Sunspot Bank Runs and Fragility: The Role of Financial Sector Competition*

Jiahong Gao Robert R. Reed

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Abstract

What is the role of financial sector competition for instability when the potential for run behavior occurs with a non-trivial probability? How does the degree of fragility vary across competitive versus monopolistic banks? We study these questions in a version of the Diamond and Dybvig (1983) model of bank runs with limited commitment. In particular, we demonstrate that as the prospect of a run increases, competitive banks react prudently in issuing liabilities, leaving themselves less unstable. In contrast, if there is greater potential for consumer sentiment to deteriorate, agents' incentives to run on monopolists are stronger over initially lower run probabilities but are weaker if crises are more likely to occur. Interestingly, the monopolist is only less susceptible to a panic over lower probabilities of crises while it is as fragile as its competitive counterpart in a more crisis-prone economy. Nevertheless, once bailouts are available, a higher run probability induces competitive banks to behave imprudently due to the incentive distortion. By comparison, agents' inclination to run on monopolistic banks can either increase or decrease, depending on the structure of financial assistance. In addition, competitive intermediaries are more fragile if bailouts are large but this does not occur if fiscal transfers are small.

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Keywords: Bank Runs; Competition; Limited Commitment

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1 Introduction

In recent years, the increase in banking consolidation in many advanced economies has received a great deal of attention.[1] As the banking sector is dominated by a small number of large intermediaries, they may have an incentive to take advantage of their market power and thus the "Too-Big-To-Fail" (TBTF) problem may become more serious in the future. Given the recent trend towards increasing concentration of the banking system and the emergence of TBTF institutions, it is important to understand how the industrial organization of the banking system matters for financial stability. Notably, a large body of literature examines the effects of competition on bank credit risk exposure and suggests that there exist concentration-stability versus concentration-instability trade-offs.[2] Yet, less attention has been paid to other important sources of risk such as liquidity risk, stemming from the liability side of bank balance sheet, that affect the stability of the banking system.[3] In particular, as highlighted in the seminal paper by Diamond and Dybvig (1983), banks are intrinsically fragile because of the liquidity and maturity transformation functions they perform. By impacting the degree of liquidity transformation, competition affects banks' exposure to liquidity risk and, hence, financial fragility.

The objective of this paper is to develop a framework to study the role of financial sector competition for instability. To this end, we build on a version of the Diamond and Dybvig (1983) model of financial intermediation with limited commitment as in Ennis and Keister (2009). Notably, banks are unable to commit to a course of action but issue payments to withdrawing consumers given the situation at hand. In this setting, we examine how competitive and monopolistic banking systems alter their portfolio choices which in turn affect depositors' incentives to panic and compare their

[1] For example, according to Janicki and Prescott (2006), the largest banks, which make up less than 1% of active financial intermediaries in the United States, held over three-quarters of assets in the banking system. In addition, McCord and Prescott (2014) argue that the long-term pattern of banking consolidation will continue in the U.S. Similar transitions have also occurred in other developed countries such as the largest economies in the OECD. See also Ghossoub and Reed (2019, 2021) and others for a discussion of the recent trend towards increasing banking concentration in the U.S. and across countries.

[2] Specifically, one strand of evidence favors the concentration-stability hypothesis in which less competition encourages banks to become more cautious to protect the present value of the future rents. For example, see Marcus (1984), Keeley (1990), Allen and Gale (2004), Beck et al. (2006), and Bretschger et al. (2012), among others. In contrast, another strand of evidence supports the concentration-instability hypothesis in which risk-taking actions may be induced by the market power of large concentrated banks, undermining the stability of the banking sector. See Boyd and De Nicolo (2005). Other recent contributions obtain mixed results. For instance, Boyd et al. (2004) show that if the nominal rate of interest is below (above) some threshold, a monopolistic banking system will always result in a higher (lower) probability of a crisis. Further, a recent paper by Martinez-Miera and Repullo (2010) highlights the existence of a non-monotonic, U-shaped relationship between competition and stability. Lastly, from an empirical perspective, Berger et al. (2009) find that banks with a higher degree of market power have less overall risk exposure but market power can also increase loan portfolio risk.

[3] One recent exception, among others, is the work by Carletti and Leonello (2019) which is based on a version of the Diamond-Dybvig monopolistic bank with commitment and cash-in-the-market pricing. In particular, the authors restrict their attention to fundamental runs to study the effects of credit market competition and aggregate liquidity risk on banks' risk-taking behavior and thus financial stability.

relative susceptibility to fragility. While few banking economies are highly competitive or monopolistic, studying these two systems highlights the differences in their behavior and also the degree of instability due to competition. More importantly, we study not only consumers' inclination to run but also their participation decisions in a pre-deposit game that justifies the existence of a bank.

We now turn to address the primary questions of interest. To begin, *what is the role of financial sector competition for fragility?* Interestingly, our model suggests that as the probability of a crisis is higher, competitive banks behave prudently in issuing liabilities, leaving themselves less unstable. By comparison, in economies where there is greater potential for consumer sentiment to deteriorate, monopolistic banks respond aggressively by increasing short-term obligations. Yet, a non-monotonicity of agents' incentives to run with respect to the likelihood of a run arises. In particular, the monopolist appears to be most prone to a panic when the prospect of a crisis is intermediate. To provide deeper insights into our results, we next outline the details of our framework.

As a starting point, we characterize the fragility of an active competitive banking system that maximizes the expected utility of depositors. In this setting, a competitive bank does not hold additional liquid assets to fulfill unexpected withdrawals when runs are less likely to occur. Nevertheless, if the potential for run behavior is higher, the bank finds it welfare-improving to hold a more liquid portfolio in order to mitigate costly liquidation should a crisis emerge. Notably, regardless of portfolio choices, the bank issues liabilities in a cautious manner as the run probability rises. Such prudent behavior decreases depositors' incentives to panic and thus promotes the stability of the banking system, implying that there may exist a maximum run probability with which a panic can occur. Yet, an underprovision of risk-pooling services in a more crisis-prone economy may suggest that consumers will not deposit their funds in the initial period. Thus, in contrast to previous research such as Li (2017), endogenizing consumers' financial participation in a pre-deposit game not only justifies the existence of banks but also provides a more precise characterization of financial fragility.

After examining competitive banks, we focus on the analysis of fragility under a monopolistic banking system when the "TBTF" problem is a serious concern for the policymaker. Unlike a competitive intermediary, the risk-neutral monopolist aims to extract as much surplus from depositors as possible. Yet, the bank needs to offer services such that depositors derive expected utility that is not lower than the value of their outside options. In this manner, consumers must have an incentive to deposit their funds and thus the depositors' participation constraint is an important consideration in rent-seeking, strategic behavior of the monopolist. Furthermore, once a crisis occurs, monopolists are assumed to be placed in resolution, which can be viewed as a stylized version of the Orderly Liquidation Authority (OLA) of the Dodd-Frank Act of 2010.[4] As a result, the banker

[4]Though our analysis of orderly liquidation is similar to that in Ennis and Keister (2009), Cooper and Kempf

focuses on maximizing monopoly rents in normal times but must provide depositors with higher returns on deposits than expected payoffs in resolution. In this vein, distortions on payments due to market power are mitigated and the monopolist raises depositors' welfare above what they would have achieved under financial autarky should no runs occur. Therefore, the banking authority has no incentive to intervene in normal times.

