking sector. Keeping the set up the same allows us to compare the predictions of our model to previous work in a straightforward manner. Second, if individuals engage in intertemporal decisions, then the individual households would also attempt to insure themselves against liquidity risk through their choice of savings. As a result, the predictions about the level of economic activity in relationship to the degree of banking concentration may not line-up with the empirical observations cited. For example, if households anticipate that their expected utility in old-age would be lower in a concentrated banking sector (which we demonstrate is particularly relevant for settings with more liquidity risk), then they would consume less when young in order to smooth their consumption over time. As a result, total savings and capital investment may be higher in a concentrated banking system than under more competitive systems. In this way, it is questionable how

agents.

There are two types of assets in this economy: money (fiat currency) and physical capital. The total stock of outside (fiat) money which is put into circulation is equal to M_t where each unit is a liability of the central bank.[7] Further, denote the aggregate capital stock available in period t as K_t. One unit of goods invested by a young worker in period t yields one unit of capital in $t+1$. Moreover, at the initial date 0, the generation of old workers at each location is endowed with the initial aggregate stocks of capital and money (K_0 and M_0). Since the population of workers is equal to one, these variables also represent their values per worker. Assuming that the price level is common across locations, we refer to P_t as the number of units of currency per unit of goods at time t.

The consumption good is produced by a representative firm using capital and labor as inputs. The production function is of the Cobb-Douglas form, $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where Y_t and L_t are period t aggregate output and labor, respectively. In addition, A is a technology parameter and $\alpha\epsilon\,(0,1)$ reflects the capital intensity. Further, the capital stock depreciates completely in the production process.

Following previous work such as Champ, Smith, and Williamson (1996) and Schreft and Smith (1997, 1998), private information and limited communication between locations require workers to use cash if they move to a different location. The communication friction provides a rationale for the circulation of fiat money – due to incomplete information across islands, individuals will not accept privately issued liabilities. Thus, in some trades between agents, only fiat money will be accepted. Moreover, workers in the economy are subject to relocation shocks. After exchange occurs in the first period, a fraction $\pi \in (0,1)$ of agents is randomly chosen to relocate. These agents are called "movers." By comparison, individuals who remain on the same location are "non-movers." While π is known at the beginning of the period, agents are privately informed about their types at the end of the period.[8]

Unlike workers, bankers are not subject to relocation shocks. Moreover, as in Diamond and Dybvig (1983), all of workers' savings are intermediated as financial intermediaries are able to provide their depositors with insurance against idiosyncratic liquidity risk. Each banker j allocates its deposits into money and capital. By construction, there is a fixed number of firms in the

much the predictions of the model would be empirically relevant and how much they would driven by intertemporal consumption versus the degree of banking concentration – thus, the trade-offs between risk-sharing and capital formation would not be as clean and as intuitive as the current approach.

[7] For a discussion of private money creation in the banking system, please see Monnet and Sanches (2015).

[8] There are also numerous papers in which the relocation probability is stochastic. Models generally introduce aggregate uncertainty in in order to study banking crises and lender of last resort policies. For example, see Antinolfi, Huybens and Keister (2001) who show that introducing a discount window may achieve complete risk sharing but also subjects the economy to a continuum of inflationary equilibria. In addition, Boyd, de Nicoló, and Smith (2004) study the probabilities of banking crises across monopolistic and perfectly competitive banking systems.

banking sector. Moreover, in contrast to previous work such as Schreft and Smith, banks are Cournot competitors in capital markets. In addition, banks are subject to legal reserve or liquidity requirements. In particular, a bank is required to hold at least a fraction of its deposits (ρ) in cash reserves. All cash reserves are held at the monetary authority's vaults and earn interest. In particular, the central bank pays a gross nominal interest rate I_i for reserves held between t and $t+1$, where $i = R, e$ designating whether reserves are required or excess, respectively. The interest rate on required reserves is higher than on excess reserves. That is, $I_R \geq I_e$.

The central bank imposes the nominal returns on cash reserves once and for all at the beginning of time.[9] In this manner, interest rates on reserves are the primary policy instruments of the central bank. At the beginning of period $t+1$, the payments to the banking system for parking deposits at the central bank are paid from new money created and proportional income taxes on young agents. The tax rate, τ, is assumed to be constant over time. In nominal terms, the central authority's budget constraint in period $t+1$ is as follows:

$$M_{t+1} + P_{t+1}\tau w_{t+1} = I_R M_{R,t} + I_e M_{e,t} \tag{1}$$

where $M_{R,t} + M_{e,t} = M_t$, with $M_{R,t}$ and $M_{e,t}$ being the nominal stocks of required and excess cash reserves held by the banking system in period t. In real terms:

$$\tilde{m}_{t+1} + \tau w_{t+1} = \tilde{m}_{R,t} R_{R,t} + \tilde{m}_{e,t} R_{e,t} \tag{2}$$

where $\tilde{m}_{t+1} = M_{t+1}/P_{t+1}$ is the real aggregate stock of money in $t+1$ and $R_{i,t} = I_i \frac{P_t}{P_{t+1}}$ is the real return on cash reserves.

In sum, every period, the central authority adjusts its nominal money stock to satisfy its budget constraint. In this way, the money supply responds endogenously to economic conditions, depending on the amount of required and (endogenous) excess reserves that are held by individual institutions. Thus, money growth is not the primary tool of the central bank – it is simply a means to implement its interest rate policy – financing interest payments on bank reserves in the same way that expanding the money supply in numerous developed countries allowed central banks to pay interest on reserves and finance large scale asset purchases in recent years.

4 Trade

4.1 Factor Markets

In period t, a representative firm rents capital and hires workers in factor markets at rates r_t and w_t, respectively. Both workers and firms view that their

[9] As there are two different locations, one can view that the overall central bank functions on both islands in a way that is similar to the different regional banks in the federal reserve system.

actions do not affect market prices. Thus, the inverse demand functions for labor and capital by a typical firm are expressed by:

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{3}$$

and

$$r_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} \tag{4}$$

where $L_t = 1$ in equilibrium.

4.2 A Typical Worker

At the beginning of period t, each worker receives her labor income. A fraction of the income, τ, is paid in taxes to the government. Given that agents only value old-age consumption, all income is saved. Furthermore, as agents are subject to relocation shocks, all savings are intermediated. As the population size of workers is equal to mass one, the total amount of deposits in the banking system is: $(1-\tau) w_t$.

4.3 A Typical Bank's Problem

At the beginning of period t, each banker announces deposit rates taking the announced rates of return of other banks as given. A bank promises a gross real return on deposits, r_t^m if a young individual is relocated and a gross real return r_t^n if not. As we explain further below, banks offer similar financial services so each bank receives the same market share in the deposit market, attracting $1/N$ depositors or $\frac{1}{N}(1-\tau) w_t$ in deposits. Each bank allocates its deposits towards cash reserves and capital goods. Let m_t and k_{t+1}, respectively, denote the real amount of cash balances and capital goods held by each bank.[10]

Furthermore, unlike previous work such as Schreft and Smith (1997) and Ghossoub (2012), the rental market is characterized by Cournot (quantity) competition. That is, each bank recognizes that its own decisions about the amount of capital supplied will affect the market rental rate but that its choice does not affect that of other banks. In this manner, each intermediary faces the inverse demand for capital:

$$r\left(k_{t+1}, K_{t+1}^{N-1}\right) = \alpha A \left(K_{t+1}^{N-1} + k_{t+1}\right)^{\alpha-1} \tag{5}$$

where $K_{t+1}^{N-1} = \sum_{j=1}^{N-1} k_{t+1}^j$ is the amount of capital provided by all other intermediaries in the banking system. Banks act as Cournot competitors in the market for capital where a representative bank treats K_{t+1}^{N-1} as given. Moreover, $K_{t+1} = K_{t+1}^{N-1} + k_{t+1}$.

[10] Given that banks solve the same problem, we omit the indexation for each bank.

The role of market power by an intermediary enters in the choice of investment by an individual firm, k_{t+1}. Under a perfectly competitive capital market, intermediaries do not have any market power. Consequently, the marginal revenue from supplying resources to the capital market would simply be equal to r_{t+1}. For example, the rental rate represents marginal income earned from investment in Schreft and Smith (1997).

However, in an imperfectly competitive market, each intermediary is aware that they face a downward-sloping demand curve for capital. As a result, marginal revenue is given by $\frac{\partial r\left(k_{t+1},K_{t+1}^{N-1}\right)}{\partial k_{t+1}}k_{t+1}+r\left(k_{t+1},K_{t+1}^{N-1}\right)$. Therefore, due to the distortions from market power, the marginal income earned in the capital market is lower than under perfect competition.

We turn now to the deposit sector. Among the finite population, N, of identical banking firms, banks engage in Bertrand competition over the rates offered to depositors. In this setting, each bank has an incentive to offer a better interest rate schedule to attract deposit funding away from other institutions – thus, competitive pricing is the Nash equilibrium outcome in this game. If a bank does not offer a schedule which maximizes depositors' expected utility, it would not attract any deposits. Consequently, banks do not earn any profits due to Bertrand competition over deposit rates even though banks are Cournot competitors in the capital section. In turn, each bank receives the same amount of deposits, $\frac{1}{N}(1-\tau)w_t$. The same approach is adopted in Ghossoub and Reed (2015) and Ghossoub and Reed (forthcoming). Therefore, a bank's objective function is:

$$\underset{r_t^m,r_t^n,m_t,k_{t+1}}{Max}\ \pi\ln\left[r_t^m(1-\tau)w_t\right]+(1-\pi)\ln\left[r_t^n(1-\tau)w_t\right] \tag{6}$$

subject to the following constraints.

First, a bank's balance sheet at the beginning of period t is expressed by:

$$\frac{1}{N}(1-\tau)w_t=m_t+k_{t+1} \tag{7}$$

where $m_t=m_{R,t}+m_{e,t}$ and $m_{R,t}=\rho\frac{1}{N}(w_t(1-\tau))$. Furthermore, as relocated agents need cash to transact, total payments made to movers satisfy:

$$\frac{\pi}{N}r_t^m(1-\tau)w_t=m_{R,t}R_{R,t}+m_{e,t}R_{e,t} \tag{8}$$

Defining γ_t to be the fraction of deposits held by a bank in period t, the bank's cash holdings must satisfy liquidity requirements imposed by the monetary authority:

$$\gamma_t\geq\rho \tag{9}$$

We begin by studying portfolio allocations in which money is strictly dominated in rate of return and bankers' payments to non-movers are paid out of revenue from renting capital to firms in $t+1$. As banks are required to hold a

minimum amount of money cash reserves, money is strictly dominated in rate of return if $I_t > I_e$. The constraint on payments to non-movers is therefore:

$$\frac{1-\pi}{N} r_t^n (1-\tau) w_t = r\left(k_{t+1}, K_{t+1}^{N-1}\right) k_{t+1} \tag{10}$$

Finally, given that the realization of the shock is private information, banks need to offer deposit contracts that are incentive compatible to prevent agents from misrepresenting their realizations of the relocation shock. That is:

$$r_t^m \leq r_t^n \tag{11}$$

The assumption about private information of the liquidity shock is standard in models of financial intermediation going back to initial contributions by Diamond and Dybvig (1983).

In seeking to determine a bank's portfolio allocation, suppose initially that the incentive compatibility constraint does not bind. The choice of capital investment by a single financial institution is such that:[11]

$$(1-\pi) \frac{\left[\frac{\partial r\left(k_{t+1}, K_{t+1}^{N-1}\right)}{\partial k_{t+1}} k_{t+1} + r\left(k_{t+1}, K_{t+1}^{N-1}\right)\right]}{r\left(k_{t+1}, K_{t+1}^{N-1}\right) k_{t+1}} = \pi \frac{R_{e,t}}{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}} \tag{12}$$

where the term on the left-hand-side of (12) is the additional gain in utility to non-relocated depositors from a marginal increase in capital investment. The term on the right-hand-side is the loss in utility to relocated depositors when the bank increases its capital investment by one unit. Upon using (5), the choice of capital can be expressed as:

$$(1-\pi) \frac{\left[1 - (1-\alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}\right]}{k_{t+1}} = \frac{\pi R_{e,t}}{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}} \tag{13}$$

where the term $\left[1 - (1-\alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}\right]$ represents the extent of market power by a representative intermediary in the capital market. Moreover, $\frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}$ is the market share in the capital market of a typical bank. Solving for $m_{e,t}$ from the above condition indicates that the amount of excess reserves held by a representative bank is:

$$m_{e,t} = \frac{\frac{1}{N}(1-\tau) w_t - \left(1 + \frac{1-\pi}{\pi}\left[1 - (1-\alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}\right] \frac{R_{R,t}}{R_{e,t}}\right) m_{R,t}}{1 + \frac{1-\pi}{\pi}\left[1 - (1-\alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}\right]} \tag{14}$$

[11] The complete solution to the bank's problem is provided in the Appendix.

Or, equivalently, the fraction of deposits allocated towards excess reserves is:

$$\gamma_{e,t} = \frac{1 - \left(1 + \frac{1-\pi}{\pi}\left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]\frac{R_{R,t}}{R_{e,t}}\right)\gamma_{R,t}}{1 + \frac{1-\pi}{\pi}\left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]} \tag{15}$$

where $\gamma_{e,t} = m_{e,t}/\left((1-\tau)\, w_t/N\right)$ and $\gamma_{R,t} = m_{R,t}/\left((1-\tau)\, w_t/N\right) = \rho$.

In this manner, $\gamma_{e,t} > 0$ if:

$$\rho < \frac{1}{1 + \frac{1-\pi}{\pi}\left[1 - (1-\alpha) * \frac{k_{t+1}}{K_{t+1}}\right]\frac{R_{R,t}}{R_{e,t}}} \tag{16}$$

Thus, if reserve requirements are not particularly tight, then banks will tend to hold excess reserves.

Alternatively, the condition may be written in terms of the relative interest rate paid on required reserves. Let $\mu = \frac{I_R}{I_e}$ denote the relative rate paid to required reserves. Then, (16) can be expressed as:

$$\mu < \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{\left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]} = \mu_2 \tag{17}$$

Notably, the reserve constraint binds ($\gamma_t = \rho$) if $\mu \geq \mu_2$ – if the relative rate paid to *excess* reserves is low enough, banks do not hold excess reserves. From (7) - (10) the relative return paid to depositors when banks do not hold excess reserves is:

$$\frac{r_t^n}{r_t^m} = \frac{\pi}{1-\pi}\frac{(1-\rho)}{\rho}\frac{I_t}{I_R} \tag{18}$$

By comparison when $\mu < \mu_2$, the relative rate of return paid to excess (required) reserves is sufficiently high (low) so that banks hold excess reserves and the total reserves-deposit ratio comes from both required reserves and excess reserves:

$$\gamma_t = \gamma_{e,t} + \gamma_{R,t} \tag{19}$$

Substituting from the expression for the fraction of excess reserves ($\gamma_{e,t}$), the reserves to deposit ratio is:

$$\gamma_t = \frac{1 - \rho\frac{1-\pi}{\pi}\left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]\left(\frac{I_R}{I_e} - 1\right)}{1 + \frac{1-\pi}{\pi}\left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]} \tag{20}$$

It is clear that $\frac{\partial \gamma_t}{\partial \mu} < 0$. Intuitively, a higher interest rate on *excess* reserves (lower μ) raises the marginal cost of capital investment (lowers the cost of holding money) which encourages banks to hold a more liquid portfolio. In contrast,

a higher interest rate on required reserves lowers the marginal cost of capital investment as banks have more income to pay relocated agents. This, in turn, leads banks to allocate fewer resources towards money balances.

Furthermore, using (7) - (10) and (20), the relative return to depositors is such that:

$$\frac{r_t^n}{r_t^m} = \left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}\right]\frac{I_t}{I_e} \tag{21}$$

where $I_t = r\left(k_{t+1}, K_{t+1}^{N-1}\right)\frac{P_{t+1}}{P_t}$ is the nominal return to capital between periods t and $t+1$. In this manner, the incentive compatibility constraint is non-binding if: $I_t > I_e\left[1-(1-\alpha)\frac{k_{t+1}}{K_{t+1}}\right]^{-1} = \underline{I}_1$.

On the other hand, the incentive compatibility constraint is binding to the point where depositors obtain full insurance against liquidity risk if $I_t \in \left[I_e, I_e\left[1-(1-\alpha)\frac{k_{t+1}}{K_{t+1}}\right]^{-1}\right]$. Two cases arise here. Under case # 1, excess reserves are strictly dominated in rate of return where $I_t \in \left(I_e, I_e\left[1-(1-\alpha)\frac{k_{t+1}}{K_{t+1}}\right]^{-1}\right]$.[12] Then, using (7) - (10) and the fact that $\frac{r_t^n}{r_t^m} = 1$, a bank's liquidity holdings are such that:

$$\gamma_t = \frac{\frac{\pi}{1-\pi}I_t + (I_e - I_R)\rho}{\left(I_e + \frac{\pi}{1-\pi}I_t\right)} \tag{22}$$

Under case #2 where $I_t = I_e < I_R$, excess reserves and capital yield the same real return. Given the high return to money, banks hold part of their cash reserves for non-relocated agents *in addition to movers.* In particular, define λ_t to be the fraction of money balances paid to movers and $(1-\lambda_t)$ the fraction given to non-movers. Therefore, payments to movers and non-movers are given respectively by:

$$\frac{\pi}{N}r_t^m(1-\tau)w_t = \lambda_t\left[m_{R,t}R_{R,t} + m_{e,t}R_{e,t}\right] \tag{23}$$

and

$$\frac{1-\pi}{N}r_t^n(1-\tau)w_t = r\left(k_{t+1}, K_{t+1}^{N-1}\right)k_{t+1} + (1-\lambda_t)\left[m_{R,t}R_{R,t} + m_{e,t}R_{e,t}\right] \tag{24}$$

$$r\left(k_{t+1}, K_{t+1}^{N-1}\right) = I_e\frac{P_t}{P_{t+1}} \tag{25}$$

where from (2)

$$\frac{P_{t+1}}{P_t} = \frac{\tilde{m}_{R,t}I_R + \tilde{m}_{e,t}I_e}{\tilde{m}_{t+1} + \tau_{t+1}w_{t+1}} \tag{26}$$

[12] As previously mentioned, we only study equilibria where money is dominated in rate of return ($I \geq I_e$).

4.4 General Equilibrium

In a symmetric Nash equilibrium, each intermediary makes the same choice of investment. We define the symmetric Nash equilibrium level of investment of an individual intermediary as $k_{t+1}^* \equiv k_{t+1}(w_t, \frac{P_{t+1}}{P_t}; N, \mu)$. Thus, $K_{t+1}^* = Nk_{t+1}(w_t, \frac{P_{t+1}}{P_t}; N, \mu)$. In equilibrium, all markets will clear. In particular, labor receives its marginal product, (3), and the labor market clears with $L_t = 1$. In this manuscript we focus on the behavior of the economy in the steady-state, where $\tilde{m}_{t+1} = \tilde{m}_t = \tilde{m}$ and $K_{t+1} = K_t = K$. Imposing steady-state on (26), the steady-state inflation rate is:

$$\frac{P_{t+1}}{P_t} = \frac{\rho I_R + (\gamma^* - \rho) I_e}{\gamma^* + \frac{\tau}{1-\tau}} \tag{27}$$

where

$$\gamma^* = \begin{cases} \rho & \text{if } \mu \geq \mu_2 = \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1-\frac{(1-\alpha)}{N}} \\ \frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]} & \text{if } \mu \in \left(\mu_1 = \frac{1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}, \mu_2\right) \\ \frac{\frac{\pi}{1-\pi}I+(I_e-I_R)\rho}{I_e+\frac{\pi}{1-\pi}I} & \text{if } \mu \in \left(\mu_0 = \frac{\alpha\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right)}{(1-\pi)(1-\tau)(1-\alpha)\rho} - \frac{(1-\rho)}{\rho}, \mu_1\right] \\ \frac{\rho(\mu-1)-\frac{1}{r}\frac{\tau}{1-\tau}}{\left(\frac{1}{r}-1\right)} & \text{if } \mu \in [1, \mu_0] \end{cases} \tag{28}$$

In the first scenario where $\mu \geq \mu_2$, the relative return to excess reserves is so low that banks only hold required reserves. In the second situation in which $\mu \in (\mu_1, \mu_2)$, the relative interest rate on excess reserves is slightly higher and banks start to hold excess reserves. However, as the amount of money balances is relatively low, $r_t^n > r_t^m$. For relative returns such that $\mu \in (\mu_0, \mu_1]$, depositors obtain full insurance but money is still dominated in rate of return. In the final scenario, depositors obtain full insurance and $I_t^* = I_e$.

Finally, as we demonstrate in the appendix, using the pricing equation, (4), the definition of I, along with a bank's balance sheet, (7) the equilibrium aggregate stock of capital is such that:

$$(K^*)^{1-\alpha} = \begin{cases} (1-\rho)(1-\tau)(1-\alpha)A & \text{if } \mu \geq \mu_2 \\ \left[\frac{1+\rho(\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}\right]\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](1-\tau)(1-\alpha)A & \text{if } \mu \in [\mu_1, \mu_2) \\ \alpha A\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right) & \text{if } \mu \in (\mu_0, \mu_1] \\ \alpha A\left(\frac{1}{(1-\tau)[1+\rho(\mu-1)]}\frac{1}{1-\alpha}\frac{1+\sqrt{1-4(1-\tau)[1+\rho(\mu-1)](1-\alpha)\alpha}}{2}\right)^{-1} & \text{if } \mu \in [1, \mu_0] \end{cases} \tag{29}$$

We proceed to establish existence and uniqueness of monetary steady-state equilibria in the following proposition. All the details and complete derivations are provided in the Appendix.

Proposition 1.

i. Suppose $\mu \in [1, \mu_0]$, *where* $\mu_0 \geq 1$ *if* $\tau \geq \frac{\pi}{1-\pi}\left[\frac{1}{1-\pi}\frac{\alpha}{1-\alpha} - 1\right] = \tilde{\tau}$. *In addition, suppose that* $\tau < \frac{1-2\alpha}{1-\alpha}$ *and* $\mu_0 < \tilde{\mu}_1 = 1 + \frac{\tau}{\rho}\frac{1-\alpha}{\alpha}$. *Under these conditions, a steady-state where* $\gamma^* > \rho$, $\frac{r^{n^*}}{r^{m^*}} = 1$, *and* $I^* = I_e$ *exists and is unique.*

ii. Suppose $\mu \in (\mu_0, \mu_1]$, *with* $\mu_0 < \mu_1$. *Under this condition, a steady-state where* $\gamma^* > \rho$, $\frac{r^{n^*}}{r^{m^*}} = 1$, *and* $I^* > I_e$ *exists and is unique.*

iii. Suppose $\mu \in [\mu_1, \mu_2)$, *where* $\mu_1 \leq \mu_2$ *if* $\rho \leq \frac{1}{1-\tau}\left(\frac{\alpha}{1-\alpha}\frac{\pi}{1-\pi} - \tau\right) = \hat{\rho}$. *Under this condition, a steady-state where* $\gamma^* > \rho$, $\frac{r^{n^*}}{r^{m^*}} > 1$, *and* $I^* > I_e$ *exists and is unique.*

iv. Suppose $\mu \geq \mu_2$. *Under this condition, a steady-state where* $\gamma^* = \rho$, $\frac{r^{n^*}}{r^{m^*}} > 1$, *and* $I^* > I_e$ *exists and is unique.*

In the first scenario, the interest rate paid to excess reserves is at its highest level, yet the rate paid on required reserves is at least as high as excess reserves. This scenario is possible if taxes on wage income are sufficiently high which provides the central authority with enough revenue to pay the amount of interest on reserves. On the other hand, the tax rate has an upper bound so that the large amount of interest payments on reserves does not lead to multiple steady-states. As the nominal interest rate on excess reserves is so high, then $I^* = I_e$. Moreover, if μ were to be less than 1, then money would no longer be dominated in rate of return.

In the second scenario, the relative return paid to required reserves is somewhat higher but banks still hold some excess reserves. In turn, depositors continue to receive full insurance against liquidity risk. The third scenario also involves banks holding excess reserves but depositors do not obtain full insurance because the reserve requirement is not particularly tight and the interest rate on excess reserves is lower than in the second scenario. Finally, in the last case, the interest rate on excess reserves is so low that banks only hold required reserves.

We proceed to examine how the degree of banking competition affects the economy (including the consumption of non-movers, c^{n^*}, and movers, c^{m^*}) in the following Proposition:

Proposition 2.

i. γ^*, K^*, $\tilde{m}^*$, c^{n^*}, c^{m^*}, *and* $\frac{r^{n^*}}{r^{m^*}}$ *do not depend on* N *if* $\mu \in [1, \mu_1]$.

ii. Suppose $\mu \in (\mu_1, \mu_2)$. *Under this condition,* γ^* *is decreasing in* N, *but* K^*, c^{n^*}, *and* $\frac{r^{n^*}}{r^{m^*}}$ *are increasing in* N. *The effects of* N *on* c^{m^*} *and* $\tilde{m}^*$ *depend on* π. *If* $\pi < \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, c^{m^*} *and* $\tilde{m}^*$ *are decreasing in* N. *By comparison, if* $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ *and* $\rho < \acute{\rho} = \left(1 - \frac{\alpha}{1-\tau}\right)$, c^{m^*} *is increasing in* N *up to* $\widehat{N}$, *then decreasing in* N.[13] *Similarly, suppose that* $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, *and* $\rho < \ddot{\rho}$ *where* $(1-\alpha) = \ddot{\rho} < \hat{\rho}$ *if* $\frac{1}{1-\alpha}\frac{\pi}{1-\pi} - \frac{1-\alpha}{\alpha} > \tau$. *Under these conditions,* $\tilde{m}^*$ *is also increasing in* N, *then decreasing in* N. *Finally,* c^{m^*} *is increasing in* N *if* $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ *and* $\rho \in (\acute{\rho}, \hat{\rho})$. *Also, if* $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ *and* $\rho \in (\ddot{\rho}, \hat{\rho})$, $\tilde{m}^*$ *is increasing in* N.

iii. γ^*, $K^*, \tilde{m}^*$, c^{n^*}, c^{m^*}, *and* $\frac{r^{n^*}}{r^{m^*}}$ *do not depend on* N *if* $\mu \geq \mu_2$.

Intuitively, when the relative return on excess reserves is high ($\mu \leq \mu_1$), banks hold a lot of excess reserves and provide full insurance against liquidity risk. Further, because the return to excess reserves is so high, there is little incentive for banks to engage in strategic behavior by withholding resources from the capital sector. Consequently, a change in the degree of banking competition will not produce any real effects.

Alternatively, if the return on excess reserves is over an intermediate range, $\mu \in (\mu_1, \mu_2)$, the return from holding excess money balances is low enough that banks exploit their market power in the capital sector. Thus, there are important direct effects from the degree of banking concentration on banks' portfolios. First, when the degree of concentration in capital markets weakens, each intermediary recognizes that it has less market power and therefore allocates a larger fraction of its deposits towards capital investment. The higher stock of capital puts downwards pressure on its return and upward pressure on wages and deposits. The increase in deposits further contributes towards capital accumulation. The effects on the consumption of non-movers directly follows the effects for the capital stock.

However, the impact of the degree of competition on the consumption of movers and the level of real money balances is non-monotonic and strongly depends on the degree of liquidity risk. First, as previously stated, banks allocate a smaller *proportion* of deposits to cash balances (γ^* is lower) as the degree of competition increases. If liquidity risk is relatively low, the total amount of capital accumulation would tend to be relatively high. In turn, an increase in the degree of competition relaxes banks' strategic incentives and more investment in capital takes place. As a result, banks hold less money balances and movers obtain less consumption. Moreover, whether the condition for case (ii) to hold depends on the share of capital in production. Simply put, if the capital share is higher, then the increase in wages would be greater as the banking sector becomes more competitive. Thus, the increase in deposit formation would stimulate both investment and the demand for money balances.[14]

[13] Please note that $\acute{\rho} < \hat{\rho}$ if $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$.

[14] Previous work such as Ghossoub and Reed (2015) studies imperfectly competitive banking

By comparison, the role of competition is more complicated when there is more liquidity risk. If reserve requirements are not particularly tight ($\rho < \hat{\rho} < \hat{\rho}$), then the amount of excess reserves would be relatively high. As a result, when N is relatively low, the degree of concentration in the capital sector is relatively high which leads to not only less capital accumulation but also lower wages. As the deposit base is relatively low, the level of real money balances and the consumption of movers would also tend to be low.

Consequently, an increase in competition (from a highly concentrated sector) has a large impact on wages and leads to an increase in money holdings because of greater deposit formation which promotes the consumption of movers. Yet, once the degree of competition is high enough, the amount of capital accumulation is sufficiently high that the direct portfolio composition effect of greater competition on the amount of capital investment dominates so that $\tilde{m}^*$ and c^{m^*} are both lower.

Moving from these conditions of relatively high liquidity risk but weak reserve requirements to a banking system with larger reserve requirements, more seigniorage is required to finance the relatively high interest rate on reserves. As a result, the amount of capital accumulation is initially low enough that the increase in deposits due to higher wages pulls up real money balances along with the consumption of movers.

Finally, if the return on excess reserves is too low, $\mu \geq \mu_2$, the liquidity constraint is binding and banks only hold required reserves. In this manner, banks do not distort capital markets and a change in the degree of banking competition has no real aggregate effects.

While the preceding analysis describes how the effects of banking concentration depend on monetary policy, the effects of changes in monetary policy have yet to be mentioned. In the following Proposition, we discuss the effects of monetary policy:

Proposition 3.

i. Suppose $\mu \in [1, \mu_0)$. *Then,* $\frac{d\gamma^*}{d\mu} < 0$, $\frac{d\tilde{m}^*}{d\mu} < 0$, $\frac{dK^*}{d\mu} > 0$, *and* $\frac{d\frac{r^{n*}}{r^m}}{d\mu} = 0$. *In addition,* $\frac{dc^*}{d\mu} \geq (<)\, 0$ *if* $\mu \leq (>)\, \tilde{\mu}_0$, *where* $1 < \tilde{\mu}_0 < \mu_0$ *if* $\pi < \frac{1-2\alpha}{1-\alpha}$. *However, if* $\pi > \frac{1-2\alpha}{1-\alpha}$, $\frac{dc^*}{d\mu} > 0$.

ii. Suppose $\mu \in [\mu_0, \mu_1)$. *Then,* $\frac{d\gamma^*}{d\mu} = 0$, $\frac{d\tilde{m}^*}{d\mu} = 0$, $\frac{dK^*}{d\mu} = 0$, $\frac{d\frac{r^{n*}}{r^m}}{d\mu} = 0$, *and* $\frac{dc^*}{d\mu} = 0$.

iii. Suppose $\mu \in [\mu_1, \mu_2)$. *Under this condition,* $\frac{d\gamma^*}{d\mu} < 0$, $\frac{dK^*}{d\mu} > 0$, $\frac{d\frac{r^{n*}}{r^m}}{d\mu} > 0$, *and* $\frac{dc^{n^*}}{d\mu} > 0$. *However, the effects of* μ *on* c^{m^*} *and* $\tilde{m}^*$ *depend on* π. *If* $\pi < \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, $\frac{d\tilde{m}^*}{d\mu} < 0$ *and* $\frac{dc^{m^*}}{d\mu} < 0$. *Next, suppose* $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ *holds. If* $\rho < \hat{\rho} < \hat{\rho}$, $\frac{dc^{m^*}}{d\mu} \geq (<)\, 0$ *if* $\mu \leq (>)\, \acute{\mu}$. *Similarly,* $\frac{d\tilde{m}^*}{d\mu} \geq (<)\, 0$ *when*

systems in endowment economies. Thus, such connections between the aggregate deposit base and the degree of competition in the banking sector do not take place as they do in this model that looks at a production economy.

$\mu \leq (>) \ddot{\mu}$ *if* $\rho < \ddot{\rho} < \hat{\rho}$. *By comparison,* $\frac{dc^{m^*}}{d\mu} > 0$ *if* $\rho \in (\hat{\rho}, \hat{\rho})$. *Moreover,* $\frac{d\tilde{m}^*}{d\mu} > 0$ *if* $\rho \in (\ddot{\rho}, \hat{\rho})$.

iv. Suppose $\mu \geq \mu_2$ $\frac{d\gamma^*}{d\mu} = 0$, $\frac{d\tilde{m}^*}{d\mu} = 0$, $\frac{dK^*}{d\mu} = 0$, $\frac{d\frac{r^{n*}}{r^{m}}}{d\mu} = 0$, $\frac{dc^{m^*}}{d\mu} = 0$, *and* $\frac{dc^{n^*}}{d\mu} = 0$.

Interestingly, the results in Proposition 3 indicate that the effects of monetary policy vary across μ. In particular, when the return on excess reserves is sufficiently high such that $\mu \in [1, \mu_0)$, the economy is at the Friedman rule and banks hold cash reserves on behalf of both non-movers and movers. In this case, lowering the return on excess reserves generates a substitution from money to physical capital – banks can continue to provide a complete risk sharing contract as they pay only a fraction of their cash reserves to movers given that the return on excess reserves is in the range where $\mu \in [1, \mu_0)$.

Yet, even in the range where $\mu \in [1, \mu_0)$, the effects on consumption across agents depend on the level of μ and the extent of liquidity risk. We first discuss the impact of monetary policy on consumption if liquidity risk is relatively low. Starting from a position where $\mu \in [1, \mu_0)$ is relatively low, banks would be holding a lot of money balances since the return on excess reserves is so high. As a result, the capital stock would be relatively low. Therefore, the increase in capital accumulation due to a lower return on excess reserves promotes deposit formation and allows consumption to increase. But, once the relative return to excess reserves falls enough, the decrease in returns to money balances lowers income from the bank's overall portfolio and and consumption begins to decline. By comparison, if liquidity risk is sufficiently high, then banks would even more money balances and inherently, the amount of capital accumulation would be relatively low. In turn, the increase in capital formation promotes deposit formation and consumption over the entire range of $\mu \in [1, \mu_0)$ for similar reasons as when liquidity risk is relatively low and $\mu < \tilde{\mu}_0$.

However, when the return on excess reserves is below some level, $\mu \geq \mu_0$, all payments to non-relocated agents are made from the return to physical capital. Thus, if $\mu \in [\mu_0, \mu_1)$, a higher return on required reserves implies a higher return to non-movers which promotes capital accumulation. In order to satisfy the complete risk sharing contract, banks must hold more money balances. As we show in the appendix, both effects cancel out each other and monetary policy does not influence capital formation or consumption.

When the return to money balances is over the intermediate range $[\mu_1, \mu_2)$, the incentive compatibility constraint is relaxed and banks do not offer full insurance to depositors. Instead, under a lower return to money, banks hold a less liquid portfolio, which results in higher capital formation. The consumption of non-movers follows the increase in capital accumulation.

As in the effects of banking concentration described in Proposition 2, the effects of monetary policy on the consumption of movers depend on the extent of liquidity risk. If liquidity risk is low, the effects of the interest rate on reserves which stimulate asset substitution towards investment in physical capital

carry over to the consumption of movers since banks hold less cash balances. However, if liquidity risk is high, the effects of increased capital accumulation stimulate deposit formation so that banks hold more money balances. In turn, the consumption of movers increases.