In this setup, we show the monopolistic bank's portfolio choice has a non-trivial effect on depositors' incentives to run on the banking system. However, it is first helpful to note that in a more crisis-prone economy, the expected utility derived from a certain level of bank payments and the resulting payoffs in resolution would be lower. Thus, to keep consumers indifferent to financial autarky, the bank must offer larger early deposit returns when the potential for run behavior is higher. Given such an aggressive response and the monopolist's portfolio choice of not holding precautionary liquid assets against crises over lower run probabilities, agents' incentives to panic are unambiguously stronger. In this manner, there may exist a *minimum* probability of a run with which panics can occur under a monopoly. Importantly, this unveils a novel source of fragility in the banking system – if there is greater potential for consumer sentiment to deteriorate, depositors' participation constraints facing the monopolist are tightened, inducing the bank to become more illiquid and unstable.

Yet, once the likelihood of a run is sufficiently high, the bank holds a more liquid portfolio and agents' incentives to panic decrease as crises become more likely. In particular, there are two competing effects at work. First, similar to that when the monopolist does not hold excess liquidity, the early payment is raised owing to the fact that depositors' participation constraints are tightened, which tends to make the monopolist less stable. Nevertheless, lower liquidation costs are incurred should a panic emerge and comparatively more illiquid assets are held to maturity to augment late payoffs in the aftermath of a crisis. Notably, this latter effect dominates over higher run probabilities whenever precautionary liquidity is held, undermining the overall degree of financial fragility. This further suggests that there may exist a maximum probability of a run with which the monopolistic bank is fragile. Interestingly, such a threshold probability is exactly the same across banking economies if both types of intermediaries possess liquid assets as a precaution to mitigate the effects of costly liquidation. This is the case because among these situations, the degree of instability depends only on exogenous parameters but not on different levels of liquidity transformation implemented by banks.

It is also worth emphasizing that bank resolution in this environment is necessary for rent-seeking, strategic behavior of monopolists with limited commitment to arise in equilibrium. To

(2016), and Keister and Mitkov (2021), our environment differs from theirs. In particular, all of the papers mentioned above focus on perfectly competitive banks, but we offer the first analysis of the effects of the resolution process on banks with market power in the tradition of Diamond and Dybvig (1983).

understand why, suppose that monopolistic banks are able to choose the payment schedule in all states, where the only constraint is to deliver the expected utility amounting to financial autarky to consumers. However, should the good aggregate state not occur, the participation constraint involving payments in both normal and crisis times would be undefined. That is, in the absence of a banking regulator, payoffs in the inferred panic state are indeterminate as there is nothing to constrain the bank's actions. From this perspective, an equilibrium with a monopolist would not exist as, ex-ante, the bank would not be able to attract any deposits.

Further, given the different best responses to an incipient crisis and patterns of instability, *how does the degree of fragility vary across competitive versus monopolistic banking systems?* Our work shows that though the monopolist remains relatively less illiquid due to the fact that it delivers lower welfare than the competitive bank, the relative stability of the monopolist is not necessarily augmented. In particular, the monopolistic bank is better for stability over lower crisis probabilities but it is as fragile as its competitive counterpart when the potential for run behavior is higher. To see this, if panics are less likely to occur, both banking structures do not hold excess liquidity to meet unexpected withdrawals in the event of a run. Among these situations, less illiquidity of the monopolist implies that the bank is able to offer higher ex-post payoffs on deposits during a panic, leaving itself less susceptible to a crisis. On the other hand, as both economies possessing extra liquid assets admit a same maximum probability with which a run can occur, the degree of fragility is the same when the likelihood of a panic is higher. Overall, the scope for instability under a monopolistic bank is weakly contained in that under its competitive counterpart.

To deepen our understanding of the role of competition for financial fragility, we conclude by extending the benchmark framework to incorporate government bailouts à la Keister (2016) and Keister and Narasiman (2016). Notably, the current structure is the first to examine the implications of banking competition in a model with limited commitment where long-term assets are illiquid and there exists public insurance. In particular, when fiscal transfers are available, competitive banks behave imprudently by ignoring their activity in times of crisis due to the moral hazard associated with bailouts. By comparison, as the probability of a run increases, agents' inclination to run on a monopolistic bank can either increase or decrease, depending on the structure of financial assistance. Yet, what remains robust is that banks subject to fragility may only emerge in equilibrium if and only if the potential for run behavior is not too high. Moreover, when a crisis becomes more likely to occur, bailouts affect a competitive and a monopolistic bank's responses through different channels. In particular, unlike competitive intermediaries whose incentives are distorted due to the provision of bailouts, monopolists never completely ignore but discount possible activity in times of crises. This necessarily happens as depositors' expected utility in the event of a crisis enters through the binding participation constraint and bailouts only tighten or relax such a constraint. In this vein, competitive banks may be more fragile if bailouts are large but the

reverse can be the case if fiscal transfers are small.

The remainder of the paper is organized as follows. In the next section, we turn to the literature review. We then present the baseline model and discuss its key assumptions and the definition of financial fragility in Section 3. In Section 4, we analyze competitive banks. In Section 5, we conduct the analysis with monopolists and further compare the degree of fragility and welfare across these two banking economies in Section 6. Section 7 extends the model to examine banking activity and instability in the presence of government bailouts. Section 8 concludes. Details of formal arguments are presented in the appendix.

2 Related literature

The model presented in this paper relates to a strand of literature that studies financial fragility in the Diamond-Dybvig tradition. In particular, Ennis and Keister (2009) highlight the role of limited commitment on the part of the banking authority or the bank for inducing a panic equilibrium, which would otherwise be prevented by an off-equilibrium suspension. Notably, a growing literature extending Ennis and Keister (2009) largely focuses on a perfectly competitive banking system while less attention has been paid to concentrated banking systems. One recent exception is the work by Gao and Reed (2021) that extends the Ennis and Keister (2009) model to examine time-consistent suspensions of convertibility under a monopolist. The authors highlight how the preferences of a regulator matter for stability: appointing an authority who puts sufficient weight on a banker's welfare acts as a commitment device that implements the unique good equilibrium.

However, the modelling strategy of Gao and Reed (2021) differs from the current paper in several critical dimensions. Most importantly, as Gao and Reed attempt to seek for substitutes for commitment that eliminate runs, they assume that the monopolist offers simple "contracts" that stipulate consumption in different periods and runs are zero-probability events. Yet, the present paper aims to study the consequences of instability and hence we show how the bank engages in rent-seeking, strategic behavior when the crisis probability is non-trivial.

Next, our analysis also relates to studies that examine portfolio choices with a positive probability of a crisis in the standard Diamond-Dybvig framework. For example, Ennis and Keister (2006) clarify part of the analysis in Cooper and Ross (1998) and demonstrate how the run possibility affects investment decisions made by competitive banks. Specifically, the authors show that unlike that in Cooper and Ross, precautionary liquid assets against runs will not be held and the effects of a higher run probability on the level of investment critically depend on the liquidation cost. Though both contributions shed light on the important relationship between economic conditions and financial instability, banks are assumed to be able make some type of commitment to their payment scheme. More recently, Li (2017) studies the relationship among interest rates,

portfolio choices, and banking instability in a version of Diamond and Dybvig with limited commitment under perfect competition. While Li offers a new method for measuring financial fragility, depositors' financial participation, which matters for the scope for the bank's vulnerability to a run, is not taken into account. In contrast to the existing literature, we develop frameworks with both competitive and monopolistic banks with limited commitment where consumers' deposit decisions in a pre-deposit game are endogenized.

Regarding the relative fragility across competitive structures, our work contributes to the existing literature such as Chang and Velasco (2001) and Matsuoka (2013) who show that the parameter space in which a run equilibrium exists is always larger for a competitive bank than for a monopolistic bank. Nevertheless, all of the papers mentioned above again place restrictions on the bank's payoffs on deposits and the probability of a run is equal to zero. In contrast, a crisis occurs with a non-trivial probability and renegotiating obligations is allowed by the bank subject to fragility in the present paper. In this manner, we are able to account for the impact of flexible payment schemes which adjust to available information at each point in time on relative instability. Interestingly, our result that banks with market power can be as fragile as competitive intermediaries has not been shown in a variant of Diamond and Dybvig (1983).

Last but not least, our paper also closely relates to that by Andolfatto and Nosal (2008) which is based on a version of the Diamond-Dybvig model developed by Green and Lin (2003) and Peck and Shell (2003). The authors study the design of bank contracts and the possibility of bank run equilibria of a self-interested banker in the presence of aggregate uncertainty about the total fundamental demand for early consumption. In particular, they establish that though the banker can commit, he may have an incentive to fabricate the situation in order to maximize profits. To investigate alternative schemes that could be used to give proper incentives to the banker, Andolfatto and Nosal construct the optimal incentive-feasible allocation. Further, the interesting implication for financial fragility is that bank run equilibria under a monopoly do not exist. By comparison, in a setting without uncertainty about consumers' total need for liquidity in the short term and with limited commitment of the monopolist, equilibria in which patient depositors misrepresent their types may still exist.

3 The model

The model is based on a version of Diamond and Dybvig (1983) where intermediaries or banking authorities have limited commitment as in Ennis and Keister (2009) but consumers' deposit decisions in a pre-deposit game are also considered. We begin by describing the physical environment and defining banking fragility in this framework.

3.1 The environment

3.1.1 Time

There are three time periods indexed by $t = 0, 1, 2$.

3.1.2 Consumers

The economy is populated by a $[0, 1]$ continuum of ex-ante identical consumers indexed by i. Each consumer is endowed with one unit of the all-purpose good at the start of period 0 and the utility function of an agent is expressed by:

$$u\left(c_1, c_2; \omega_i\right) = \frac{\left(c_1 + \omega_i c_2\right)^{1-\gamma}}{1-\gamma},$$

where c_t denotes the consumption level in period $t = 1, 2$. For each consumer, the parameter ω_i represents her type and is a privately observed binomial random variable with support $\Omega \equiv \{0, 1\}$. With probability π, an individual i is an impatient consumer, i.e., $\omega_i = 0$, and values consumption in period 1 only; with probability $1 - \pi$, an individual i is patient, i.e., $\omega_i = 1$, and values consumption equally in both periods. Further, consumers' preference types are independent and identically distributed (i.i.d). Due to the law of large numbers, the fraction of impatient or patient consumers in the population is also π or $1 - \pi$. Following Diamond and Dybvig (1983), the coefficient of relative risk aversion is greater than unity.

3.1.3 Technologies, financial intermediation, and limited commitment

There are two technologies in the economy. For example, one unit of the good invested in a simple storage technology yields one unit of the consumption good in the following period. There is also a constant-returns-to-scale investment technology in which one unit of the good invested in period 0 yields $R > 1$ units of goods in period 2, but $r \in [0, 1]$ units if liquidated early. Thus, $1 - r$ can be interpreted as the liquidation cost per unit of goods invested, as in Cooper and Ross (1998).

As a benchmark, consumers may form coalitions referred to as banks that invest in liquid and illiquid assets in order to insure themselves against idiosyncratic liquidity risk, as in Diamond and Dybvig (1983). Thus, the objective of intermediary j is to maximize the expected utility of its depositors. Importantly, welfare derived from banking services should not be lower than the value of consumers' outside option. That is, resources are not pooled if the bank delivers lower welfare to depositors than financial autarky.[5] Note that consumers also have access to both technologies which affect their reservation utility. In particular, under autarky, individuals hold a portfolio

[5] See also the pre-deposit game in Peck and Shell (2003), among others.

consisting of $1-x$ units of liquid assets and x units of illiquid assets to maximize:

$$W = \max_{\{x\}} \pi u\left(1-x+rx\right) + \left(1-\pi\right) u\left(1-x+Rx\right).$$

Notably, let x^A and W^A denote the optimal level of illiquid assets and autarky welfare respectively. In particular, when the probability of receiving a liquidity shock is small enough, i.e., $\pi \in \left(0, \frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}\right]$, the portfolio choice under autarky would be such that all resources are invested in the illiquid technology. That is, $x^A = 1$. Yet, over intermediate levels of liquidity risk, i.e., $\pi \in \left(\frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}, \frac{R-1}{R-r}\right)$, agents hold a more liquid portfolio where $x^A = \frac{\left(\frac{R-1}{1-r}\frac{1-\pi}{\pi}\right)^{\frac{1}{\gamma}}-1}{\left(\frac{R-1}{1-r}\frac{1-\pi}{\pi}\right)^{\frac{1}{\gamma}}(1-r)+(R-1)}$. On the other extreme, if consumers' degree of exposure to liquidity risk is sufficiently high, i.e., $\pi \in \left[\frac{R-1}{R-r}, 1\right)$, $x^A = 0$, implying that consumers keep all funds in the storage technology to avoid costly liquidation. Therefore, autarkic utility is summarized as follows:

$$W^A = \begin{cases} \frac{\pi r^{1-\gamma}+(1-\pi)R^{1-\gamma}}{1-\gamma} & \text{if } \pi \in \left(0, \frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}\right] \\ \frac{(R-r)^{1-\gamma}\left[\pi+(1-\pi)\left(\frac{R-1}{1-r}\frac{1-\pi}{\pi}\right)^{\frac{1-\gamma}{\gamma}}\right]}{(1-\gamma)\left[\left(\frac{R-1}{1-r}\frac{1-\pi}{\pi}\right)^{\frac{1}{\gamma}}(1-r)+(R-1)\right]^{1-\gamma}} & \text{if } \pi \in \left(\frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}, \frac{R-1}{R-r}\right) \\ \frac{1}{1-\gamma} & \text{if } \pi \in \left[\frac{R-1}{R-r}, 1\right) \end{cases}, \tag{1}$$

where $r \in (0,1)$. In addition, it is straightforward that if long-term assets are completely illiquid, i.e., $r = 0$, it would never be optimal for agents to invest all their endowments in the illiquid technology. Yet, if the liquidation cost is negligible, i.e., $r = 1$, the portfolio choice is trivial where total funds are kept with the productive technology.

In response to increasing concern about market power, we further study rent-seeking, strategic behavior by monopolists. Thus, in contrast to the previous benchmark, we alternatively consider that there are risk-neutral bankers who have the exclusive right to offer banking services. While the objective of monopolist j is to maximize its own profits, the bank needs to keep depositors indifferent to financial autarky so that consumers must have an incentive to deposit their funds.