Finally, banks' portfolios are constrained when the return to money is sufficiently low as they only hold required reserves. Thus, over the range $\mu \geq \mu_2$, monetary policy does not produce any real effects. *That is, changes in the interest rate on excess reserves do not affect investment or consumption when banks do not have any incentive to hold excess reserves.*

How do the effects of monetary policy depend on the degree of concentration in the financial sector? Does an increase in concentration render monetary transmission to be more or less effective in stimulating investment? A quick inspection of the effects of monetary policy over $\mu \in [\mu_1, \mu_2)$ indicates that:

$$\frac{dK^*}{d\mu} = \left[\frac{\frac{1-\pi}{\pi}}{\frac{1}{\left(1-\frac{1-\alpha}{N}\right)} + \frac{1-\pi}{\pi}} (1-\tau)(1-\alpha) A \right]^{\frac{1}{1-\alpha}} \frac{\rho}{1-\alpha} \left(1+\rho(\mu-1)\right)^{\frac{1}{1-\alpha}-1} > 0 \tag{30}$$

Clearly, the efficacy of monetary policy is strictly increasing with the degree of banking competition. As the degree of banking concentration rises, banks internalize that they have a lot of market power. As a result, the impact of monetary policy on economic activity is weaker.

In order to examine how the marginal effects of monetary policy on consumption vary with the degree of banking competition, we consider the following numerical example that corresponds to case where $\pi < \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$. In particular, suppose the parameter values are such that: $\alpha = .34$, $A = 10$, $\tau = 0.015$, $I_e = 1.02$, $\pi = .49$, and $\rho = .1$.

μ	$dc^m/d\mu$	$dc^n/d\mu$	$dc/d\mu$
1.40	-0.632	0.621	0.007
1.50	-0.648	0.618	-0.002
1.60	-0.663	0.615	-0.011
1.70	-0.678	0.612	-0.020
1.80	-0.692	0.610	-0.028
1.90	-0.707	0.607	-0.037
2.00	-0.722	0.604	-0.046

Table 1. Effects of Monetary Policy on Consumption, N=20

μ	$dc^m/d\mu$	$dc^n/d\mu$	$dc/d\mu$
1.40	-0.662	0.625	-0.006
1.50	-0.678	0.622	-0.015
1.60	-0.693	0.619	-0.024
1.70	-0.708	0.617	-0.033
1.80	-0.723	0.614	-0.041
1.90	-0.738	0.611	-0.050
2.00	-0.753	0.608	-0.059

Table 2. Effects of Monetary Policy on Consumption, N=100

As in the case of the capital stock, it can be observed that the effects of monetary policy on the amount of consumption by each type of agent are stronger when the banking system is more competitive. The effects for the aggregate amount of consumption are harder to interpret since monetary policy generates asymmetric effects on consumption across individuals.

5 Welfare Analysis

We proceed to examine how the degree of competition and monetary policy affect economic welfare. In particular, we study the interaction between optimal monetary policy and banking competition. Following previous work such as Williamson (1986) and Ghossoub and Reed (2010), we use the expected utility of a typical generation of depositors as a proxy for welfare. As we demonstrate in the appendix, the expected utility of a typical depositor in the steady-state can be expressed as:

$$U^* = \begin{cases} \pi \ln\left(\left[\frac{\rho+\frac{\tau}{1-\tau}}{\pi}\right](1-\tau)\,w\left(K^*\right)\right) + (1-\pi)\ln\frac{r(K^*)K^*}{(1-\pi)} & \text{if } \mu \geq \mu_2 \\ \pi \ln\left(\left[\frac{\gamma^*+\frac{\tau}{1-\tau}}{\pi}\right](1-\tau)\,w\left(K^*\right)\right) + (1-\pi)\ln\frac{r(K^*)K^*}{(1-\pi)} & \text{if } \mu \in (\mu_1, \mu_2) \\ \ln\left(\frac{r(K^*)K^*}{(1-\pi)}\right) & \text{if } \mu \in (\mu_0, \mu_1] \\ \ln\left(\frac{(1-\tau)\alpha A^2(1-\alpha)}{(\alpha A)^{\frac{1-2\alpha}{1-\alpha}}}\left(1+\rho\mu-\rho\right)\left(r\left(K^*\right)\right)^{\frac{1-2\alpha}{1-\alpha}}\right) & \text{if } \mu \in [1, \mu_0] \end{cases} \tag{31}$$

Our analysis begins by studying the optimal interest rate policy on reserves,

taking reserve requirements as given.[15] The following Proposition sheds some light on this issue:

Proposition 4.

i. Suppose $\pi < \frac{1-2\alpha}{1-\alpha}$ *and* $\rho < \hat{\rho}$. *Under these conditions,* $\mu^* = 1 + \frac{\tau}{1-\tau}\frac{1}{\rho}$, *with* $\mu^* \in [1, \mu_0]$.

ii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ *and* $\rho < \tilde{\rho} < \hat{\rho}$. *Under these conditions, we have a unique interior optimum,* $\mu^* \in (\mu_1, \mu_2)$, *where* μ^* *is decreasing in* N.

iii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ *and* $\rho \in (\tilde{\rho}, \hat{\rho})$. *Under these conditions,* μ^* *takes any value* $\in [\mu_2, \infty]$.

Interestingly, the optimal policy depends on the degree of liquidity risk in the economy. Figure 1 shows how welfare depends on the interest rate policy in the first case:

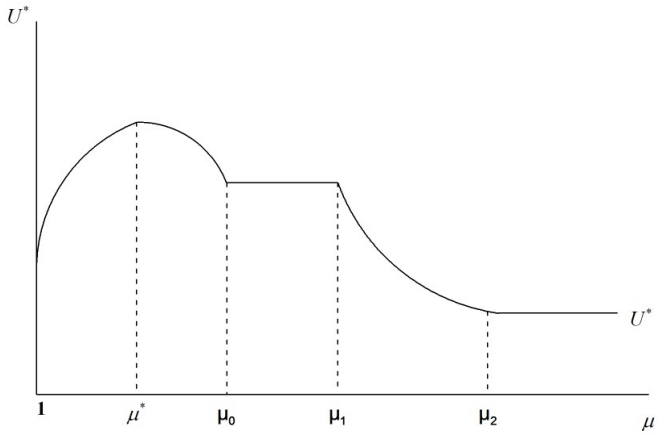

Figure 1. Welfare and Monetary Policy, $\pi < \frac{1-2\alpha}{1-\alpha}$ and $\rho < \widehat{\rho}$.

In particular, when the degree of liquidity risk is low ($\pi < \frac{1-2\alpha}{1-\alpha}$) and reserve requirements are not tight ($\rho < \hat{\rho}$), banks allocate a lot of resources towards capital formation. As a result, the return to capital is relatively low. Thus, there is little gain from lowering the value of money to promote capital investment relative to the loss in risk sharing if the relative return paid to excess reserves is already low. Therefore, welfare is decreasing in μ over the range, $\mu \in (\mu_1, \mu_2)$. Instead, as capital formation is high, it is welfare-improving to raise the value

[15] In our analysis, the frictions in the model are beyond the influence of the policymaker. Thus, the only tools available would be to implement an interest rate policy and structure the number of banks to maximize social welfare.

of money by increasing the return on excess reserves so that μ approaches μ_1 – in doing so, the return to capital would also rise.

Next, over the range $\mu \in [\mu_0, \mu_1]$, banks provide full insurance to depositors and the level of welfare is invariant to μ. As discussed following Proposition 3, at rates on reserves where $\mu \in [1, \mu_0]$, the return on money is so high that banks hold cash balances on behalf of non-movers in addition to movers. Over this range, there is a trade-off between capital formation and payments on reserves. At lower values of μ below μ_0, banks substitute away from investing in capital to holding more excess reserves. In fact, there is an optimal interest rate policy (μ^*) which balances the different rates of return to both assets (money and capital).

Interestingly, the optimal interest rate rule is increasing in the tax rate on wages but decreasing in the reserve requirement. At higher tax rates, the after-tax income of depositors is lower. Consequently, in order to maximize welfare, it is optimal to pay a relatively lower rate on excess reserves so that capital formation would be higher – and therefore, wages would be higher – in order to help offset the loss of income from higher taxes. It is also decreasing in the fraction of required reserves – if required reserves are higher, then the amount of excess reserves would tend to be lower. An increase in μ^* would raise the returns on required reserves and promote welfare.

We next turn to case (ii) where there is relatively more liquidity risk than in case (i). When the degree of liquidity risk is above a certain level, capital investment is low. In this case, the effects of monetary policy on welfare depend on the required reserves ratio. Further, from our discussion of Proposition 3, monetary policy only has real effects when $\mu \in (\mu_1, \mu_2)$. Notably, in such a setting, a lower return on excess reserves (higher value of μ) promotes capital formation and wages, which improves welfare. However, depositors also receive less insurance against relocation shocks which adversely affects their welfare. The optimal policy balances these effects. That is, there is an interior optimal policy on reserves when there is sufficient liquidity risk in the economy and reserve requirements are not particularly tight. As illustrated in Figure 2, $\mu^* \in (\mu_1, \mu_2)$:

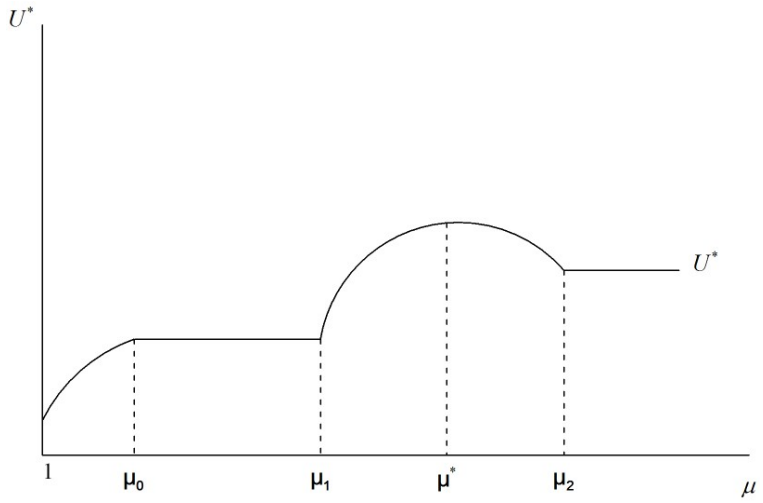

Figure 2. Welfare and Monetary Policy, $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho < \tilde{\rho} < \hat{\rho}$.

In addition, the optimal policy is inversely related to the degree of banking competition. Intuitively, when the banking system gets more concentrated, banks hold more liquid portfolios and provide more insurance against liquidity shocks. This raises the need to lower the value of money to promote capital formation and welfare.

When ρ is high, there are significant gains from promoting capital formation as the capital stock in the economy is low. Therefore, welfare is strictly increasing with μ, when $\mu \in (\mu_1, \mu_2)$. The optimal policy would be any value, $\mu^* \geq \mu_2$ as illustrated in Figure 3.

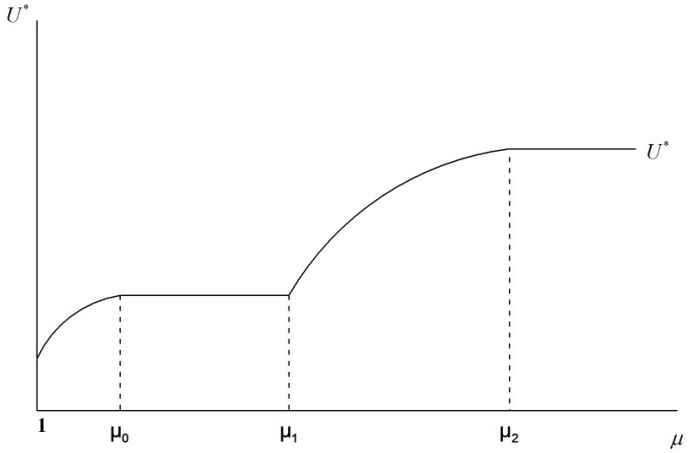

Figure 3. Welfare and Monetary Policy, $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho \in (\tilde{\rho}, \hat{\rho})$.

That is, if liquidity risk is high and reserve requirements are also relatively tight, the optimal policy attempts to remove incentives of banks to carry excess reserves. In turn, any interest rate policy where $\mu \geq \mu_2$ provides the same level of welfare since banks would only hold required reserves.

Does more competition in the banking sector promote welfare? What is the welfare maximizing degree of banking competition, N^*? The following Proposition sheds some light on these issues.

Proposition 5.

i. Suppose $\pi < \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho}$. Under this condition, N^* takes any value $\in (1, \underline{N}]$.

ii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho < \tilde{\rho} < \hat{\rho}$. Under these conditions, we have a unique interior optimum, $N^* \in \left(\underline{N}, \bar{N}\right)$, with $\frac{dN^*}{d\mu} < 0$.

iii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho \in (\tilde{\rho}, \hat{\rho})$. Under these conditions, N^* takes any value $\in \left[\bar{N}, \infty\right]$.

We begin by discussing case (i) where there is little liquidity risk. When the degree of liquidity risk is low, capital formation would tend to be high. Therefore, it is optimal to have a highly concentrated system as the gains in capital formation from more competition are small – this implies that the optimal size of the banking system is not any larger than $\underline{N}$. In order to gain additional insight into the optimal range for N, it is important to note that the conditions for case (i) in Proposition 2 can alternatively be written in terms of N rather than $\mu \in [1, \mu_1]$. Notably, we could instead write that the degree of concentration does not affect real outcomes for $N \in (1, \underline{N}]$ as depositors receive full insurance

against liquidity risk. Thus, any level of concentration along this range maximizes welfare. In particular, perfect competition under these conditions is not optimal – instead, a relatively concentrated banking system maximizes welfare.

However, when the degree of liquidity risk is high, the optimal degree of competition depends on the required reserves ratio. In particular, when banks are not required to hold much liquidity, too much competition can harm total welfare as the loss in insurance outweighs the gains in capital formation. From our discussion of Proposition 2, the degree of banking competition only matters for the real economy when $\mu \in (\mu_1, \mu_2)$, which can be written as a condition on N, with $N \in \left(\underline{N}, \bar{N}\right)$. Over this range, a more competitive banking system promotes capital formation. However, banks in a less concentrated system provide less insurance against liquidity risk, which adversely affects welfare. The optimal degree of banking competition balances these trade-offs. In fact, the optimal degree of concentration under case (ii) is inversely related to the relative return on money. Intuitively, when the return on excess reserves is low (high μ), banks provide little insurance to their risk averse depositors. Therefore, it is optimal to have a more concentrated banking system to promote risk sharing. As in case (i), again, perfect competition is not optimal. Yet, if banks are forced to hold a high amount of reserves, perfect competition can be optimal.

Given that the monetary authority has multiple tools at its discretion, we now turn to studying optimal reserve requirements in addition to the optimal interest rate policy. We examine this issue using numerical examples. The parameter values for the construction of the first two tables are: $\alpha = .34$, $A = 10$, $N = 20$, $\tau = 0.015$, and $I_e = 1.02$. In Table 3, $\pi = .4$, which corresponds to the case where $\pi < \frac{1-2\alpha}{1-\alpha}$. In this case, the Friedman rule is optimal. As shown in Table 3, there appears to be a continuum of optimal policies – the optimal

ρ^*	μ^*
0.02	1.7614
0.04	1.3807
0.06	1.2538
0.08	1.1904
0.10	1.1523
0.12	1.1269
0.14	1.1088
0.16	1.0952

Table 3.
Optimal Policy
Under $\pi = .4$

combination favors paying relatively higher interest on excess reserves as reserve requirements are tighter. When the degree of liquidity risk is low, there will

be relatively less individuals who will transact using money balances. As a result, the optimal interest rate policy favors paying low rates on reserves. If reserve requirements are not particularly tight, the optimal policy is to pay a relatively low interest rate on excess reserves since banks will tend to hold more cash on their balance sheets than required by the central bank. On the other hand, as reserve requirements increase, banks will hold more required reserves. As the optimal policy does not involve paying high rates on money balances, the optimal relative interest rate on required reserves falls. The degree of concentration does not play a role because banks provide full insurance to their depositors.

In contrast to the examples in Table 3, Table 4 looks at policy when there is more liquidity risk, $\pi = .49 > \frac{1-2\alpha}{1-\alpha}$. In this case, incomplete risk sharing is optimal but we continue to have a range of policies that maximizes social welfare:

ρ^*	μ^*
0.02	2.8643
0.04	1.9322
0.06	1.6215
0.08	1.4661
0.10	1.3729
0.12	1.3107
0.14	1.2663
0.16	1.2330

Table 4.
Optimal Policy
under $\pi = .49$
and $N = 20$

In comparison to the previous case, there will be more individuals who are exposed to liquidity shocks and derive income from the returns to money balances. Thus, the optimal policy is to pay relatively higher rates on reserves than in settings where there is little liquidity risk. As reserve requirements increase, the degree of risk-sharing improves and it is optimal to pay lower rates on required reserves.

Finally, Table 5 below looks at optimal policy when there is greater competition in the banking sector and $N = 100$:

ρ^*	μ^*
0.02	2.1775
0.04	1.5887
0.06	1.3925
0.08	1.2944
0.10	1.2355
0.12	1.1962
0.14	1.1682
0.16	1.1472

Table 5.
Optimal Policy
under $\pi = .49$
and $N = 100$

In this scenario, the strategic incentives of banks to withhold resources from the capital sector are weaker and there will be less incentive to hoard cash reserves. Thus, their holdings of excess reserves are not as high as when the banking sector is more concentrated. In order to encourage banks to hold more excess reserves, the relative interest rate on required reserves is lower – in other words, the optimal interest rate on excess reserves is higher when the banking sector is more competitive.

6 Endogenous Intermediation of Savings

In the previous section, we exogenously assumed that agents intermediate their savings. This approach follows nearly all of the existing literature using the Schreft-Smith framework – the only exception is Bencivenga and Smith (2003). In comparison to much of previous research, we extend our analysis to endogenously determine conditions under which agents participate in the financial system. In this manner, our work provides a unique contribution to the literature as we endogenize intermediation in economies where the monetary authority uses interest on reserves as its primary policy tool rather than the rate of money growth which is standard in prior contributions.

To begin, suppose that the banking sector is active but an individual agent considers the possibility of self-insuring against relocation shocks rather than depositing their funds at a bank. In such a case, the individual would lose the opportunity to obtain interest from cash reserves. Moreover, under financial autarky, any capital investment conducted by an agent is lost if she has to relocate to the other location.

We denote the amount of money balances and capital investment by the

individual as m_t^a and k_{t+1}^a, respectively. The agent's budget constraint is such that:

$$(1-\tau)\, w_t = m_t^a + k_{t+1}^a \tag{32}$$

An agent's consumption if relocated stems from the amount of cash balances on hand:

$$c_{t+1}^m = m_t^a \frac{P_t}{P_{t+1}} \tag{33}$$

In addition, the consumption of a non-relocated agent comes from the returns on cash reserves and physical capital:

$$c_{t+1}^n = m_t^a \frac{P_t}{P_{t+1}} + r_{t+1} k_{t+1}^a \tag{34}$$

Obviously, the individual would make their portfolio choice to maximize their expected utility. Consequently, the problem facing a typical agent under autarky is summarized by:

$$\underset{c_{t+1}^m, c_{t+1}^n, m_t^a, k_{t+1}^a}{Max} \pi \ln c_{t+1}^m + (1-\pi) \ln c_{t+1}^n \tag{35}$$

subject to $(32) - (34)$. From $(32) - (34)$ into (35):

$$\underset{m_t^a}{Max}\ \pi \ln \left(m_t^a \frac{P_t}{P_{t+1}} \right) + (1-\pi) \ln \left(m_t^a \frac{P_t}{P_{t+1}} + r_{t+1} \left((1-\tau)\, w_t - m_t^a \right) \right) \tag{36}$$

The solution to the problem yields the fraction of savings allocated towards cash balances:

$$\gamma_t^a = \frac{\pi}{\left(1 - \frac{1}{I_t}\right)} \tag{37}$$

Next, substituting (33) - (35) and (37) into (36), the expected utility of a potential depositor under self-insurance is:

$$u_t^a = \pi \ln \frac{\pi}{(I_t - 1)} + (1-\pi) \ln \left[1 - \pi\right] + \ln (1-\tau)\, w_t + \ln I_t \frac{P_t}{P_{t+1}} \tag{38}$$

In this manner, for a given nominal return to capital and wages, agents participate in the banking system if:

$$u_t^b > u_t^a \tag{39}$$

where u_t^b is the expected utility from intermediating savings.

Since the solution to the bank's problem and therefore the return on deposits depends on the relative return on cash reserves, it is imperative to evaluate

the participation constraint relative to all four possibilities discussed in the previous sections. First, from (7)−(10), it is easily verified that the consumption of each type of depositor under returns offered by the bank under cases 1-3 listed in Proposition 1 is: $c_{t+1}^m = \frac{1}{\pi}\left(\gamma_{R,t} I_R + \gamma_{e,t} I_e\right) \frac{P_t}{P_{t+1}} (1-\tau) w_t$ and $c_{t+1}^n = \frac{1}{1-\pi} R_{t+1} \left(1-\gamma_t^b\right)(1-\tau) w_t$.

As we demonstrate in the appendix, the net gain from a depositor's participation in the banking system in the steady-state is such that:

$$u^b - u^a = \begin{cases} \ln\left[\frac{\rho I_R}{\pi^2}\left(\frac{I-1}{I}\right)\right]^{\pi} \frac{(1-\rho)^{1-\pi}}{(1-\pi)^{2(1-\pi)}} & \text{if } \mu \geq \mu_2 \\ \ln \frac{1}{\pi^{2\pi}} \left(\rho\left(I_R - I_e\right) + \gamma^b I_e\right)^{\pi} \left(1-\frac{1}{I}\right)^{\pi} \frac{\left(1-\gamma^b\right)^{(1-\pi)}}{(1-\pi)^{2(1-\pi)}} & \text{if } \mu \in [\mu_1, \mu_2) \\ \ln \frac{\left(\frac{I_e-(I_e-I_R)\rho}{(1-\pi)I_e+\pi I}\right)}{\frac{\pi^{\pi}}{(I-1)^{\pi}}[1-\pi]^{1-\pi}} & \text{if } \mu \in (\mu_0, \mu_1] \\ \ln \frac{[1+\rho(\mu-1)](I_e-1)^{\pi}}{\pi^{\pi}[1-\pi]^{1-\pi}} & \text{if } \mu \in [1, \mu_0] \end{cases} \tag{40}$$

Moreover, the following Proposition provides conditions in which agents will participate in the financial system:

Proposition 6.

i. Suppose $\mu \in [1, \mu_0]$. *Under this condition,* $u^b - u^a > 0$ if $I_e > 1 + \pi\left[1-\pi\right]^{\frac{1-\pi}{\pi}}$

ii. Suppose $\mu \in (\mu_0, \mu_1]$. *Under this condition,* $u^b - u^a > 0$ if $I_e > 1 + \left[\frac{(1-\pi)\frac{(1-\tau)(1-\alpha)}{\alpha}\pi^{\pi}[1-\pi]^{1-\pi}}{\left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right)}\right]^{\frac{1}{\pi}}$

iii. Suppose $\mu \in [\mu_1, \mu_2)$. *Under this condition,* $u^b - u^a > 0$ if $I_e > \left[1-\frac{1-\alpha}{N}\right] + \pi^2 \left[\frac{\pi(1-\pi)}{\left[1-\frac{1-\alpha}{N}\right]}\right]^{\frac{1-\pi}{\pi}} \left(\frac{\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{[\rho(\mu_1-1)+1]}\right)^{\frac{1}{\pi}}$

iv. Suppose $\mu \geq \mu_2$. *Under this condition,* $u_t^b - u_t^a > 0$ if $I_R > \left(\frac{\left(\rho+\frac{\tau}{1-\tau}\right)}{\rho}\frac{(1-\rho)(1-\tau)(1-\alpha)}{\alpha} + \frac{\pi^2}{\rho}\frac{(1-\pi)^2\frac{(1-\pi)}{\pi}}{(1-\rho)^{\frac{1-\pi}{\pi}}}\right)$

It is clear from (40) that the decision of an agent to intermediate her savings depends on the returns to reserves. For instance, when $\mu \in [\mu_1, \mu_2)$, banks do not offer complete risk-sharing, but they do hold excess reserves. Given that agents do not receive interest on their money holdings under autarky, they would intermediate their savings if the return on reserves is high enough.

7 Conclusions

In recent years, the increased concentration of activity in the banking system has received much attention. However, such consolidation is part of a long-term

pattern of consolidation activity. For example, Janicki and Prescott (2006) present evidence that the largest banks which made up less than 1% of active institutions in the United States held over 75% of assets in the banking system prior to the crisis. With this large of a role in economic activity, it is hard to believe that these institutions would not take advantage of their market power in the financial system. Moreover, institutions grew to be so large that they were deemed "too big to fail" during the crisis. Banking regulators and monetary policy authorities must confront the challenges associated with an increasing concentration of activity in the banking sector.

Our analysis demonstrates that banks in more concentrated systems have a tendency to hoard cash reserves in order to exploit their strategic advantages in the market for capital. However, Bencivenga and Smith (1991) argue that one of the key roles of intermediaries is to facilitate risk-sharing and promote investment activity. In stark contrast, concentrated banking systems deviate from this function. Yet, that does not mean that perfect competition should be a regulatory goal. In some settings, depositors would benefit from consolidation. Nevertheless, the model also demonstrates that the optimal rate on excess reserves is generally lower as the banking sector consolidates due to the tendency of concentrated banks to distort the amount of capital investment. In this manner, we view that our work contributes to recent research that examines the implications of the industrial organization of the banking system. It also adds to the emerging literature which studies the evolution of monetary policy tools that has emerged out of the global financial crisis.[16]

There is important work that remains. In particular, the model is based on fixed entry which limits the ability to study the impact of rules restricting bank entry and other regulations on banking activity. Notably, Beck et al. (2004) point out that lower barriers to entry and efforts to promote competition have an impact on banking fragility even when controlling for the degree of concentration in the banking sector. That is, they affect outcomes in the banking sector above their role for the number of banks. Moreover, Adams and Gramlich (2014) argue that weak economic conditions have played a major role in the lack of entry in the U.S. banking system in recent times. In this regard, the model could be extended to allow for endogenous entry which would further give us the ability to address the variety of regulatory and institutional changes that have occurred across countries since the end of the financial crisis. Interestingly, the impact of interest on reserves and reserve requirements on the equilibrium number of banks could be studied. As a result, the effects of monetary policy would depend on the (endogenous) degree of concentration, banking regulations, and economic conditions such as income and the degree of liquidity risk.

[16] Tarullo (2011) calls for further research into the industrial organization of the financial system.

References

Adams, R.M. and J.R. Gramlich, 2014. Where Are All the New Banks? The Role of Regulatory Burden in New Charter Creation. Finance and Economics Discussion Series: Federal Reserve Board, Working Paper #2014-113.

Ahmed, S., and J.H. Rogers, 2000. Inflation and the Great Ratios: Long Term Evidence from the U.S. Journal of Monetary Economics 45, 3-35

Allen, F. and Gale, D., 2000. *Comparing Financial Systems* (Cambridge, MA: MIT Press).

Antinolfi, G., E. Huybens, and T. Keister, 2001. Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort. Journal of Economic Theory 99, 187-219.

Bank for International Settlements, 2001. Report on Consolidation in the Financial Sector. January.

Barth, J.R., G. Caprio, and R. Levine, 2013. Bank Regulation and Supervision in 180 Countries from 1999 to 2011. NBER Working Paper #18733.

Beck, T., A. Demirguc-Kunt, and V. Maksimovic, 2004. Bank Competition, Financing and Access to Credit. Journal of Money, Credit and Banking 36(3), 627-648.

_____, _____, and _____, 2006. Bank Concentration, Competition, and Crises: First Results. Journal of Banking and Finance 30, 1581-1603.

Bencivenga, V. and B.D. Smith, 1991. Financial Intermediation and Endogenous Growth. Review of Economic Studies 58, 195-209.

_____ and _____, 2003. Monetary Policy and Financial Market Evolution. Federal Reserve Bank of St. Louis *Review*, July/August, 7-26.

Berger, A.N. and T.H. Hannan, 1991. The Rigidity of Prices: Evidence from the Banking Industry. American Economic Reveiw 81, 938-945.

Boyd, J., G. De Nicolo, and B.D. Smith, 2004. Crises in Competitive versus Monopolistic Banking Systems. Journal of Money, Credit and Banking 36, 487-506.

Boyd, J., De Nicolo, G., 2005. The Theory of Bank Risk Taking and Competition Revisited. Journal of Finance 60, 1329-1343.

Cavalcanti, R. and N. Wallace, 1999a. A Model of Private Bank-note Issue. Review of Economic Dynamics 2, 104-36.

_____ and _____, 1999b. Inside and Outside Money as Alternative Media of Exchange 31, 443-457.

Cechetti, S., 1999. Legal Structure, Financial Structure, and the Monetary Policy Transmission Mechanism. Federal Reserve Bank of New York Economic Policy Review, 9-28.

Cetorelli, N., 2004. Real Effects of Bank Competition. Journal of Money, Credit and Banking 36, 543-558.

Cetorelli, N. and P. Peretto, 2012. Credit Quantity and Credit Quality: Bank Competition and Capital Accumulation. Journal of Economic Theory 147, 967-998.

Champ, B., B.D. Smith, and S.D. Williamson, 1996. Currency Elasticity and Banking Panics: Theory and Evidence. Canadian Journal of Economics, 828-864.

Corvosier, S. and R. Gropp, 2002. Bank Concentration and Retail Interest Rates. Journal of Banking and Finance 26, 2155-2189.

Diamond, D. and P. Dybvig, 1983. Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 85, 191-206.

Financial Services Authority, 2013. A Review of Requirements for Firms Entering Into or Expanding in the Banking Sector. March.

Fisher, R.W. and H. Rosenblum., 2009. 'Too Big To Fail' Weakened Monetary Policy. Wall Street Journal, September 27.

Gandel, S. 2013. By Every Measure, the Big Banks are Bigger. CNN Money. September 13.

Gerali, A., S. Neri, and F.M. Signoretti, 2010. Credit and Banking in a DSGE Model of the Euro Area. Journal of Money, Credit and Banking 42, 107-141.

Ghossoub, E. and R. Reed, 2010. Liquidity Risk, Economic Development, and the Effects of Monetary Policy. European Economic Review 54, 252-268.

Ghossoub, E., 2012. Liquidity Risk and Financial Competition: Implications for Asset Prices and Monetary Policy. European Economic Review 56, 155-173.

Ghossoub, E., T. Laosuthi, and R. Reed, 2012. The Role of Financial Sector Competition for Monetary Policy. Canadian Journal of Economics 45, 270-287.

_ _ _ _ _ and R. Reed, 2015. The Size Distribution of the Banking Sector and the Effects of Monetary Policy. European Economic Review 75, 156-176.

_ _ _ _ _ and _ _ _ _ _, 2017. Banking Competition, Production Externalities, and the Effects of Monetary Policy. Forthcoming, Economic Theory.

Gongloff, M., J. Diehm, and K. Hall., 2013. After the Crisis, Big Banks are Bigger Than Ever. Huffington Post. November 19, 2013.

Hannan, T.H., 1991. Bank Commercial Loan Markets and the Role of Market Structure: Evidence From Surveys of Commercial Lending, Journal of Banking and Finance 15, 133-149.

Janicki, H.P. and E.S. Prescott. 2006. Changes in the Size Distribution of U.S. Banks: 1960-2005. Federal Reserve Bank of Richmond Economic Quarterly 92, 291-316.

Jimenez, G., S. Ongena, J. Peydro, and J. Saurina, 2012. Credit Supply and Monetary Policy: Identifying the Bank Balance Sheet Channel with Loan Applications. American Economic Review 102, 2301-2326.

Kashyap, A. and J. Stein, 1997. The Role of Banks in Monetary Policy: A Survey with Implications for the European Monetary Union. Federal Reserve Bank of Chicago Economic Perspectives September - October, 2-18.

Lagos, R. and R. Wright, 2005. A Unified Framework for Monetary Theory and Policy Analysis. Journal of Political Economy 113, 463-84.

Mandelman, F., 2011. Business Cycles and the Role of Imperfect Competition in the Banking System. International Finance 14, 103-133.

Matsuoka, T., 2011. Monetary Policy and Banking Structure. Journal of Money, Credit and Banking 43, 1109-1129.

McCord, R. and E.S. Prescott, 2014. The Financial Crisis, the Collapse of Bank Entry, and Changes in the Size Distribution of Banks. Economic Quarterly 100, 23-50.

Monnet, C. and D.R. Sanches, 2015. Private Money and Banking Regulation. Journal of Money, Credit and Banking 47, 1031-1062.

Olivero, M., 2010. Market Power in Banking, Countercyclical Margins and the International Transmission of Business Cycles. Journal of International Economics 80, 292-301.

Paal, B., B. Smith, and K. Wang, 2005. Monopoly versus Competition in Banking: Some Implications for Growth and Welfare. Mimeo, University of Tokyo.

Peersman, G., 2004. The Transmission of Monetary Policy in the Euro Area: Are the Effects Different Across Countries? Oxford Bulletin of Economics and Statistics 66, 285-308.

Schreft, S.L. and B.D. Smith, 1997. Money, Banking, and Capital Formation. Journal of Economic Theory 73, 157-182.

Schreft, S. and B.D. Smith, 1998. The Effects of Open Market Operations in a Model of Intermediation and Growth. Review of Economic Studies 65, 519-50.

Tarullo, D.K., 2011. Industrial Organization and Systemic Risk: An Agenda for Further Research. Remarks at the Conference on Regulating Systemic Risk. September 15, 2011.

Williamson, S.D., 1986. Increasing Returns to Scale in Financial Intermediation and the Non-Neutrality of Government Policy. Review of Economic Studies 53, 863-875.