In a similar spirit to that of Ennis and Keister (2009), both competitive and monopolistic banks with limited commitment offer flexible payment schedules which adjust to available information at each point in time. It is worth noting that with full commitment, runs would not be an issue in this setting as the bank could always eliminate run equilibria by threatening to suspend redemptions once self-fulfilling behavior is identified.

3.2 Bank panics and resolution

Following Cooper and Ross (1998) and Peck and Shell (2003), among others, we introduce an exogenous sunspot signal on which depositors can condition their actions. In particular, there are two states of the world, $s \in S \equiv \{\alpha, \beta\}$, with respective probabilities $\{1-q, q\}$. The value of q is assigned strictly between 0 and 1, so the potential for a run occurs with a non-trivial probability. Note that the realization of the sunspot state is revealed to depositors immediately at the beginning of period 1 whereas the bank needs to infer the aggregate state from the flow of withdrawals.

Specifically, since a fraction π of depositors in a bank are impatient and have an urgent need to consume in period 1, they will choose to withdraw early with certainty. Thus, if only π depositors show up in period 1, runs do not take place and therefore both competitive and monopolistic banks would resume their operations without any public intervention. That is, as the bank is still able to raise depositors' welfare above what they would have achieved under financial autarky should no runs occur, the banking authority has no incentive to intervene.

Nevertheless, the bank experiences runs if more than a fraction π of depositors request withdrawals before illiquid assets mature. Among these situations, we attempt to discuss two cases: an intermediary with and without a bank authority. If the authority intervenes, for example, she will: (i) take control of the bank, (ii) decide how much long-term assets to liquidate, and (iii) dictate the remaining payments to maximize returns to creditors who have not contacted the bank. That is, the regulator will implement the efficient continuation allocation of resources and all banks earn zero profits in the process of a resolution. More importantly, as we will demonstrate later, incorporating bank resolution is necessary for the behavior of the monopolistic bank with limited commitment to arise in equilibrium.

3.3 Equilibrium of the withdrawal game and partial runs

As discussed above, at the beginning of period 1, depositors decide whether to contact the bank in period 1 or to wait until period 2 after learning their liquidity needs and the aggregate state. In particular, for each combination of consumers' preference type and the sunspot state, the function for a withdrawal strategy for an agent i at bank j is:

$$y_i^j : \{0,1\} \times \{\alpha, \beta\} \mapsto \{0,1\}.$$

Specifically, $y_i^j = 0$ and $y_i^j = 1$ correspond to withdrawing in periods 1 and 2 respectively. Moreover, let y^j denote a complete profile of strategies for all depositors at bank j and χ^j the repayment strategy for bank j. Notably, we study equilibria of such a withdrawal game where: (i) each depositor determines y_i^j to maximize her expected utility $\sigma_i^j\left(\chi^j, \left(y_i^j, y_{-i}^j\right)\right)$; (ii) competitive bank j

(monopolistic bank j) chooses χ^j to maximize the expected utility of its depositors (its own profits) $U^j\left(\chi^j, y^j\right)$; (iii) each agent's strategy is a best response to the strategies of others. Thus, the formal definition of an equilibrium is as follows:

Definition 1. *The equilibrium of the withdrawal game at bank j is the profile of strategies (χ^{j*}, y^{j*}) such that:*

i. $\sigma_i^j\left(\chi^{j*}, \left(y_i^{j*}, y_{-i}^{j*}\right)\right) \geq \sigma_i^j\left(\chi^{j*}, \left(y_i^j, y_{-i}^{j*}\right)\right)$ *for all y_i^j, all $i \in [0,1]$, and all j;*

ii. $U^j\left(\chi^{j*}, y^{j*}\right) \geq U^j\left(\chi^j, y^{j*}\right)$ *for all χ^j and all j.*

To proceed, without loss of generality, depositors withdraw funds according to their preference types in state α, which is labeled the good state. Nevertheless, runs may occur in state β, which is called the panic or crisis state. We focus on actions taken by patient depositors, since impatient agents only value consumption in period 1 and will choose to withdraw early with certainty. In this manner, we consider a specific partial-run strategy profile for depositors at bank j as follows:

$$\begin{aligned} & y_i^j\left(\omega_i, \alpha\right) = \omega_i \qquad \text{for } i \in [0,1] \text{ and} \\ & y_i^j(\omega_i, \beta) = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \quad \text{for } \left\{ \begin{array}{c} i \in [0, \pi] \\ i \in (\pi, 1] \end{array} \right\}. \end{aligned} \tag{2}$$

In this profile, the patient consumer with $i \leq \pi$ at bank j chooses to run in period 1 in the crisis state. As we will show below in our analysis, once the remaining consumption levels are rescheduled after the first π withdrawals, all remaining patient depositors have no incentive to run.[6] In this vein, we next state the formal definition of financial fragility.

Definition 2. *If the strategy profile (2) is part of an equilibrium, the banking system is defined to be fragile.*

Note also that this form of fragility is different from that in Diamond and Dybvig (1983) where depositors run en masse and there is no payment rescheduling. However, the interpretation of relative fragility across banking structures in the present paper would be the same as that in a model where full bank runs are possible. That is, if the parameter space in which a run equilibrium exists is larger, then a banking economy is said to be more fragile.

[6] As in Keister (2016) and Li (2017), we restrict our attention to a single wave run to simplify notation and concentrate on the role of competition for financial instability. One could generalize the model by introducing multiple waves of runs in an environment of a richer sunspot variable as in Ennis and Keister (2010). However, employing such an approach does not alter the main mechanisms and results in the model qualitatively.

3.4 Timeline of events

The timing of events is summarized in Figure 1. In period 0, after receiving their endowments, consumers decide whether to use the banking service. If the expected utility of the risk-sharing arrangement offered by the intermediary is not strictly worse than autarky, agents then deposit their funds. Furthermore, they choose state-contingent withdrawal strategies after which the bank makes a portfolio choice about how much of its attracted deposits are placed into productive investment.

At the beginning of period 1, depositors follow their withdrawal plan once they learn their liquidity needs and the realized aggregate state. For tractability, if individuals decide to withdraw in period 1, depositors know their position in the queue, following Green and Lin (2000), Andolfatto et al. (2007), and Ennis and Keister (2010). Thus, depositors will arrive one at a time in the order given by their index and must consume immediately upon arrival. Following Wallace (1988, 1990), the bank is subject to this sequential service constraint where it only observes that a depositor has come and thus the payment to each withdrawing individual can only depend on the number of total withdrawals that have been made. As discussed above, the bank with limited commitment offers a flexible payout scheme. That is, once the bank infers the aggregate state after a fraction π of depositors have withdrawn, payments to the remaining depositors will be rescheduled based on the updated information.

In period 2, depositors who have not withdrawn contact their bank and the banker will access its profits, if any, after making payments to all depositors.