Technical Appendix

1. A typical bank's problem. To begin, we solve the bank's problem when the incentive compatibility constraint is relaxed. Upon using (7) – (10), along with (5) into the objective function and some algebra, we get:

$$\underset{k_{t+1}}{Max}\pi \ln\left[R_{e,t}\left(\frac{1}{N}(1-\tau)w_t - k_{t+1}\right) - (R_{e,t}-R_{R,t})m_{R,t}\right] + (1-\pi)\ln\left[r\left(k_{t+1}, K_{t+1}^{N-1}\right)k_{t+1}\right] + \pi\ln\frac{N}{\pi} + (1-\pi)\ln\frac{N}{1-\pi}$$

It is easily verified that the first order condition yields (12) in the text:

$$-\frac{\pi R_{e,t}}{m_{R,t}R_{R,t}+m_{e,t}R_{e,t}} + (1-\pi)\frac{\left[\frac{\partial r\left(k_{t+1},K_{t+1}^{N-1}\right)}{\partial k_{t+1}}k_{t+1} + r\left(k_{t+1},K_{t+1}^{N-1}\right)\right]}{r\left(k_{t+1},K_{t+1}^{N-1}\right)k_{t+1}} = 0$$

where $m_{R,t}R_{R,t} + m_{e,t}R_{e,t} = R_{e,t}\left(\frac{1}{N}(1-\tau)w_t - k_{t+1}\right) - (R_{e,t}-R_{R,t})m_{R,t}$ given that $m_t = m_{e,t} + m_{R,t}$ and $\frac{1}{N}(1-\tau)w_t - k_{t+1} = m_t$. Upon using (5), the first order condition can be written as:

$$R_{e,t}k_{t+1} = \frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right][m_{R,t}R_{R,t}+m_{e,t}R_{e,t}]$$

which is (13) in the text. Finally, using the bank's balance sheet condition, (7), where $k_{t+1} = \frac{1}{N}(1-\tau)w_t - m_{R,t} - m_{e,t}$ into (13) with some algebra to get:

$$m_{e,t} = \frac{\frac{1}{N}(1-\tau)w_t - \left(1+\frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]\frac{R_{R,t}}{R_{e,t}}\right)m_{R,t}}{1+\frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]}$$

Given that $m_{R,t} = \gamma_{R,t}\frac{1}{N}(1-\tau)w_t$, with $\gamma_{R,t} = \rho$, it directly follows that $\gamma_{e,t} = \frac{m_{e,t}}{\frac{1}{N}(1-\tau)w_t}$ is:

$$\gamma_{e,t} = \frac{1-\left(1+\frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]\frac{R_{R,t}}{R_{e,t}}\right)\gamma_{R,t}}{1+\frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]}$$

In this manner, $\gamma_{e,t} \geq 0$ if:

$$\frac{R_{R,t}}{R_{e,t}} \leq \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}}$$

where $\frac{R_{R,t}}{R_{e,t}} = \frac{I_R \frac{P_t}{P_{t+1}}}{I_e \frac{P_t}{P_{t+1}}} = \mu$. Therefore, banks hold excess reserves as long as:

$$\mu < \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}} = \mu_2$$

Next, the equilibrium amount of money is such that:

$$\gamma_t = \gamma_{e,t} + \gamma_{R,t}$$

where $\gamma_{R,t} = \rho$. Upon using the expression for $\gamma_{e,t}$ above, we get:

$$\gamma_t = \frac{1-\rho\frac{1-\pi}{\pi}\left[(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]\left(\frac{I_R}{I_e}-1\right)}{1+\frac{1-\pi}{\pi}\left[(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]}$$

which is (20) in the text. Next, the bank's balance sheet condition, $k_{t+1} = \frac{1}{N}(1-\tau)w_t - m_t$ can be equivalently stated as:

$$\frac{N}{(1-\tau)}\frac{k_{t+1}}{w_t} = 1-\gamma_t$$

$$\frac{N}{(1-\tau)}\frac{k_{t+1}}{w_t} = 1 - \frac{1-\rho\frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]\left(\frac{I_R}{I_e}-1\right)}{1+\frac{1-\pi}{\pi}\left[1-(1-\alpha)*\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]} \tag{41}$$

Next, using (8) and (10), the relative return to depositors is such that:

$$\frac{\frac{1-\pi}{N}r_t^n(1-\tau)w_t}{\frac{\pi}{N}r_t^m(1-\tau)w_t} = \frac{r\left(k_{t+1},K_{t+1}^{N-1}\right)k_{t+1}}{m_{R,t}R_{R,t}+m_{e,t}R_{e,t}} \tag{42}$$

$$\frac{r_t^n(1-\tau)w_t}{r_t^m(1-\tau)w_t} = \frac{\pi}{1-\pi}\frac{k_{t+1}}{m_{R,t}R_{R,t}+m_{e,t}R_{e,t}}r\left(k_{t+1},K_{t+1}^{N-1}\right) \tag{43}$$

Using the definition of γ_t, along with the bank's balance sheet, (7) to get:

$$\frac{r_t^n}{r_t^m} = \frac{\pi}{1-\pi}\frac{1-\gamma_t}{\rho I_R+\gamma_{e,t}I_e}r\left(k_{t+1},K_{t+1}^{N-1}\right)\frac{P_{t+1}}{P_t} \tag{44}$$

or equivalently:

$$\frac{r_t^n}{r_t^m} = \frac{\pi}{1-\pi}\frac{1-\gamma_t}{\rho I_R+\gamma_{e,t}I_e}I_t \tag{45}$$

where $I_t = r\left(k_{t+1},K_{t+1}^{N-1}\right)\frac{P_{t+1}}{P_t}$. Next, using the fact that $\gamma_t = \gamma_{e,t}+\gamma_{R,t}$, the relative return to depositors can be expressed as:

$$\frac{r_t^n}{r_t^m} = \frac{\pi}{1-\pi} \frac{1-\gamma_t}{\gamma_t + \rho\left(\frac{I_R}{I_e} - 1\right)} \frac{I_t}{I_e} \tag{46}$$

Finally, we substitute for γ_t from (20) along with some simplification to get:

$$\frac{r_t^n}{r_t^m} = \left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}}\right] \frac{I_t}{I_e}$$

which is equation (21) in the text. Clearly, $\frac{r_t^n}{r_t^m} > 1$ if $I_t > \frac{I_e}{1-(1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}} > I_e$.

Next, suppose $\mu \geq \mu_2$. As discussed above, under this condition, $\gamma_{e,t} = 0$ and $\gamma_t = \rho$. Therefore, from the bank's balance sheet, (7) and the definition of γ_t, the fraction of deposits allocated towards capital is such that:

$$\frac{k_{t+1}}{\frac{1}{N}(1-\tau)w_t} = 1 - \rho \tag{47}$$

Moreover, from (45) and the fact that $\gamma_t = \rho$, the relative return to depositors is such that:

$$\frac{r_t^n}{r_t^m} = \frac{\pi}{1-\pi} \frac{(1-\rho)}{\rho} \frac{I_t}{I_R} \tag{48}$$

Subsequently, suppose $I_t \in \left(I_e, I_e\left[1 - (1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}\right]^{-1}\right]$. In this case, the bank's incentive compatibility constraint is binding, with $\frac{r_t^n}{r_t^m} = 1$ and excess reserves are dominated in rate of return. From (45) and the fact that $\frac{r_t^n}{r_t^m} = 1$, it is easy to verify that:

$$\gamma_t = \frac{\frac{\pi}{1-\pi} I_t - \rho\left(I_R - I_e\right)}{\left(I_e + \frac{\pi}{1-\pi} I_t\right)}$$

which is (22) in the text. In addition:

$$\gamma_{e,t} = \gamma_t - \rho = \frac{\frac{\pi}{1-\pi}(1-\rho) I_t - \rho I_R}{\left(I_e + \frac{\pi}{1-\pi} I_t\right)} \tag{49}$$

Moreover, $I_t = r\left(k_{t+1}, K_{t+1}^{N-1}\right)\frac{P_{t+1}}{P_t}$, where

$$\frac{P_{t+1}}{P_t} = \frac{\rho I_R + \gamma_{e,t} I_e}{\gamma_t + \frac{\tau}{1-\tau}}$$

Using the expressions for $\gamma_{e,t}$ and γ_t:

$$\frac{P_{t+1}}{P_t} = \frac{\left(\rho I_R + \frac{\frac{\pi}{1-\pi}(1-\rho)I_t - \rho I_R}{\left(I_e + \frac{\pi}{1-\pi}I_t\right)} I_e\right)}{\frac{\frac{\pi}{1-\pi}I_t + (I_e - I_R)\rho}{\left(I_e + \frac{\pi}{1-\pi}I_t\right)} + \frac{\tau}{1-\tau}}$$

which can be written as:

$$\frac{P_{t+1}}{P_t} = \frac{\rho I_R + (1-\rho) I_e}{\frac{1}{1-\tau}I_t + \frac{1-\pi}{\pi}\left[\left(\rho + \frac{\tau}{1-\tau}\right) I_e - \rho I_R\right]} I_t$$

Plugging this information into the expression for the nominal return to capital, $I_t = r\left(k_{t+1}, K_{t+1}^{N-1}\right)\frac{P_{t+1}}{P_t}$:

$$I_t = (1-\tau)\left[\rho I_R + (1-\rho) I_e\right] r\left(k_{t+1}, K_{t+1}^{N-1}\right) - (1-\tau)\frac{1-\pi}{\pi}\left[\left(\rho + \frac{\tau}{1-\tau}\right) I_e - \rho I_R\right] \tag{50}$$

2. Proof of Proposition 1. We begin by providing the conditions for case (iii). Notably, suppose that $\frac{r_t^n}{r_t^m} > 1$ and $\gamma_t^* > \rho$. First, from the derivation of the bank's problem, we showed that $\gamma^* > \rho$ if $\mu < \mu_2$. Next, we established that $\frac{r_t^n}{r_t^m} > 1$ if $I_t > \frac{I_e}{1-(1-\alpha)\frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}}}$. In a symmetric steady-state Nash equilibrium, this condition can be written as:

$$I > \frac{I_e}{1 - \frac{1-\alpha}{N}}$$

where

$$I = r\frac{P_{t+1}}{P_t}$$

and $\frac{P_{t+1}}{P_t}$ is given by (27) and $r = \alpha A K^{\alpha-1}$ under the functional form for the production technology. Using the expressions for γ_e and γ, from (15) and (20), along with some algebra:

$$\frac{P_{t+1}}{P_t} = \frac{\rho I_R + (1-\rho) I_e}{1 - \rho\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\left(\frac{I_R}{I_e} - 1\right) + \frac{\tau}{1-\tau}\left(1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\right)} \tag{51}$$

In addition, imposing symmetry and steady-state on (41):

$$\frac{1}{(1-\tau)}\frac{K}{w} = 1 - \frac{1 - \rho\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right](\mu - 1)}{1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]} \tag{52}$$

where $\frac{K}{w} = \frac{K^{1-\alpha}}{(1-\alpha)A} = \frac{\alpha}{(1-\alpha)r}$. Substituting this information into (52) :

$$r = \frac{1}{(1-\tau)}\frac{\alpha}{(1-\alpha)}\frac{1}{1-\frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}} \tag{53}$$

Equivalently:

$$K^{1-\alpha} = (1-\tau)(1-\alpha)A\left(1-\frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}\right) \tag{54}$$

which can be written as:

$$K^{1-\alpha} = (1-\tau)(1-\alpha)A\left(\frac{\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right][1+\rho(\mu-1)]}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}\right)$$

which is the expression found in the text.

Using (51) and (53) and the definition of I:

$$I = \frac{\alpha}{\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](1-\tau)(1-\alpha)}\frac{1}{\frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}+\frac{\tau}{1-\tau}}I_e \tag{55}$$

In this manner, $I > \frac{I_e}{1-\frac{1-\alpha}{N}}$ if:

$$\frac{\alpha}{\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](1-\tau)(1-\alpha)}\frac{1}{\frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}+\frac{\tau}{1-\tau}}I_e > \frac{I_e}{1-\frac{1-\alpha}{N}} \tag{56}$$

With some simplification, (56) can be written as:

$$\mu > 1+\frac{1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]} = \mu_1$$

where $\mu_2 > \mu_1$ if:

$$\frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1-\frac{1-\alpha}{N}} > 1+\frac{1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}$$

which can be reduced to:

$$\rho < \frac{1}{1-\tau}\left(\frac{\alpha}{1-\alpha}\frac{\pi}{1-\pi}-\tau\right) = \hat{\rho}$$

This provides the proof of case (iii) in the Proposition.

Next, we move to case (ii). Suppose $\mu \leq \mu_1$, where $r^n = r^m$. We proceed to find conditions under which $I > I_e$. From (50) above, in the steady-state:

$$I = (1-\tau)\left[\rho I_R + (1-\rho)\, I_e\right] r - (1-\tau)\,\frac{1-\pi}{\pi}\left[\left(\rho + \frac{\tau}{1-\tau}\right) I_e - \rho I_R\right] \quad (57)$$

Using (7) and (22), with some simplification, we have:

$$\frac{K}{(1-\tau)\, w} = \frac{(1-\rho)\, I_e + \rho I_R}{\left(I_e + \frac{\pi}{1-\pi} I\right)} \quad (58)$$

From our work above, $\frac{K}{w} = \frac{1}{(1-\alpha)AK^{\alpha-1}} = \frac{\alpha}{(1-\alpha)r}$. Plugging this information into (58) :

$$I = \frac{1-\pi}{\pi}\frac{(1-\rho)\, I_e + \rho I_R}{\frac{\alpha}{(1-\tau)(1-\alpha)r}} - \frac{1-\pi}{\pi} I_e \quad (59)$$

Setting (50) = (59) along with some algebra:

$$r = \left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right)^{-1} \quad (60)$$

or equivalently:

$$K^{1-\alpha} = \alpha A \left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right) \quad (61)$$

where $\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi} > 0$ is assumed to hold.

In regards to the nominal return to capital, using (60) into (59) we get:

$$I = \frac{1-\pi}{\pi}\frac{(1-\tau)\,(1-\alpha)}{\alpha}\frac{(1-\rho)\, I_e + \rho I_R}{\left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right)} - \frac{1-\pi}{\pi} I_e \quad (62)$$

From (62), it is trivial to show that $I > I_e$ if:

$$\mu > \frac{\alpha\left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right)}{(1-\pi)\,(1-\tau)\,(1-\alpha)\,\rho} - \frac{(1-\rho)}{\rho} = \mu_0$$

Next, $\mu_0 \leq \mu_1$ if

$$\frac{\alpha\left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right)}{(1-\pi)\,(1-\tau)\,(1-\alpha)\,\rho} - \frac{(1-\rho)}{\rho} \leq 1 + \frac{1 - \left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)} - \frac{\tau}{1-\tau}\right)\left[1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\right]}{\rho\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]}$$

With a few lines of algebra, this condition can be written as:

$$1 - \frac{1-\alpha}{N} \leq 1$$

which always holds as $N > 1$. This completes the proof for case (ii).

We continue by moving on to case (i). Clearly, at $\mu = \mu_0$, $I = I_e$. In addition, $\mu_0 \geq 1$ if:

$$\tau \geq \frac{\frac{\alpha}{(1-\alpha)}\frac{\pi}{1-\pi} - \pi}{(1-\pi)}$$

We proceed to get an expression for the real return to capital. From (25), in the steady-state, the Friedman rule implies that:

$$r\frac{P_{t+1}}{P_t} = I_e \tag{63}$$

Using the expression for inflation from (27) :

$$\frac{P_{t+1}}{P_t} = \frac{(\rho I_R + \gamma_e I_e)}{\gamma + \frac{\tau}{1-\tau}}$$

Upon substituting into (63):

$$\gamma = \frac{\rho(\mu - 1) - \frac{1}{r}\frac{\tau}{1-\tau}}{\left(\frac{1}{r} - 1\right)} \tag{64}$$

Moreover, from the bank's balance sheet, (7):

$$\gamma = 1 - \frac{\alpha}{(1-\alpha)(1-\tau)r} \tag{65}$$

The equilibrium value(s) of r at the Friedman rule solve the system of equations given by (64) and (65).

We begin by characterizing the (65) locus. The locus (65) satisfies the following: $\gamma = 0$ when $r = r_0 = \frac{\alpha}{(1-\alpha)(1-\tau)}$ and $\frac{d\gamma}{dr} > 0$. Moreover, $\lim_{r\to\infty} \gamma \to 1$.

Next, we characterize the (64) locus. The sign of γ depends on the relationship between r and $\hat{r} = \frac{\frac{\tau}{1-\tau}}{\rho(\mu-1)}$. On one hand, $\hat{r}$ may be larger than 1. Under this case, if $r \in (1, \hat{r})$, then $\gamma > 0$. Alternatively, $\hat{r}$ may be less than 1. In this case, $\gamma > 0$ if $r \in (\hat{r}, 1)$.

Note that for $r < 1 < \hat{r}$, $\gamma < 0$. If $\hat{r} > 1$, it must be that in an equilibrium $r \in (1, \hat{r})$. Moreover, $\lim_{r\to 1}\gamma \to \infty$ and $\frac{d\gamma}{dr} < 0$. In addition, $\gamma(\hat{r}) = 0$. By comparison, if $\hat{r} < 1$, it must be that in equilibrium, $r < 1$. For $r \in (\hat{r}, 1)$, $\gamma > 0$, $\frac{d\gamma}{dr} > 0$, $\lim_{r\to 1}\gamma \to \infty$ and $\gamma(\hat{r}) = 0$.

Given the characterization of each locus, a unique intersection between both loci occurs when $r_0 < \hat{r}$ (we can get up to two intersections if $\hat{r} < r_0$). Clearly, $r_0 \leq \hat{r}$ if:

$$\frac{\alpha}{(1-\alpha)(1-\tau)} \leq \frac{\frac{\tau}{1-\tau}}{\rho(\mu-1)}$$

which can be written as:

$$\mu \leq 1 + \frac{\tau}{\rho}\frac{1-\alpha}{\alpha} = \tilde{\mu}_1 \tag{66}$$

Moreover, $\hat{r} > 1$ if:

$$\mu < 1 + \frac{\tau}{\rho\left(1-\tau\right)} = \tilde{\mu}_0 \tag{67}$$

Next, combining, (64) and (65) with some algebra, the following polynomial yields the values of r:

$$r^2 + \frac{1}{\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]}\frac{\alpha}{\left(1-\alpha\right)} - \frac{1}{\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]}\frac{1}{1-\alpha}r = 0$$

which has two roots:

$$r_1 = \frac{1}{\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]}\frac{1}{1-\alpha}\frac{1+\sqrt{1-4\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]\left(1-\alpha\right)\alpha}}{2} \tag{68}$$

$$r_2 = \frac{1}{\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]}\frac{1}{1-\alpha}\frac{1-\sqrt{1-4\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]\left(1-\alpha\right)\alpha}}{2} \tag{69}$$

The, discriminant under the square root is positive if:

$$\mu < \left(\frac{1}{\left(1-\alpha\right)\alpha 4\left(1-\tau\right)} - 1\right)\frac{1}{\rho} + 1 = \tilde{\mu}_2$$

where $\tilde{\mu}_2 > \mu_0$ if:

$$\left(\frac{1}{\left(1-\alpha\right)\alpha 4\left(1-\tau\right)} - 1\right)\frac{1}{\rho} + 1 > \frac{\rho + \frac{\frac{\pi}{1-\pi}+\tau}{\left(1-\tau\right)} - \frac{\frac{\pi}{1-\pi}\left(1-\rho\right)}{\frac{\left(1-\alpha\right)}{\alpha}-\frac{\pi}{1-\pi}}}{\rho\left(1+\frac{\frac{\pi}{1-\pi}}{\frac{\left(1-\alpha\right)}{\alpha}-\frac{\pi}{1-\pi}}\right)}$$

with some algebra this condition can be written as:

$$\alpha^2 - \alpha\left(1-\pi\right) + \frac{\left(1-\pi\right)^2}{4} > 0$$

which holds for all $\alpha \in [0,1]$. In this manner for all $\mu < \mu_0$, the discriminant in the expressions for r is positive.