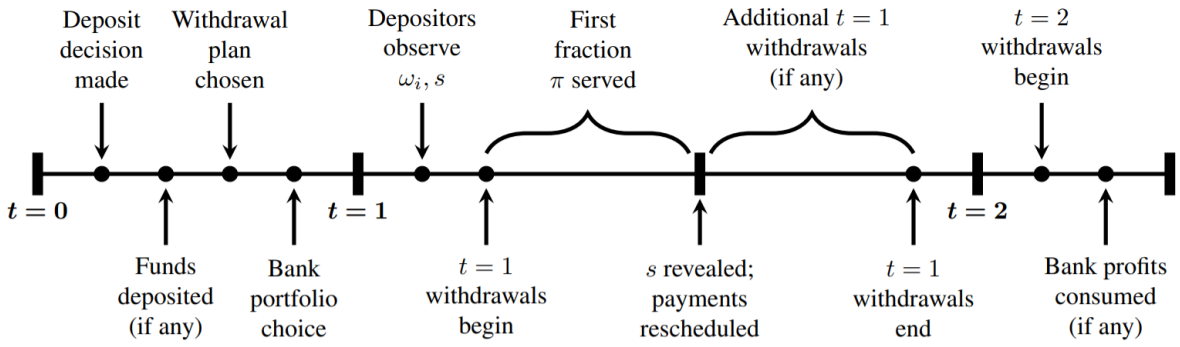

Figure 1: Timeline

4 Equilibrium under a competitive banking system

We begin our analysis with competitive banks. Note that a burgeoning literature extending Ennis and Keister (2009) exogenously assumes that agents always pool their resources to form coalitions,

irrespective of the probability of a crisis. In comparison to prior contributions such as Li (2017), we endogenize consumers' deposit decisions in a pre-deposit game, which allows us to characterize financial fragility more precisely. Note also that whether an intermediary subject to instability is active in equilibrium depends on how both consumers and the bank respond in periods 1 and 2. Consequently, we need to assume that a bank exists in period 0 and analyze backwards. Thus, in the next subsection, we first derive the best responses of a competitive bank to the strategy profile (2) where the approach follows that of Keister (2016).[7] Specifically, we investigate how intermediaries adjust their payments to depositors after the sunspot state is revealed. Subsequently, we look at what banks would choose to offer to the first fraction π of depositors and their portfolio choices. We then study how the probability of a crisis affects a bank's susceptibility to a panic and thus the existence of run equilibria. Since all banks face the same decision problem, we omit the indexation j for each intermediary to simplify the notation unless otherwise noted.

4.1 The best-response allocation

4.1.1 Ex-post allocation of consumption

To begin, the first fraction π of a bank's depositors who withdraw in period 1 will receive payment c_1. As a competitive bank aims to maximize depositors' welfare, it seeks to reallocate the remaining resources efficiently. Specifically, after the bank has inferred the sunspot signal, it can calculate the fraction of remaining depositors who are impatient and will contact the bank in period 1 based on the strategy profile (2). In particular, since runs do not occur in the good state and all impatient depositors have withdrawn from their bank, no remaining depositors will contact intermediaries. Yet, an extra fraction $(1-\pi)\pi$ of depositors will still demand period-1 consumption if a run has occurred. In this vein, upon the realization of the crisis state, intermediaries will choose to give a common amount $c_{1\beta}$ to each remaining impatient depositor in period 1. In addition, each remaining patient depositor who withdraws in period 2 will receive a common amount c_{2s} for $s = \alpha, \beta$.[8] Thus, consumption levels will be chosen to solve:

$$V_\alpha \equiv \max_{\{c_{2\alpha}\}} (1-\pi)\, u\left(c_{2\alpha}\right) \quad \text{and} \quad V_\beta \equiv \max_{\left\{c_{1\beta}, c_{2\beta}\right\}} (1-\pi)\left[\pi u\left(c_{1\beta}\right) + (1-\pi)\, u\left(c_{2\beta}\right)\right]. \quad (3)$$

To determine the bank's constraint set, note that it will never be optimal for banks to resort

[7] It is worth emphasizing again that while the backwards induction approach we use here is similar to that in Keister (2016), we do not include taxes or public goods. In this respect, our benchmark model with perfect competition looks at fragility free of any influence from the fiscal authority that is present in the Keister (2016) model.

[8] Note that if a run is underway, a competitive bank with limited commitment will implement the efficient continuation allocation of resources. Such an outcome would be the same if a benevolent banking authority takes control of the bank and then dictates payments to maximize welfare of agents who have not contacted the bank. Consequently, our approach here without introducing the authority and bank resolution serves only to simplify the analysis.

to costly liquidation in normal times. Otherwise, the bank could hold more liquid assets so that higher consumption levels are made to all depositors. Furthermore, it would be inefficient for intermediaries to hold liquid assets until period 2 in the crisis state, as intermediaries could augment payoffs by investing more in productive assets. In turn, banks may hold liquid assets in state α while liquidating long-term investment to meet extra period-1 withdrawals upon the revelation of state β. Letting x represent units of deposits invested in the illiquid technology, the competitive bank's resource constraints can be expressed as:

$$\begin{aligned} 0 &\leq 1 - x - \pi c_1 && (4) \\ (1-\pi)\, c_{2\alpha} &= 1 - x - \pi c_1 + Rx && (5) \\ 1 - x - \pi c_1 &\leq (1-\pi)\, \pi c_{1\beta} && (6) \\ (1-\pi)^2\, c_{2\beta} &= R\left\{ x - \frac{1}{r}\left[(1-\pi)\, \pi c_{1\beta} - (1 - x - \pi c_1) \right] \right\} && (7) \end{aligned}$$

First, all payments made to the first fraction π of depositors come from liquid assets, as suggested by the constraint (4). Furthermore, if such a constraint binds, the bank exhausts all of its holdings of liquid assets upon the revelation of the sunspot state and this is referred to as a no-excess liquidity state. Otherwise, the bank is still able to make payments in either period 1 in state β or in period 2 in state α using the remaining liquid assets and among these cases, it is said that the bank holds excess liquidity. Once the good aggregate state is inferred, the constraint (5) states that the return on deposits in normal times is determined by the standard pro rata division of excess liquid assets held until period 2, if any, and matured long-term assets. Yet, if a run is identified, the constraint (6) means that all resources held in storage must be consumed. Moreover, the bank needs to liquidate illiquid investment to fulfill additional withdrawal requests in period 1 if this constraint does not bind. Lastly, the priod-2 consumption level in the panic state is an even share of the bank's remaining resources, as implied by the constraint (7).

Thus, in solving this optimization problem, we divide the above constraint set into three distinct scenarios as described in Table 1 below. Under Scenario I, the bank only holds liquid assets for the first fraction π of agents in state α and liquidates long-term investment for any additional period-1 withdrawals beyond the first π depositors in state β. Under Scenario II, some of the returns from holdings of short-term assets are paid to patient consumers in period 2 in normal times but costly liquidation occurs should a run emerge. Under Scenario III, the bank holds a sufficiently liquid portfolio so that any withdrawal demand is met without liquidation of productive assets.

	State α	State β
Scenario I	No-excess liquidity	Liquidation
Scenario II	Excess liquidity	Liquidation
Scenario III	Excess liquidity	No liquidation

Table 1: Three scenarios

Letting μ_1, $\mu_{2\alpha}$, $\mu_{1\beta}$, and $\mu_{2\beta}$ denote the multipliers on the resource constraints (4) to (7), the first-order conditions characterizing the payments $c_{2\alpha}$, $c_{1\beta}$, and $c_{2\beta}$ are given by:

$$u'(c_{2\alpha}) = \mu_{2\alpha}, \quad u'(c_{1\beta}) = \frac{R}{r}\mu_{2\beta} - \mu_{1\beta}, \quad \text{and} \quad u'(c_{2\beta}) = \mu_{2\beta}. \tag{8}$$

The above conditions imply that the ex-post optimal payout scheme entails: $\frac{c_{2\beta}}{c_{1\beta}} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}}$ or $\frac{c_{2\beta}}{c_{1\beta}} < \left(\frac{R}{r}\right)^{\frac{1}{\gamma}}$, depending on whether long-term assets are liquidated should the crisis state be revealed.

4.1.2 Early payments

We move backwards to examine how much consumption the bank must give to each of the first fraction π of depositors, recognizing that ex-post it would reallocate the remaining resources according to (8). Since the bank has not inferred the sunspot signal and depositors are risk averse, it would choose to offer the same payment c_1 to maximize:

$$W \equiv \max_{\{c_1\}} \pi u(c_1) + (1-q) V_\alpha + q V_\beta, \tag{9}$$

subject to the resource constraints (4) to (7).

The first-order condition for this problem is:

$$u'(c_1) = (1-q)(\mu_1 + \mu_{2\alpha}) + q\left(\frac{R}{r}\mu_{2\beta} - \mu_{1\beta}\right), \tag{10}$$

which implies that the bank equates the marginal utility of early payments to the expected marginal value of resources taking into account all possible outcomes in both sunspot states. In particular, the term $\left(\frac{R}{r}\mu_{2\beta} - \mu_{1\beta}\right)$ on the right-hand-side of equation (10) indicates that the bank weighs the shadow value of resources in period 2 against the possibility of costly liquidation in period 1. Specifically, if the shadow value of resources in period 2 in state β is high and costly liquidation occurs, then payments to the first fraction π of consumers would be lower. However, if the bank can pay to all impatient consumers based strictly on its holdings of liquid assets, then payments to the first π agents would be higher.

4.1.3 Portfolio choice

We now examine how the intermediary makes its portfolio choice. In particular, x will be chosen to maximize (9) subject to the resource constraints (4) to (7). The optimality condition regarding the portfolio choice is as follows:

$$(1-q)\left(\mu_1+\mu_{2\alpha}\right)+q\left(\frac{R}{r}\mu_{2\beta}-\mu_{1\beta}\right)=R\left[(1-q)\,\mu_{2\alpha}+q\mu_{2\beta}\right]. \tag{11}$$

That is, a portfolio is held to balance the expected marginal utility in all states and the average of period-2 marginal values of resources across states.

4.2 Equilibrium fragility and allocations

The preceding analysis demonstrates the best responses of competitive intermediaries to the strategy profile (2), and there are three possible allocations in total. Letting the superscripts I, II, and III designate the outcomes under each scenario, we can express the best-response allocations as three different vectors, where each vector contains information about the bank's portfolio choice and payments to depositors:

$$c^*\equiv\left\{x^*,c_1^*,c_{2\alpha}^*,c_{1\beta}^*,c_{2\beta}^*\right\}\quad\text{for}*=I,II,III.^{9}$$

Note that the portfolio choice and consumption levels specified above are jointly defined by conditions (8), (10), (11), and the resource constraints (4) to (7).

In what follows, we discuss under what conditions the strategy profile (2) is part of an equilibrium. To begin, we only focus on actions taken by patient depositors since impatient agents with an urgent need to consume will choose to withdraw in period 1 with certainty. We then begin by considering the behavior of patient individuals with $i>\pi$. Specifically, as there is no uncertainty once the aggregate state is identified, the bank sets returns on deposits such that $c_{2\beta}^*>c_{1\beta}^*$ under the ex-post allocation of consumption. As a result, withdrawing in period 2 is a strictly dominant strategy for a patient depositor $i>\pi$.

To proceed, we consider separately two possible cases for patient agents who have the chance to withdraw before the state of nature is inferred by the bank. In particular, a patient depositor $i\leq\pi$ receives $c_{2\alpha}^*$ should the good state be realized and c_1^* if she contacts the bank in period 1. In this regard, it can be shown that $c_{2\alpha}^*>c_1^*$ across all scenarios and thus no runs occur in normal times. By comparison, if the panic state is realized at the start of period 1, a patient agent $i\leq\pi$ anticipates to receive $c_{2\beta}^*$ if she waits until period 2 but c_1^* if she joins the run. Consequently, the

[9] See the Appendix for the calculated consumption levels.

partial-run strategy profile specified in (2) is part of an equilibrium if and only if $c_1^* \geq c_{2\beta}^*$.

Notably, allocations under Scenarios I and II can be run prone in the sense that there is room for the condition $c_1^* \geq c_{2\beta}^*$ to hold whereas Scenario III is not. To see why this is the case, note that if the bank holds a sufficiently liquid portfolio so that liquidation is not needed in state β, $1-x^{III}-\pi c_1^{III} = (1-\pi)\,\pi c_{1\beta}^{III}$ and $(1-\pi)^2\, c_{2\beta}^{III} = Rx^{III}$. Substituting these two conditions into the constraint (5) and some simplifying algebra yields $\frac{c_{2\alpha}^{III}}{c_{2\beta}^{III}} = \pi \frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} + (1-\pi) < 1$, as $c_{1\beta}^{III} < c_{2\beta}^{III}$. Furthermore, $c_{2\alpha}^{III} < c_{2\beta}^{III}$ and $c_1^{III} < c_{2\alpha}^{III}$ jointly imply that $c_1^{III} < c_{2\beta}^{III}$, which violates the incentive to run condition $c_1^* \geq c_{2\beta}^*$. It also follows that if $r = 0$, the solution to the bank's optimization problem lies under this scenario. Consequently, we restrict our attention to Scenarios I and II and $r \in (0, 1]$ throughout the remainder of the analysis of the competitive bank.

Lemma 1. *If the competitive banking system is fragile, the solution to the bank's optimization problem belongs to either Scenario I or II.*

It is also worth noting that when runs are less likely to occur, it would be optimal for competitive banks to hold liquid assets for the first fraction π of agents only while liquidating investment if some depositors run on the bank. Nevertheless, if the potential for run behavior is higher, banks find it efficient to hold excess liquid assets as a precaution to mitigate the effects of costly liquidation should the crisis state be identified.

4.3 The impact of a higher run probability on fragility

We now examine our primary question of interest: Given the portfolio choice of banks, what are the effects of a higher run probability on depositors' incentives to panic? The following proposition demonstrates that over higher probabilities of a crisis, competitive banks are less vulnerable to a run.

Proposition 1. *Suppose that run equilibria exist. Under this condition, regardless of the competitive bank's portfolio choice, depositors have less incentives to panic if the probability of a run is higher.*

When the potential for a crisis is more likely, the bank allocates resources in a cautious manner. In particular, the bank reduces the early payments while setting aside more income for depositors to be paid once long-term assets mature in the crisis state. As a result, such prudent behavior mitigates depositors' incentive to run on the banking system.