Next, $\tilde{\mu}_0 < \tilde{\mu}_1$ if:

$$1 + \frac{\tau}{1-\tau}\frac{1}{\rho} < 1 + \frac{\tau}{\rho}\frac{1-\alpha}{\alpha}$$

which can be written as:

$$\tau < \frac{1-2\alpha}{1-\alpha}$$

As we focus on the case of a unique equilibrium at the Friedman rule, our analysis above requires that: $r_0 \leq \hat{r}$ or equivalently: $\mu \leq 1 + \frac{\tau}{\rho}\frac{1-\alpha}{\alpha} = \tilde{\mu}_1$. A

sufficient condition is that $\tilde{\mu}_1 > \mu_0$. Upon using the expressions for $\tilde{\mu}_1$ and μ_0, this condition is written as:

$$\tau \left(\frac{1-\alpha}{\alpha} \right)^2 \left[\frac{(1-2\alpha)}{(1-\alpha)} - \tau \right] + \frac{\pi}{1-\pi} \left(\frac{1}{1-\pi} - \frac{(1-\alpha)}{\alpha} \right) > 0$$

which always holds when $\pi \geq \frac{1-2\alpha}{1-\alpha}$. We assume that this condition also holds when $\pi < \frac{1-2\alpha}{1-\alpha}$. Given this analysis, a unique Friedman rule equilibrium exists if $\mu \in [1, \mu_0]$. Notably, if $\mu_0 < \tilde{\mu}_0$, the Friedman rule equilibrium is such that $r > 1$. By comparison, if $\mu_0 > \tilde{\mu}_0$, the Friedman rule equilibrium is such that $r > 1$ when $\mu \in [1, \tilde{\mu}_0]$ and $r < 1$ if $\mu \in [\tilde{\mu}_0, \mu_0]$. Upon using the definitions of μ_0 and $\tilde{\mu}_0$, $\mu_0 < \tilde{\mu}_0$ if:

$$\frac{\alpha \left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi} \right)}{(1-\pi)(1-\tau)(1-\alpha)\rho} - \frac{(1-\rho)}{\rho} < 1 + \frac{\tau}{1-\tau} \frac{1}{\rho}$$

which can be written as: $\pi > \frac{1-2\alpha}{1-\alpha}$. Finally, it is easily verified that $\mu_0 \geq 1$ if $\tau \geq \frac{\pi}{1-\pi} \left[\frac{1}{1-\pi} \frac{\alpha}{1-\alpha} - 1 \right] = \tilde{\tau}$, with $\tilde{\tau} > 0$ if $\pi \geq \frac{1-2\alpha}{1-\alpha} = \tilde{\pi}$. In this manner, when the conditions in Proposition 1, case i hold, a unique Friedman rule equilibrium exists. The equilibrium is represented by r_1 above. Upon substituting for the production technology, the expression for K in (29) is easily obtained. This completes the proof of Proposition 1.

3. Proof of Proposition 2. Most of the results in Proposition 2 can be inferred from the main text. In regards to the effects of N on c^n under cases *ii* and *iii*, it is easily verified from (10) in the steady-state that $c^n = r^n w (1-\tau) = \frac{rK}{1-\pi}$. Under Cobb-Douglas, $c^n = \frac{\alpha A K^\alpha}{1-\pi}$. Therefore, the impact of N on c^n directly follows the impact of N on K. However, the impact of N on c^m is less clear under case *ii*. From (8) in the steady-state:

$$c^m = \frac{N}{\pi} m_R R_R + \frac{N}{\pi} m_e R_e \tag{70}$$

where $c^m = r^m w (1-\tau)$. Using the fact that $R_i = I_i \frac{P_t}{P_{t+1}}$ where $\frac{P_{t+1}}{P_t} = \frac{(\rho I_R + \gamma_e I_e)}{\gamma + \frac{\tau}{1-\tau}}$, and (28), (70) can be written as:

$$c^m = \frac{1}{\pi} \left(\frac{1 - \rho \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right] (\mu - 1)}{1 + \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]} + \frac{\tau}{1-\tau} \right) (1-\tau)(1-\alpha) A K^\alpha \tag{71}$$

where $K^{1-\alpha} = \left[\frac{1+\rho(\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]} \right] \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right] (1-\tau)(1-\alpha) A$ from (29). Differentiating (71) with respect to N and some simplification, to get:

$$\frac{dc^m}{dN} =$$

$$\frac{(1-\tau)(1-\alpha)AK^{\alpha}}{\pi\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)N^2}\left\{\begin{array}{c}\frac{-[1+\rho(\mu-1)](1-\pi)(1-\alpha)}{\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)\pi}+\\ \left(\frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}+\frac{\tau}{1-\tau}\right)\frac{\alpha}{\left[1-\frac{1-\alpha}{N}\right]}\end{array}\right\} \quad (72)$$

It is clear that the sign of $\frac{dc^m}{dN}$ depends on the term in the curly brackets. With few lines of algebra, $\frac{dc^m}{dN} \geq 0$ if:

$$\frac{1}{1-\tau} \geq \left([1+\rho(\mu-1)]\frac{1}{\alpha}-\frac{1}{1-\tau}\right)\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right] = \Phi_c(N) \quad (73)$$

where $\Phi_c'(N) > 0$. Under case ii, we have $\mu \in (\mu_1, \mu_2)$. Recall that the incentive compatibility constraint is non-binding if:

$$\mu > \mu_1 = 1 + \frac{1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}$$

which can be expressed as a condition on N:

$$N > (1-\alpha)\frac{\left(1+\rho(\mu-1)-\frac{\alpha}{1-\alpha}\frac{1}{1-\tau}\left(\frac{1}{\alpha}-\frac{1}{1-\pi}\right)\right)}{\left[1+\rho(\mu-1)-\frac{1}{1-\pi}\frac{1}{1-\tau}\frac{\alpha}{1-\alpha}\left(\frac{1}{\alpha}-\frac{1}{1-\pi}\right)\right]} = \underline{N}$$

where $\frac{d\underline{N}}{d\mu} < 0$. Analogously, the reserves constraint is non-binding if : $\mu < \mu_2 = \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1-\frac{1-\alpha}{N}}$ which can be written as a condition on N:

$$N < \frac{1-\alpha}{1-\frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{\mu}} = \bar{N}$$

with $\frac{d\bar{N}}{d\mu} < 0$. Given that μ_1 and μ_2 are both decreasing functions with N, with $\mu_1 < \mu_2$. Then, it must that for a given μ, $\underline{N} < \bar{N}$. In this manner, $\mu \in (\mu_1, \mu_2) \Longleftrightarrow N \in \left(\underline{N}, \bar{N}\right)$. That is, the degree of concentration only matters for real activity if N lies in this range. Further, using the expression for $\underline{N}$, it can be shown that $\underline{N} > 1$ if $\mu < 1 + \left(\left[\frac{1}{1-\pi}-(1-\alpha)\right]\frac{1}{1-\tau}\frac{1}{1-\alpha}\left(\frac{1}{\alpha}-\frac{1}{1-\pi}\right)-1\right)\frac{1}{\rho}$.

Using the definition of $\underline{N}$ and some algebra, $\Phi_c(\underline{N}) > \frac{1}{1-\tau}$ if:

$$\pi < \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$$

Analogously, $\Phi_c\left(\bar{N}\right) > \frac{1}{1-\tau}$ if:

$$\left(\frac{1}{\alpha}-\frac{1}{1-\rho}\frac{1}{1-\tau}\right)\frac{1}{1-\rho}\rho\mu>-\left(\frac{1}{\alpha}-\frac{1}{1-\rho}\frac{1}{1-\tau}\right)$$

It is easily verified that this condition always holds if:

$$\rho<\left(1-\frac{\alpha}{1-\tau}\right)=\acute{\rho}$$

The opposite is also true. From our work above, $\mu_1 \leq \mu_2$ if:

$$\rho\leq\left(\frac{\alpha}{1-\alpha}\frac{\pi}{1-\pi}\frac{1}{1-\tau}-\tau\frac{1}{1-\tau}\right)=\hat{\rho}$$

In this manner, $\hat{\rho}>\acute{\rho}$ if:

$$\pi>\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$$

Given the characterization of $\Phi_c(N)$ over the $N\in\left(\underline{N},\bar{N}\right)$ range, $\Phi_c(N)>\frac{1}{1-\tau}$if $\pi<\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ since $\Phi_c\left(\underline{N}\right)>\frac{1}{1-\tau}$ and and $\Phi_c'(N)>0$. This implies that c^m is decreasing in N. However, if $\pi>\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, $\Phi_c\left(\underline{N}\right)<\frac{1}{1-\tau}$. Two possibilities emerge here. First, if $\rho<\acute{\rho}<\hat{\rho}$, $\Phi_c\left(\bar{N}\right)>\frac{1}{1-\tau}$, and there is a unique N, such that (73) holds with equality. Therefore, c^m is increasing, then decreasing in N. By comparison, if $\rho\in(\acute{\rho},\hat{\rho})$, $\Phi_c\left(\bar{N}\right)<\frac{1}{1-\tau}$. This implies that $\Phi_c(N)<\frac{1}{1-\tau}$ over the range $N\in\left(\underline{N},\bar{N}\right)$. Therefore, c^m is increasing in N.

We proceed to examine the effects of banking competition on the aggregate real money stock, $\tilde{m}$. Imposing steady-state on a bank's balance sheet condition, (7) , to get:

$$\frac{(1-\tau)w}{N}=m+k$$

Aggregating and imposing symmetry:

$$\tilde{m}=(1-\tau)w-K$$

which under Cobb-Douglas becomes:

$$\tilde{m}=(1-\tau)(1-\alpha)AK^{\alpha}-K \tag{74}$$

$$\frac{d\tilde{m}}{dN}=(1-\tau)(1-\alpha)A\alpha K^{\alpha-1}\frac{dK}{dN}-\frac{dK}{dN}$$

$$\frac{d\tilde{m}}{dN}=\left(\frac{(1-\tau)(1-\alpha)A\alpha}{K^{1-\alpha}}-1\right)\frac{dK}{dN} \tag{75}$$

Clearly, when $\frac{dK}{dN}=0$, as under cases i and iii, we have $\frac{d\tilde{m}}{dN}=0$. However, under case ii, we have $\frac{dK}{dN}>0$. Therefore, $\frac{d\tilde{m}}{dN}>0$ if:

$$\frac{(1-\tau)(1-\alpha)A\alpha}{K^{1-\alpha}} - 1 > 0$$

Upon substituting for $K^{1-\alpha}$ from (29), $\frac{d\tilde{m}}{dN} > 0$ if:

$$\alpha > (1-\alpha)\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right] + \rho(\mu - 1)\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right] \equiv \Phi_{\tilde{m}}(N) \quad (76)$$

where $\Phi'_{\tilde{m}}(N) > 0$. We evaluate whether this condition can be satisfied under the lower and upper bounds on N under which case ii applies, $\underline{N}$ and $\bar{N}$. Using the definition of $\underline{N}$ and some algebra, $\Phi_{\tilde{m}}(\underline{N}) < \alpha$ if:

$$\tau < -\frac{1-\alpha}{\alpha} + \frac{\pi}{1-\pi}\frac{1}{(1-\alpha)}$$

where $-\frac{1-\alpha}{\alpha} + \frac{\pi}{1-\pi}\frac{1}{(1-\alpha)} > 0$ if:

$$\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2 + \alpha}$$

We next evaluate the condition in (76) at the upper bound, $\bar{N}$. From the definition of $\bar{N}$ with a few lines of algebra, $\Phi_{\tilde{m}}\left(\bar{N}\right) < \alpha$ iff:

$$\rho > 1 - \alpha = \ddot{\rho}$$

where $\ddot{\rho} < \hat{\rho}$ if $\tau < -\frac{1-\alpha}{\alpha} + \frac{\pi}{1-\pi}\frac{1}{(1-\alpha)}$. By comparison, if $\rho < \ddot{\rho} < \hat{\rho}$, $\Phi_{\tilde{m}}\left(\bar{N}\right) > \alpha$. Therefore, $\Phi_{\tilde{m}}(N) = \alpha$ has a unique solution, $\ddot{N}$. Consequently, for $\underline{N} < N < \ddot{N}$, $\tilde{m}$ is increasing in N, while it is decreasing in N for $\ddot{N} < N < \bar{N}$ if $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ and $\rho < \ddot{\rho} < \hat{\rho}$.

Alternatively, if $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$and $\rho \in (\ddot{\rho}, \hat{\rho})$, then $\Phi_{\tilde{m}}(N) < \alpha$ for $N \in \left(\underline{N}, \bar{N}\right)$, which implies that $\tilde{m}$ is increasing in N. Finally, if $\pi < \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, $\Phi_{\tilde{m}}(\underline{N}) > \alpha$ and $\Phi_{\tilde{m}}(N) > \alpha$ for $N \in \left(\underline{N}, \bar{N}\right)$.since $\Phi'_{\tilde{m}}(N) > 0$. Thus, under such conditions, $\tilde{m}$ is decreasing in N. This completes the proof of Proposition 2.

4. Proof of Proposition 3. As for Proposition 2, most of the results for Proposition 3 can be observed from the explicit solutions in the main text. For the effects of monetary policy on the consumption of depositors at the Friedman rule, case i, we begin by deriving an expression for the consumption of depositors.

Using (23) and (24) under complete risk sharing we have:

$$\frac{\lambda_t N\left[m_{R,t}R_{R,t} + m_{e,t}R_{e,t}\right]}{\pi} = N\frac{r\left(k_{t+1}, K_{t+1}^{N-1}\right)k_{t+1} + (1-\lambda_t)\left[m_{R,t}R_{R,t} + m_{e,t}R_{e,t}\right]}{1-\pi} \quad (77)$$

With a few lines of algebra, (77) becomes:

$$\lambda_t = \pi \left(\frac{r\left(k_{t+1}, K_{t+1}^{N-1}\right) k_{t+1}}{\left[m_{R,t} R_{R,t} + m_{e,t} R_{e,t}\right]} + 1 \right) \tag{78}$$

At the Friedman rule, $r\left(k_{t+1}, K_{t+1}^{N-1}\right) = R_{e,t}$. Therefore, (78) is:

$$\lambda_t = \pi \left(\frac{k_{t+1}}{\left[m_{R,t}\mu + m_{e,t}\right]} + 1 \right)$$

From (23) in the steady-state:

$$c = r^m \left(1-\tau\right) w = \frac{\lambda N \left[m_R R_R + m_e R_e\right]}{\pi} \tag{79}$$

Upon substituting for λ_t in the steady-state and imposing symmetry:

$$c = \left(K + N\left[m_R \mu + m_e\right]\right) R_e \tag{80}$$

which can be written as:

$$c = \left(1 - \gamma + \rho\mu + \gamma_e\right) r\left(K\right) \left(1-\tau\right) w \tag{81}$$

Substituting for the functional form from the production function and utilizing $\gamma_e = \gamma - \rho$:

$$c = \frac{\left(1-\tau\right)\left(1+\rho\mu-\rho\right)\alpha A \left(1-\alpha\right) A}{K^{1-2\alpha}}$$

Equivalently:

$$c = \frac{\left(1-\tau\right)\alpha A \left(1-\alpha\right) A}{\left(\alpha A\right)^{\frac{1-2\alpha}{1-\alpha}}} \left(1+\rho\mu-\rho\right) r^{\frac{1-2\alpha}{1-\alpha}} \tag{82}$$

where $r = r_1$ derived above. Simple differentiation of (82), we get:

$$\frac{dc}{d\mu} = \frac{\left(1-\tau\right)\alpha A \left(1-\alpha\right) A}{\left(\alpha A\right)^{\frac{1-2\alpha}{1-\alpha}}} r^{\frac{1-2\alpha}{1-\alpha}} \left(\rho + \frac{1-2\alpha}{1-\alpha} \left(1+\rho\mu-\rho\right) \frac{1}{r} \frac{dr}{d\mu} \right)$$

In this manner, $\frac{dc}{d\mu} \geq 0$ if:

$$1 \geq -\frac{1-2\alpha}{1-\alpha} \frac{\left[1+\rho\left(\mu-1\right)\right]}{\rho} \frac{1}{r} \frac{dr}{d\mu} \tag{83}$$

From the expression for r at the Friedman rule:

$$r_1 = \frac{1}{2\left(1-\alpha\right)\left(1-\tau\right)} \frac{1 + \left[1 - 4\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]\left(1-\alpha\right)\alpha\right]^{\frac{1}{2}}}{\left[1+\rho\left(\mu-1\right)\right]}$$

Upon differentiating with respect to μ and simplifying, we get:

$$\frac{dr}{d\mu} = \frac{\frac{-\rho+2\rho(1-\tau)[1+\rho(\mu-1)](1-\alpha)\alpha}{[1-4(1-\tau)[1+\rho(\mu-1)](1-\alpha)\alpha]^{\frac{1}{2}}} - \rho}{2\left(1-\alpha\right)\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]^{2}}$$

Moreover, from the expression for r and few lines of algebra we get:

$$\frac{\left[1+\rho\left(\mu-1\right)\right]}{\rho}\frac{1}{r}\frac{dr}{d\mu} = -\frac{\left[\frac{1-2(1-\tau)[1+\rho(\mu-1)](1-\alpha)\alpha}{[1-4(1-\tau)[1+\rho(\mu-1)](1-\alpha)\alpha]^{\frac{1}{2}}} + 1\right]}{1+\left[1-4\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]\left(1-\alpha\right)\alpha\right]^{\frac{1}{2}}}$$

Using this information into (83) and simplifying, $\frac{dc}{d\mu} \geq 0$ if:

$$\left[1-4\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]\left(1-\alpha\right)\alpha\right]^{\frac{1}{2}}+\left[1-2\left(1-\tau\right)\left[1+\rho\left(\mu-1\right)\right]\left(1-\alpha\right)\right] \geq 0$$

which occurs when μ is below some level. It is easily verified that $\frac{dc^*}{d\mu} \geq (<) 0$ if $\mu \leq (>) \tilde{\mu}_0$ where $\tilde{\mu}_0 = 1 + \frac{\tau}{1-\tau}\frac{1}{\rho}$ is defined above. Recall that $\mu_0 < \tilde{\mu}_0$ if $\pi > \frac{1-2\alpha}{1-\alpha}$. In this manner, for $\pi > \frac{1-2\alpha}{1-\alpha}$, $\frac{dc^*}{d\mu} > 0$ when $\mu \in [1, \mu_0)$. By comparison, if $\pi < \frac{1-2\alpha}{1-\alpha}$, $\tilde{\mu}_0 < \mu_0$ and $\frac{dc^*}{d\mu} \geq (<) 0$ if $\mu \leq (>) \tilde{\mu}_0$.