Lastly, we turn to study the existence of run equilibria by providing a precise parameter region of the run probability in which the results in Proposition 1 hold. In this setup, Li (2017) focuses on a post-deposit game and shows that depositors would run on the bank as long as the likelihood of

a crisis is lower than a maximum probability. Nevertheless, as discussed earlier, it would be necessary to explicitly consider a pre-deposit game where competitive banks are expected to deliver welfare not lower than autarky. Only in this way can the existence of banks be justified. Moreover, as we will show below, both consumers' deposit decisions and the bank's response jointly determine the existence of a run equilibrium.

To begin, we concentrate on situations in which $q \in \left(0, q^I\right]$, where $q^I = \frac{1}{1+\left(\frac{1-r}{R-1}\right)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^{\gamma}\left(\frac{R}{r}\right)^{\gamma}}$, and therefore it would be optimal for banks not to hold excess liquid assets. Notably, it follows from Proposition 1 that intermediaries behave more cautiously as q rises and hence agents have an incentive to panic if the probability of a crisis is relatively low. Specifically, $c_1^I \geq c_{2\beta}^I$ if and only if:

$$q \leq \frac{\frac{1}{R}\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^{\gamma}\left(\frac{R}{r}\right)^{\gamma-1} - 1}{\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^{\gamma}\left(\frac{R}{r}\right)^{\gamma-1} - 1} = q_F^I. \tag{12}$$

Note that if $\pi > \pi_F^I = \frac{R^{\frac{1}{\gamma}}-1}{\left(\frac{R}{r}\right)^{\frac{\gamma-1}{\gamma}}-1}$, $q_F^I \in (0,1)$. That is, when the fraction of impatient depositors is large enough, banks may be subject to instability for any $q < q_F^I$. The intuition is as follows. The intermediary infers the sunspot state after the first π redemptions have taken place and if there are more individuals with an urgent need for their consumption, runs, if any, will be identified relatively late. In this vein, banks tend to be prone to fragility over higher values of π as they only have fewer resources left for ex-post payoffs. It is worth emphasizing that if $\gamma \in (1,2)$ and $R < r^{\frac{1-\gamma}{2-\gamma}}$ or $\gamma \geq 2$, $\pi_F^I < 1$ and thus there is room for the condition $\pi > \pi_F^I$ to hold. In this sense, runs may only occur when agents are highly risk-averse. Otherwise, the asset return R needs to be low enough to give depositors an incentive to participate in the run.

Note also that $q^I < q_F^I$ if and only if $\pi > \pi^I > \pi_F^I$, where $\pi^I = \frac{\left(\frac{R^2-2rR+r}{(1-r)R}\right)^{\frac{1}{\gamma}}-1}{\left(\frac{R}{r}\right)^{\frac{\gamma-1}{\gamma}}-1} \geq 1$ if $\gamma \leq \gamma^I = \frac{\ln\left(\frac{R^2-2rR+r}{(1-r)R}\right)}{\ln\left(\frac{R}{r}\right)} + 1$. Thus, suppose that $\gamma \in \left(1, \min\left\{\gamma^I, 2\right\}\right]$ and $R < r^{\frac{1-\gamma}{2-\gamma}}$ or $\gamma \in \left(2, \gamma^I\right)$ such that $\pi_F^I < 1 < \pi^I$ and therefore $0 < q_F^I < q^I$ if $\pi > \pi_F^I$. In this manner, bank runs may be on the equilibrium path of play $\forall q \in \left(0, q_F^I\right]$. This may also be the case when consumers' risk aversion is relatively high as long as the probability of receiving a liquidity shock is intermediate. To see precisely, suppose that $\gamma > \gamma^I$. We then have $\pi_F^I < \pi^I < 1$ and if $\pi \in \left(\pi_F^I, \pi^I\right]$, $0 < q_F^I < q^I$, implying again that crises may occur if $q < q_F^I$. By comparison, $0 < q^I < q_F^I$ if $\pi > \pi^I$. Among these situations, runs may emerge in equilibrium for any $q \in \left(0, q^I\right]$. Intuitively, if individuals are sufficiently risk-averse and the degree of liquidity risk is high enough, patient depositors have a stronger incentive to panic. As a result, the incentive to run condition $c_1^I \geq c_{2\beta}^I$ holds even though the potential for run behavior is higher over lower probabilities of state β.

In addition to consumers' inclination to panic, it is also important to acknowledge that a run equilibrium will never occur unless banks are active. In particular, the bank is expected to offer a desirable level of services to depositors and therefore is active in the initial period when the propensity to run is relatively small. To be precise, $W^I \geq W^A$ if and only if:

$$q \leq \frac{\left(\left[(1-\gamma)\,W^A\right]^{\frac{1}{\gamma}} - \pi\right)^{\gamma} - \left[(1-\pi)\,R^{\frac{1-\gamma}{\gamma}}\right]^{\gamma}}{r^{1-\gamma}\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} - \left[(1-\pi)\,R^{\frac{1-\gamma}{\gamma}}\right]^{\gamma}} = q_A^I, \tag{13}$$

where W^A is given by (1). Consequently, run equilibria exist for any $q \in \left(0, \min\left\{q_F^I, q^I, q_A^I\right\}\right]$. Recall that $\pi_F^I < 1 < \pi^I$ if and only if $\gamma \in \left(1, \min\left\{2, \gamma^I\right\}\right]$ and $R < r^{\frac{1-\gamma}{2-\gamma}}$ or $\gamma \in \left(2, \gamma^I\right]$. Moreover, $W^A = \frac{1}{1-\gamma}$ if $\pi \geq \frac{R-1}{R-r}$ and in such an environment, banks are always able to deliver higher welfare than autarky, indicating that $\max\left\{q_F^I, q^I\right\} < q_A^I$. As it is complex to characterize run equilibria analytically when liquidity risk is relatively low, we begin by examining situations where depositors' degree of exposure to liquidity risk is relatively high:

Proposition 2.

i. If $\gamma \leq \gamma^I$ and $\pi > \max\left\{\pi_F^I, \frac{R-1}{R-r}\right\}$, $q_F^I < q^I < q_A^I$ and run equilibria exist $\forall q \in \left(0, q_F^I\right]$.

ii. If $\gamma > \gamma^I$ and $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$, $q^I < q_F^I < q_A^I$ and run equilibria exist $\forall q \in \left(0, q^I\right]$.

That is, results in cases i and ii summarize respectively situations where either q_F^I or q^I is the cutoff probability of a crisis with which depositors play the run strategy profile.

To proceed, in order to study the existence of run equilibria when parameter restrictions on π are somewhat relaxed compared to those imposed in Proposition 2, we consider a case where the liquidation cost is negligible. Specifically, suppose the liquidation value of long-term assets is close to their face value such that autarkic welfare is $W^A = \pi u\,(1) + (1-\pi)\,u\,(R)$ and $q_F^I < q^I = 1$. Furthermore, $q_F^I > 0$ if and only if $\pi > \frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}$, where $\frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}} < 1$ if $\gamma > 2$. As a result, run equilibria exist $\forall q \in \left(0, \min\left\{q_F^I, q_A^I\right\}\right]$:

Proposition 3. *Suppose that $r = 1$ and $\gamma \in (2, \infty)$. Under these conditions:*

i. If $\pi \in \left(\frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}, \frac{1}{\gamma-1}\right]$, $q_F^I < q_A^I$ and run equilibria exist $\forall q \in \left(0, q_F^I\right]$.

ii. If $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$, then there exists a threshold asset return $\underline{R}^I \in (1, \infty)$ such that: (a) if $R \in \left(1, \underline{R}^I\right)$, $q_A^I < q_F^I$ and run equilibria exist $\forall q \in \left(0, q_A^I\right]$; (b) if $R \in \left(\underline{R}^I, \infty\right)$, $q_F^I < q_A^I$ and run equilibria exist $\forall q \in \left(0, q_F^I\right]$.