We proceed to sign $\frac{dc^m}{d\mu}$ under case iii. Differentiating (71) with respect to μ and some simplification to get:

$$\frac{dc^m}{d\mu} = \frac{\left(1-\tau\right)\alpha A}{\pi\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)}\left\{-\frac{1-\pi}{\pi}\frac{1-\alpha}{\alpha}\left[1-\frac{1-\alpha}{N}\right]+\frac{\frac{1}{1-\tau}+\left(\frac{\tau}{1-\tau}-\rho\left(\mu-1\right)\right)\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}{\left[1+\rho\left(\mu-1\right)\right]}\right\}\rho K^{\alpha}$$

With few lines of algebra, $\frac{dc^m}{d\mu} \geq 0$ if:

$$\mu \leq \left(\frac{1}{1-\tau}\alpha\frac{\pi}{1-\pi}\frac{\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)}{\left[1-\frac{1-\alpha}{N}\right]}-1\right)\frac{1}{\rho}+1 \equiv \acute{\mu} \tag{84}$$

We proceed to compare $\acute{\mu}$ to μ_1 and μ_2. In particular, it is easily verified that $\acute{\mu} < \mu_1$ if:

$$\pi < \frac{\left(1-\alpha\right)^2}{1+\alpha^2-\alpha}$$

whereas $\acute{\mu} < \mu_2$ if:

$$\rho < 1 - \frac{1}{\left(1-\tau\right)}\alpha = \acute{\rho}$$

In this manner, if $\pi < \frac{(1-\alpha)^2}{1+\alpha^2-\alpha}$, $\acute{\mu} < \mu_1$ and $\frac{dc^m}{d\mu} < 0$ when $\mu \in [\mu_1, \mu_2)$. However, if $\pi > \frac{(1-\alpha)^2}{1+\alpha^2-\alpha}$ and $\rho < \acute{\rho} < \hat{\rho}$, there exists an interior value of μ,

$\acute{\mu}$ that maximizes c^m. That is, $\frac{dc^m}{d\mu} \geq (<) 0$ if $\mu \leq (>) \acute{\mu}$, with $\acute{\mu} \in (\mu_1, \mu_2)$. Finally, if $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$ and $\rho \in (\acute{\rho}, \hat{\rho})$, $\frac{dc^m}{d\mu} > 0$ when $\mu \in [\mu_1, \mu_2)$ as $\acute{\mu} > \mu_2$.

As for the effects of monetary policy on aggregate real money holdings, we differentiate (74) with respect to μ to get:

$$\frac{d\tilde{m}}{d\mu} = \left(\frac{(1-\tau)(1-\alpha) A\alpha}{K^{1-\alpha}} - 1 \right) \frac{dK}{d\mu} \tag{85}$$

which suggests that $\frac{d\tilde{m}}{d\mu} = 0$ if $\frac{dK}{d\mu} = 0$ as under chases ii and iv. Under case i, re-writing (85) by using (4) to get:

$$\frac{d\tilde{m}}{d\mu} = ((1-\tau)(1-\alpha) r - 1) \frac{dK}{d\mu}$$

Clearly, $\frac{d\tilde{m}}{d\mu} > 0$ if:

$$r > \frac{1}{(1-\tau)(1-\alpha)}$$

Upon using the expression for r at the Friedman rule from our work above, this condition becomes:

$$\sqrt{1 - 4(1-\tau)[1+\rho(\mu-1)](1-\alpha)\alpha} > 1 + 2\rho(\mu-1)$$

which never holds. Therefore, $\frac{d\tilde{m}}{d\mu} < 0$ under case i. Next, we consider signing $\frac{d\tilde{m}}{d\mu}$ under case iii. Using the expression for $K^{1-\alpha}$ from (29) into (85) and some simplification, $\frac{d\tilde{m}}{d\mu} > 0$ if:

$$\mu < \left[\alpha - (1-\alpha) \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right] \right] \frac{1}{\rho \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]} + 1 = \ddot{\mu}$$

By definition of μ_1 and some algebra, $\ddot{\mu} < \mu_1$ if:

$$\tau > -\frac{1-\alpha}{\alpha} + \frac{\pi}{1-\pi} \frac{1}{(1-\alpha)} \equiv \tau_0$$

where $-\frac{1-\alpha}{\alpha} + \frac{\pi}{1-\pi} \frac{1}{(1-\alpha)} > 0$ if:

$$\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2 + \alpha}$$

Analogously, $\ddot{\mu} < \mu_2$ if:

$$\rho < 1 - \alpha$$

In this manner, if $\pi < \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, then $\tau > -\frac{1-\alpha}{\alpha} + \frac{\pi}{1-\pi} \frac{1}{(1-\alpha)}$. Consequently, under this condition on π, $\ddot{\mu} < \mu_1$, which indicates that $\frac{d\tilde{m}}{d\mu} < 0$ for $\mu \in [\mu_1, \mu_2)$. However, if $\pi > \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha}$, but $\rho < \ddot{\rho} < \hat{\rho}$, then $\ddot{\mu} \in (\mu_1, \mu_2)$, which implies

that $\frac{d\tilde{m}}{d\mu} \geq (<) 0$ when $\mu \leq (>) \ddot{\mu}$. However, if the same conditions hold, except that $\rho \in (\ddot{\rho}, \hat{\rho})$, then $\ddot{\mu} > \mu_2$. This implies that $\frac{d\tilde{m}}{d\mu} > 0$ if $\mu \in [\mu_1, \mu_2)$. This completes the proof of Proposition 3.

5. Derivation and characterization of the welfare function. To begin, consider case iii, where $\mu \in [\mu_1, \mu_2)$ under which $\frac{r^n}{r^m} > 1$ and $\gamma > \rho$. From (6), the expected utility of depositors is such that:

$$U_t = \pi \ln \left[r_t^m \left(1-\tau\right) w_t \right] + \left(1-\pi\right) \ln \left[r_t^n \left(1-\tau\right) w_t \right]$$

Substituting the binding constraints, (8) and (10) :

$$U_t = \pi \ln \left[N \frac{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}}{\pi} \right] + \left(1-\pi\right) \ln \left[N \frac{r\left(k_{t+1}, K_{t+1}^{N-1}\right) k_{t+1}}{\left(1-\pi\right)} \right]$$

Imposing steady-state and symmetry:

$$U = \pi \ln \left[\frac{N m_R R_R + N m_e R_e}{\pi} \right] + \left(1-\pi\right) \ln \frac{rK}{\left(1-\pi\right)} \tag{86}$$

Subsequently, using the definition of γ, the fact that $I_i = R_i \frac{P_t}{P_{t+1}}$, with $i = e, R$, and (27):

$$U = \pi \ln \left[\frac{\gamma + \frac{\tau}{1-\tau}}{\pi} \right] \left(1-\tau\right) w + \left(1-\pi\right) \ln \frac{rK}{\left(1-\pi\right)} \tag{87}$$

With some simplification, we have:

$$U = \alpha \ln K + \pi \ln \left[\gamma + \frac{\tau}{1-\tau} \right] + \pi \ln \frac{\left(1-\tau\right)\left(1-\alpha\right) A}{\pi} + \left(1-\pi\right) \ln \frac{\alpha A}{\left(1-\pi\right)} \tag{88}$$

where $K^{1-\alpha} = \left[\frac{1+\rho(\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]} \right] \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right] \left(1-\tau\right)\left(1-\alpha\right) A$ from (29) and:

$$\gamma = \frac{1 - \rho \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right] \left(\mu - 1\right)}{1 + \frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N}\right]}$$

Upon differentiating (88):

$$\frac{dU}{d\mu} = \alpha \frac{1}{K} \frac{dK}{d\mu} + \frac{\pi}{\gamma + \frac{\tau}{1-\tau}} \frac{d\gamma}{d\mu} \tag{89}$$

where from the expression for K,

$$\left(1-\alpha\right) K^{-\alpha} \frac{dK}{d\mu} = \frac{\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\left(1-\tau\right)\left(1-\alpha\right) A\rho}{1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]}$$

Upon using the expression for K and some simplification:

$$\frac{1}{K}\frac{dK}{d\mu} = \frac{\rho}{(1-\alpha)\left[1+\rho\left(\mu-1\right)\right]} \tag{90}$$

Next, from the expression for γ, (28) :

$$\gamma = \frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\left(\mu-1\right)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}$$

Differentiating with respect to μ:

$$\frac{d\gamma}{d\mu} = -\frac{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]} \tag{91}$$

Plugging (90) and (91) into (89) with some algebra:

$$\frac{dU}{d\mu} = \frac{\alpha\rho}{(1-\alpha)\left[1+\rho\left(\mu-1\right)\right]} - \frac{\pi\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\left(\mu-1\right)+\frac{\tau}{1-\tau}\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)} \tag{92}$$

$\frac{dU}{d\mu} \geq 0$ if:

$$\frac{\alpha\rho}{(1-\alpha)\left[1+\rho\left(\mu-1\right)\right]} - \frac{\pi\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\left(\mu-1\right)+\frac{\tau}{1-\tau}\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)} \geq 0 \tag{93}$$

which can be re-written as:

$$\mu \leq \frac{\frac{1}{\left[1-\frac{1-\alpha}{N}\right]}+\frac{\tau}{1-\tau}\left(\frac{1}{\left[1-\frac{1-\alpha}{N}\right]}+\frac{1-\pi}{\pi}\right)-\pi\frac{1-\pi}{\pi}\frac{(1-\alpha)}{\alpha}+\left[\pi\frac{(1-\alpha)}{\alpha}+1\right]\frac{1-\pi}{\pi}\rho}{\left[\pi\frac{(1-\alpha)}{\alpha}+1\right]\frac{1-\pi}{\pi}\rho} = \hat{\mu} \tag{94}$$

Recall that $\mu_2 = \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1-\frac{1-\alpha}{N}}$ and $\mu_1 = 1+\frac{1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}$.
The behavior of the welfare function with respect to μ over the $\mu \in (\mu_1, \mu_2)$ interval clearly hinges on the position of $\hat{\mu}$ relative to μ_1 and μ_2. Three possibilities may arise, one where $\hat{\mu} < \mu_1$, another where $\hat{\mu} \in (\mu_1, \mu_2)$, and a third case where $\hat{\mu} > \mu_2$. First, $\hat{\mu} \leq \mu_1$ if:

$$\frac{1+\frac{\tau}{1-\tau}\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)-\pi\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\frac{(1-\alpha)}{\alpha}}{\left[\pi\frac{(1-\alpha)}{\alpha}+1\right]\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\rho}+1 \leq 1+\frac{1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right.}{\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}$$

Upon simplifying, it can be verified that this condition becomes:

$$\pi \leq \frac{1-2\alpha}{(1-\alpha)}$$

From our work above, $\mu_1 \leq \mu_2$ if:

$$\rho \leq \left(\frac{\alpha}{1-\alpha} \frac{\pi}{1-\pi} \frac{1}{1-\tau} - \tau \frac{1}{1-\tau} \right) = \hat{\rho}$$

In this manner, whenever $\pi \leq \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho}$, $\hat{\mu} \leq \mu_1 < \mu_2$ and $\frac{dU}{d\mu} \leq 0$ for $\mu \in [\mu_1, \mu_2)$.

Now, suppose $\pi > \frac{1-2\alpha}{1-\alpha}$. Under this condition, $\hat{\mu} > \mu_1$. Next, we need to compare $\hat{\mu}$ to μ_2. In particular, $\hat{\mu} \leq \mu_2$ if:

$$\frac{1 + \frac{\tau}{1-\tau}\left(1 + \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\right) - \pi \frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\frac{(1-\alpha)}{\alpha}}{\left[\pi \frac{(1-\alpha)}{\alpha} + 1\right]\frac{1-\pi}{\pi}\left[1 - \frac{1-\alpha}{N}\right]\rho} + 1 \leq \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{1 - \frac{1-\alpha}{N}}$$

with some algebra, this condition becomes:

$$\rho \leq \frac{\pi \frac{(1-\alpha)}{\alpha} - \frac{\tau}{1-\tau}}{\left[\pi \frac{(1-\alpha)}{\alpha} + 1\right]} = \tilde{\rho}$$

Using the definitions of $\hat{\rho}$ and $\tilde{\rho}$, with a few lines of algebra, $\hat{\rho} > \tilde{\rho}$ if $\pi > \frac{1-2\alpha}{1-\alpha}$.

Two cases emerge here. First, consider that $\rho < \tilde{\rho} < \hat{\rho}$. Under this condition, $\hat{\mu} \in (\mu_1, \mu_2)$ and we have a unique interior local optimum, $\mu^* \in (\mu_1, \mu_2)$. Second, if $\rho \in (\tilde{\rho}, \hat{\rho})$, $\hat{\mu} > \mu_2$ and $\frac{dU}{d\mu} > 0$ for $\mu \in (\mu_1, \mu_2)$.

Next, consider case iv, where $\mu \geq \mu_2$, with $\frac{r^n}{r^m} > 1$ and $\gamma = \rho$. Over this parameter space, we established that $K^{1-\alpha} = (1-\rho)(1-\tau)(1-\alpha)A$ and $\frac{P_{t+1}}{P_t} = \frac{\rho I_R}{\rho + \frac{\tau}{1-\tau}}$. Plugging this information into the welfare function, (87) :

$$U = \pi \ln \left[\frac{\rho + \frac{\tau}{1-\tau}}{\pi} \right] (1-\tau) w(K) + (1-\pi) \ln \frac{r(K) K}{(1-\pi)}$$

Since $\frac{dK}{d\mu} = 0$, it directly follows that, $\frac{dU}{d\mu} = 0$.

Under case ii, where $\mu \in (\mu_0, \mu_1]$, with $\frac{r^n}{r^m} = 1$ and $I > I_e$, we established that $K^{1-\alpha} = \alpha A \left(\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi} \right)$. Given that depositors receive complete risk sharing, the welfare function is such that:

$$U = \ln \left[r^n (1-\tau) w \right]$$

By (10) in a symmetric steady-state equilibrium, $r^n (1-\tau) w = \frac{r(K)K}{(1-\pi)}$. Therefore:

$$U = \ln \frac{r(K) K}{(1-\pi)}$$

Clearly, $\frac{dU}{d\mu} = 0$ as $\frac{dK}{d\mu} = 0$.

Finally, at $\mu = \mu_0$, where $\frac{r^n}{r^m} = 1$ and $I = I_e$, from the work above, the expected utility in the steady-state is:

$$U = \ln\left[r^m\left(1-\tau\right)w\right] = \ln c$$

where c is given by (82). Differentiating U with respect to μ to get:

$$\frac{dU}{d\mu} = \frac{1}{c}\frac{dc}{d\mu} \tag{95}$$

which clearly indicates that the sign of $\frac{dU}{d\mu}$, depends on that of $\frac{dc}{d\mu}$. From our work in Proposition 3, it directly follows that $\frac{dU}{d\mu} \geq (<)\, 0$ if $\mu \leq (>)\, \tilde{\mu}_0$ where $\tilde{\mu}_0 = 1 + \frac{\tau}{1-\tau}\frac{1}{\rho}$ is defined above. Recall that $\mu_0 < \tilde{\mu}_0$ if $\pi > \frac{1-2\alpha}{1-\alpha}$. In this manner, for $\pi > \frac{1-2\alpha}{1-\alpha}$, $\frac{dU}{d\mu} > 0$ if $\mu \in [1, \mu_0)$. By comparison, if $\pi < \frac{1-2\alpha}{1-\alpha}$, $\tilde{\mu}_0$ is a local optimum over the range $\mu \in [1, \mu_0)$. This completes the characterization of the welfare function.

6. Proof of Proposition 4. From our characterization of the welfare function, $\frac{dU}{d\mu} = 0$ if $\mu \in [\mu_0, \mu_1]$ and $\frac{dU}{d\mu} = 0$ if $\mu \geq \mu_2$. Therefore, the optimal monetary policy hinges on the behavior of U at the Friedman rule and over the interval $\mu \in (\mu_1, \mu_2)$. From our work above, $\frac{dU}{d\mu} < 0$ when $\mu \in (\mu_1, \mu_2)$ if $\pi \leq \frac{1-2\alpha}{1-\alpha}$ since $\hat{\mu} < \mu_2$. However, $\frac{dU}{d\mu} \geq (<)\, 0$ if $\mu \leq (>)\, \tilde{\mu}_0$ over $\mu \in [1, \mu_0)$. Therefore, $\tilde{\mu}_0$ is the global optimum. In comparison, if $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho < \tilde{\rho} < \hat{\rho}$, $\frac{dU}{d\mu} \geq (<)\, 0$ if $\mu \leq (>)\, \hat{\mu}$, where $\hat{\mu} \in (\mu_1, \mu_2)$ under the conditions stated. In this manner, $\hat{\mu}$ is a global optimum. This necessarily happens as $\frac{dU}{d\mu} > 0$ if $\mu \in [1, \mu_0)$ when $\pi > \frac{1-2\alpha}{1-\alpha}$. Finally, suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho \in (\tilde{\rho}, \hat{\rho})$. Under these conditions, $\frac{dU}{d\mu} > 0$, when $\mu \in (\mu_1, \mu_2)$. Consequently, it is optimal to set $\mu^* \geq \mu_2$. This completes the proof of Proposition 4.

7. Proof of Proposition 5. As shown above, the degree of banking competition only matters when $\mu \in (\mu_1, \mu_2)$. From the Proof of Proposition 2, we established that the condition on μ, $\mu \in (\mu_1, \mu_2)$, can be written as a one on N, $N \in \left(\underline{N}, \bar{N}\right)$ where:

$$\underline{N} = (1-\alpha)\frac{\left(1+\rho\left(\mu-1\right) - \frac{\alpha}{1-\alpha}\frac{1}{1-\tau}\left(\frac{1}{\alpha} - \frac{1}{1-\pi}\right)\right)}{\left[1+\rho\left(\mu-1\right) - \frac{1}{1-\pi}\frac{1}{1-\tau}\frac{\alpha}{1-\alpha}\left(\frac{1}{\alpha} - \frac{1}{1-\pi}\right)\right]}$$

and

$$\bar{N} = \frac{1-\alpha}{1 - \frac{\frac{1-\rho}{\rho}\frac{\pi}{1-\pi}}{\mu}}$$

Differentiating the welfare function, (88) with respect to N to obtain:

$$\frac{dU}{dN} = \alpha\frac{1}{K}\frac{dK}{dN} + \frac{\pi}{\gamma + \frac{\tau}{1-\tau}}\frac{d\gamma}{dN}$$

where $K^{1-\alpha} = \left[\frac{1+\rho(\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}\right] \frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](1-\tau)(1-\alpha)A$ from the work above and

$$\gamma = \frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](\mu-1)}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}$$

Using the expressions for K and γ with some algebra:

$$\alpha\frac{1}{K}\frac{dK}{dN} = \frac{1}{N^2}\frac{\alpha}{\left[1-\frac{1-\alpha}{N}\right]\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)}$$

and

$$\frac{d\gamma}{dN} = -\frac{(\mu-1)\rho+1}{\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)^2}\frac{1-\pi}{\pi}\frac{1-\alpha}{N^2}$$

Substituting this information into $\frac{dU}{dN}$ to get:

$$\frac{dU}{dN} = \frac{1}{\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)N^2}\left(\frac{\alpha}{\left[1-\frac{1-\alpha}{N}\right]} - \frac{\pi}{\gamma+\frac{\tau}{1-\tau}}\frac{[(\mu-1)\rho]+1}{\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)}\frac{1-\pi}{\pi}(1-\alpha)\right)$$

In this manner, $\frac{dU}{dN} \geq 0$ if:

$$\frac{\alpha}{\left[1-\frac{1-\alpha}{N}\right]} \geq \frac{\pi}{\gamma+\frac{\tau}{1-\tau}}\frac{[(\mu-1)\rho]+1}{\left(1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right)}\frac{1-\pi}{\pi}(1-\alpha)$$

Using the expression for γ and some simplification, $\frac{dU}{dN} \geq 0$ if:

$$\frac{1}{1-\tau} > \left([(\mu-1)\rho+1]\pi\frac{(1-\alpha)}{\alpha} + \rho(\mu-1) - \frac{\tau}{1-\tau}\right)\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right] \equiv \Phi(N) \tag{96}$$

It is clear that $\Phi'(N) > 0$. Moreover,

$$\Phi(\underline{N}) = \frac{\left((\mu-1)\rho\left(\pi\frac{(1-\alpha)}{\alpha}+1\right)+\pi\frac{(1-\alpha)}{\alpha}-\frac{\tau}{1-\tau}\right)}{\left[\rho(\mu-1)+\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right]}\left[1-\left(\frac{\alpha}{\frac{1-\pi}{\pi}(1-\tau)(1-\alpha)}-\frac{\tau}{1-\tau}\right)\right] \tag{97}$$

and

$$\Phi(\bar{N}) = \left([(\mu-1)\rho+1]\pi\frac{(1-\alpha)}{\alpha} + \rho(\mu-1) - \frac{\tau}{1-\tau}\right)\frac{1}{\mu}\frac{1-\rho}{\rho} \tag{98}$$

Three cases are possible, here. First, if $\Phi(\underline{N}) > \frac{1}{1-\tau}$, then $\frac{dU}{dN} < 0$ for $N \in (\underline{N}, \bar{N})$. Second, if $\Phi(\underline{N}) < \frac{1}{1-\tau}$ but $\Phi(\bar{N}) > \frac{1}{1-\tau}$, then $\frac{dU}{dN} \geq (<) 0$ if $N \leq (>) \hat{N}$ with $\hat{N} \in (\underline{N}, \bar{N})$. Finally, if $\Phi(\bar{N}) < \frac{1}{1-\tau}$, $\frac{dU}{dN} > 0$ for $N \in (\underline{N}, \bar{N})$.

To begin, from (97), $\Phi\left(\underline{N}\right) > \frac{1}{1-\tau}$ if:

$$\left(\mu-1\right)\rho\pi\left\{\frac{\left(1-\alpha\right)}{\alpha}-\left(\frac{\pi}{\left(1-\pi\right)}+\frac{\alpha}{\left(1-\pi\right)\left(1-\alpha\right)}\right)\right\} >$$

$$\frac{\alpha}{\frac{1-\pi}{\pi}\left(1-\tau\right)\left(1-\alpha\right)}-\frac{\tau}{1-\tau}-\left[\pi\frac{\left(1-\alpha\right)}{\alpha}-\frac{\tau}{1-\tau}\right]\left[1-\frac{\alpha}{\frac{1-\pi}{\pi}\left(1-\alpha\right)}\right]$$

where $\frac{(1-\alpha)}{\alpha}-\left(\frac{\pi}{(1-\pi)}+\frac{\alpha}{(1-\pi)(1-\alpha)}\right) > 0$ if $\pi < \frac{1-2\alpha}{(1-\alpha)}$. By comparison, the term on the right-hand-side is negative if $\pi < \frac{(1-2\alpha)}{(1-\alpha)}$. In this manner, $\Phi\left(\underline{N}\right) > \frac{1}{1-\tau}$ if $\pi < \frac{(1-2\alpha)}{(1-\alpha)}$. The opposite holds. Therefore when $\pi < \frac{(1-2\alpha)}{(1-\alpha)}$, $\frac{dU}{dN} < 0$ for $N \in \left(\underline{N}, \bar{N}\right)$, and the optimal degree of banking competition, N^* is such that: $N^* \in (1, \underline{N}]$. This completes the proof of case (i).

Next, $\Phi\left(\underline{N}\right) < \frac{1}{1-\tau}$ if $\pi > \frac{(1-2\alpha)}{(1-\alpha)}$. Using the definition of $\bar{N}$, $\Phi\left(\bar{N}\right) < \frac{1}{1-\tau}$ if:

$$\left[\pi\frac{\left(1-\alpha\right)}{\alpha}+1-\frac{1}{1-\tau}\frac{1}{1-\rho}\right]\mu < 1-\frac{1}{\rho}\left[\pi\frac{\left(1-\alpha\right)}{\alpha}-\frac{\tau}{1-\tau}\right]+\pi\frac{\left(1-\alpha\right)}{\alpha}$$

where $\left[\pi\frac{(1-\alpha)}{\alpha}+1-\frac{1}{1-\tau}\frac{1}{1-\rho}\right] > 0$ if $\rho < \frac{\pi\frac{(1-\alpha)}{\alpha}-\frac{\tau}{1-\tau}}{\left[\pi\frac{(1-\alpha)}{\alpha}+1\right]} = \tilde{\rho}$, whereas the term on the right hand side is positive if $\rho > \frac{\left[\pi\frac{(1-\alpha)}{\alpha}-\frac{\tau}{1-\tau}\right]}{1+\pi\frac{(1-\alpha)}{\alpha}}$. In this manner, $\Phi\left(\bar{N}\right) < (>)\,\frac{1}{1-\tau}$ if $\rho > (<)\,\tilde{\rho}$. Therefore, if $\pi > \frac{(1-2\alpha)}{(1-\alpha)}$ and $\rho < \tilde{\rho} < \hat{\rho}$, $\frac{dU}{dN} \geq (<)\,0$ if $N \leq (>)\,\hat{N}$ with $\hat{N} \in \left(\underline{N}, \bar{N}\right)$. Under these condition, $\hat{N} = N^*$ is optimal. Finally, if $\rho \in (\tilde{\rho}, \hat{\rho})$, $\Phi\left(\bar{N}\right) < \frac{1}{1-\tau}$ and $\frac{dU}{dN} > 0$ for $N \in \left(\underline{N}, \bar{N}\right)$. Therefore, it is optimal to set $N^* : N^* \in \left[\bar{N}, \infty\right]$. This completes the proof of Proposition 5.

8. **Proof of Proposition 6.** To begin, consider case i, the Friedman rule. The expected utility of a potential depositor under self-insurance is given by (38):

$$u^a = \pi \ln \frac{\pi}{\left(I_t-1\right)} + \left(1-\pi\right)\ln\left[1-\pi\right] + \ln\left(1-\tau\right)w_t + \ln I_t \frac{P_t}{P_{t+1}}$$

From (81), the steady-state consumption of depositors at the Friedman rule is:

$$c^b = \left(1+\rho\left(\mu-1\right)\right)r\left(1-\tau\right)w$$

Therefore, the expected utility if savings are intermediated is:

$$u^b = \ln\left[1+\rho\left(\mu-1\right)\right]r + \ln\left(1-\tau\right)w$$

Simple algebra yields:

$$u^b - u^a = \ln \frac{\left[1 + \rho\left(\mu - 1\right)\right]\left(I_e - 1\right)^{\pi}}{\pi^{\pi}\left[1 - \pi\right]^{1-\pi}}$$

In this manner, $u^b - u^a > 0$ if:

$$\frac{\left[1 + \rho\left(\mu - 1\right)\right]\left(I_e - 1\right)^{\pi}}{\pi^{\pi}\left[1 - \pi\right]^{1-\pi}} > 1$$

Given that $\mu = \frac{I_R}{I_e}$, this condition always holds when I_R is high enough. Evaluating at the lower bound on μ, $\mu = 1$, this condition becomes:

$$I_e > 1 + \pi\left[1 - \pi\right]^{\frac{1-\pi}{\pi}}$$

which is the expression in the text.

Next, consider case ii, where the bank offers complete risk sharing above the Friedman rule, where:

$$\gamma_t^b = \frac{\frac{\pi}{1-\pi} I_t + \left(I_e - I_R\right)\rho}{\left(I_e + \frac{\pi}{1-\pi} I_t\right)}$$

and $c_{t+1}^m = c_{t+1}^n = \frac{1}{1-\pi} r_{t+1}\left(1 - \gamma_t^b\right)\left(1 - \tau\right) w_t$. In this manner, the expected utility under financial intermediation is:

$$u_t^b = \ln \frac{1}{1-\pi} r_{t+1}\left(1 - \gamma_t^b\right)\left(1 - \tau\right) w_t$$

Upon using the expression for γ_t^b and some algebra:

$$u_t^b = \ln \frac{1}{1-\pi} r_{t+1}\left(\frac{I_e - \left(I_e - I_R\right)\rho}{\left(I_e + \frac{\pi}{1-\pi} I_t\right)}\right) + \ln\left(1 - \tau\right) w_t$$

and

$$u_t^a = \pi \ln \gamma_t^a \frac{P_t}{P_{t+1}} + \left(1 - \pi\right) \ln\left[\gamma_t^a \frac{P_t}{P_{t+1}} + R_{t+1}\left(1 - \gamma_t^a\right)\right] + \ln\left(1 - \tau\right) w_t$$

In this manner, subsituting for γ_t^a from (37), it can be easily verified that in the steady-state:

$$u^b - u^a = \ln \frac{\left(\frac{I_e - \left(I_e - I_R\right)\rho}{\left(1-\pi\right) I_e + \pi I}\right)}{\frac{\pi^{\pi}}{\left(I-1\right)^{\pi}}\left[1 - \pi\right]^{1-\pi}}$$

where $u^b - u^a > 0$ if:

$$\frac{I_e - \left(I_e - I_R\right)\rho}{\pi^{\pi}\left[1 - \pi\right]^{1-\pi}} \frac{\left(I_t - 1\right)^{\pi}}{\left(1 - \pi\right) I_e + \pi I} > 1$$

Recall from the solution for the problem:

$$I=\frac{1-\pi}{\pi}\frac{(1-\tau)(1-\alpha)}{\alpha}\frac{(1-\rho)I_e+\rho I_R}{\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right)}-\frac{1-\pi}{\pi}I_e$$

substitute into the inequality above and simplifying, $u^b-u^a>0$ if::

$$\left(\frac{(1-\tau)(1-\alpha)}{\alpha}\frac{(1-\rho)+\rho\mu}{\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right)}-1\right)\frac{1-\pi}{\pi}I_e>1+\left[\frac{(1-\pi)\frac{(1-\tau)(1-\alpha)}{\alpha}\pi^{\pi}\left[1-\pi\right]^{1-\pi}}{\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right)}\right]^{\frac{1}{\pi}}$$

which requires I_R (μ) to be high enough. Evaluating at the lower bound, $\mu_0=\frac{\alpha\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right)}{(1-\pi)(1-\tau)(1-\alpha)\rho}-\frac{(1-\rho)}{\rho}$, the condition becomes:

$$I_e>1+\left[\frac{(1-\pi)\frac{(1-\tau)(1-\alpha)}{\alpha}\pi^{\pi}\left[1-\pi\right]^{1-\pi}}{\left(\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}\right)}\right]^{\frac{1}{\pi}}$$

which is the expression in the text.

Next, consider case iii. From (8) and (10), $c_{t+1}^m=r_t^m(1-\tau)w_t=\frac{1}{\pi}\left(\gamma_{R,t}I_R+\gamma_{e,t}I_e\right)\frac{P_t}{P_{t+1}}(1-\tau)w_t$ and $c_{t+1}^n=\frac{N}{1-\pi}r\left(k_{t+1},K_{t+1}^{N-1}\right)k_{t+1}=\frac{1}{1-\pi}r_{t+1}\left(1-\gamma_t^b\right)(1-\tau)w_t$. In this manner, the expected utility under financial intermediation is:

$$u_t^b=\pi\ln\frac{1}{\pi}\left(\gamma_{R,t}I_R+\gamma_{e,t}I_e\right)\frac{P_t}{P_{t+1}}(1-\tau)w_t+(1-\pi)\ln\frac{1}{1-\pi}R_{t+1}\left(1-\gamma_t^b\right)(1-\tau)w_t \tag{99}$$

Next, impose steady-state and some algebra:

$$u^b-u^a=\ln\frac{1}{\pi^{2\pi}}\left(\rho\left(I_R-I_e\right)+\gamma^bI_e\right)^{\pi}\left(1-\frac{1}{I^*}\right)^{\pi}\frac{\left(1-\gamma^b\right)^{(1-\pi)}}{(1-\pi)^{2(1-\pi)}}$$

which is the expression in the text. Next, $u^b-u^a>0$ if:

$$\frac{1}{\pi^{2\pi}}\left(\rho\left(I_R-I_e\right)+\gamma^bI_e\right)^{\pi}\left(1-\frac{1}{I^*}\right)^{\pi}\frac{\left(1-\gamma^b\right)^{(1-\pi)}}{(1-\pi)^{2(1-\pi)}}>1 \tag{100}$$

Using the definition of γ^b from (28), this condition becomes:

$$\left[\frac{\pi^2}{I_e}\right]^{\pi}\left[\frac{(1-\pi)^2}{\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]}\right]^{1-\pi}\frac{\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\left[\rho\left(\frac{I_R}{I_e}-1\right)+1\right]}<\left(1-\frac{1}{I^*}\right)^{\pi}$$

where under banking:

$$I^* = \frac{\alpha}{\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right](1-\tau)(1-\alpha)} \frac{1}{\frac{1-\rho\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\left(\frac{I_R}{I_e}-1\right)}{\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}+\frac{\tau}{1-\tau}} I_e$$

with $\frac{dI^*}{dI_R} > 0$. Given that the term on the left-hand-side of (100) is decreasing in I_R, the condition holds when I_R is above some level. Sufficient to evaluate the condition at the lower bound on μ, μ_1, where $I^* = \frac{I_e}{\left[1-\frac{1-\alpha}{N}\right]}$. The condition becomes:

$$I_e > \left[1-\frac{1-\alpha}{N}\right] + \pi^2 \left[\frac{\pi(1-\pi)}{\left[1-\frac{1-\alpha}{N}\right]}\right]^{\frac{1-\pi}{\pi}} \left(\frac{\left[1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]\right]}{\left[\rho(\mu_1-1)+1\right]}\right)^{\frac{1}{\pi}}$$

which is the condition in the text.

Finally, consider case iv, where $\gamma_t^b = \rho$. From (38), (99), along with the expression for γ_t^a in the steady-state:

$$u^b - u^a = \ln\left[\frac{\rho I_R}{\pi^2}\left(\frac{I-1}{I}\right)\right]^{\pi} \frac{(1-\rho)^{1-\pi}}{(1-\pi)^{2(1-\pi)}}$$

Therefore, $u^b - u^a > 0$ if:

$$\left[\frac{\rho I_R}{\pi^2}\left(\frac{I-1}{I}\right)\right]^{\pi} \frac{(1-\rho)^{1-\pi}}{(1-\pi)^{2(1-\pi)}} > 1$$

By evaluating at the equilibrium level of I, where

$$I = \frac{\rho I_R}{\rho+\frac{\tau}{1-\tau}} \frac{\alpha}{(1-\rho)(1-\tau)(1-\alpha)}$$

the inequality becomes:

$$\frac{1}{\pi^2}\left[\frac{\rho I_R}{\rho+\frac{\tau}{1-\tau}}\frac{\alpha}{(1-\rho)(1-\tau)(1-\alpha)}-1\right]\left(\rho+\frac{\tau}{1-\tau}\right)\frac{(1-\rho)(1-\tau)(1-\alpha)}{\alpha}\frac{(1-\rho)^{\frac{1-\pi}{\pi}}}{(1-\pi)^{2\frac{(1-\pi)}{\pi}}} > 1$$

By re-writing, we get:

$$I_R > \left(\frac{\left(\rho+\frac{\tau}{1-\tau}\right)}{\rho}\frac{(1-\rho)(1-\tau)(1-\alpha)}{\alpha}+\frac{\pi^2}{\rho}\frac{(1-\pi)^{2\frac{(1-\pi)}{\pi}}}{(1-\rho)^{\frac{1-\pi}{\pi}}}\right)$$

which is the condition in the text. The completes the Proof of Proposition 6.

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