Notably, when depositors' risk aversion is high enough, banks may be susceptible to instability depending on the degree of liquidity risk. On one hand, if the population of impatient agents is not particularly large, i.e., $\pi \in \left(\frac{R^{\frac{2-\gamma}{\gamma}}-R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}, \frac{1}{\gamma-1}\right]$, only a lower level of investment is liquidated for the first π withdrawals and large-scale productive assets are held to maturity. Consequently, intermediaries have the potential to offer sufficient liquidity insurance over a wider range of q. Given that consumers only have an incentive to panic if the probability of a crisis is low enough, run equilibria exist $\forall q \in \left(0, q_F^I\right]$. Conversely, suppose that depositors are highly exposed to liquidity risk, i.e., $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$, and the bank has a smaller amount of remaining resources kept in investment after meeting higher demand for early consumption. Among these situations, if the asset return is relatively low, an underprovision of risk-sharing services in a more crisis-prone economy suggests that a bank and thus run equilibria only exist when runs are unlikely, i.e., $q \in \left(0, q_A^I\right]$.

In addition, Figure 2 illustrates how consumers' deposit decisions matter for the existence of run equilibria when $r < 1$ and the degree of liquidity risk is relatively low. Specifically, we construct examples with parameters $(\pi, \gamma, R, r) = (0.5, 8, 1.15, 0.985)$ under which $\pi < \frac{R-1}{R-r}$. In this case, $q_A^I < q_F^I < q^I$ implies that run equilibria only exist if $q \in \left(0, q_A^I\right]$.

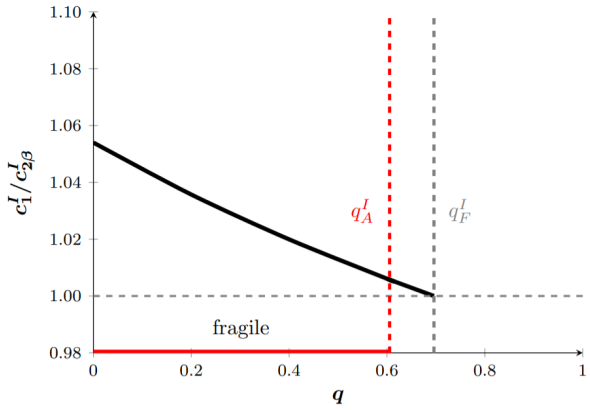

Figure 2: The existence of run equilibria under Scenario I of competitive banks

We proceed to examine run equilibria under scenarios where intermediaries subject to fragility find it optimal to hold precautionary liquid assets. In particular, if $q \in \left(q^I, q^{II}\right)$, where $q^{II} = \frac{1}{1+\left(\frac{1-r}{R-1}\right)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\right]^{\gamma}}$, the optimal allocation would be c^{II} and $c_1^{II} \geq c_{2\beta}^{II}$ if and only if:

$$q \leq \frac{r\left(R-1\right)}{R\left(R-r\right)} = q_F^{II}. \tag{14}$$

where $q_F^{II} < q^{II}$ and it can be shown that $q^I < q_F^{II}$ if and only if $\gamma > \gamma^I$ and $\pi > \pi^I$.

As before, it is also critical that banks are active in equilibrium in the sense that $W^{II} \geq W^{A}$ if and only if $q \leq q_A^{II}$, where q_A^{II} is determined by:

$$\frac{\left(\pi + (1-\pi)\left[\left(1-q_A^{II}\right)^{\frac{1}{\gamma}}\left(\frac{R-r}{1-r}\right)^{\frac{1-\gamma}{\gamma}} + \left(q_A^{II}\right)^{\frac{1}{\gamma}}\left(\frac{R-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right]\right)^{\gamma}}{1-\gamma} - W^{A} = 0. \tag{15}$$

Note that the strictly decreasing welfare level W^{II} combined with constant autarkic welfare W^{A} with respect to the probability of a run implies that the solution q_A^{II} is unique. Therefore, whenever excess liquidity is held, run equilibria exist $\forall q \in \left(q^{I}, \min\left\{q_F^{II}, q_A^{II}\right\}\right]$. Recall that $W^{A} = \frac{1}{1-\gamma}$ if $\pi \geq \frac{R-1}{R-r}$ and among these circumstances, intermediaries can always deliver higher welfare than consumers' reservation utility, indicating that $q_F^{II} < q_A^{II}$. We then summarize run equilibria under Scenario II over higher degrees of liquidity risk.

Proposition 4. *If $\gamma > \gamma^{I}$ and $\pi > \max\left\{\pi^{I}, \frac{R-1}{R-r}\right\}$, $q^{I} < q_F^{II} < q_A^{II}$ and run equilibria exist $\forall q \in \left(q^{I}, q_F^{II}\right]$.*

That is, as there exists a cutoff probability q_F^{II} below which runs occur in equilibrium, depositors play the run strategy profile for any $q < q_F^{II}$.

Furthermore, the graph below highlights the importance of agents' participation decisions in a pre-deposit game to the existence of run equilibria, especially under Scenario II. Specifically, we modify the parameter set used for Figure 2 where banks are inactive for any $q > q_A^{I}$ to $(\pi, \gamma, R, r) = (0.45, 8, 1.038, 0.985)$ under which q_A^{I} does not exist and $\pi < \frac{R-1}{R-r}$. Notably, though the bank's prudent reaction in a more crisis-prone economy reduces agents' inclination to panic, one would still expect run behavior to be on the equilibrium path of play as long as $\frac{c_1^*}{c_{2\beta}^*} \geq 1$. Nevertheless, $q_A^{II} < q_F^{II}$ implies that whenever it is optimal for intermediaries to hold excess liquid assets, run equilibria under Scenario II only exist if and only if $q \in \left(q^{I}, q_A^{II}\right]$.

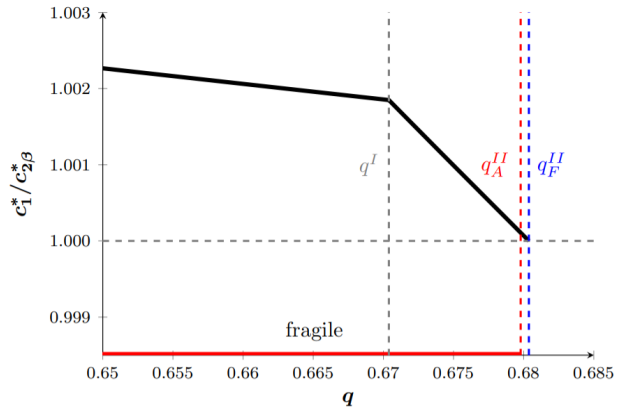

Figure 3: The existence of run equilibria under Scenarios I and II of competitive banks

5 Equilibrium under a monopolistic banking system

The model presented above assumes that the bank operates with an objective to maximize depositors' welfare. Nevertheless, as we argue in the Introduction, there also exists a great amount of concern about the power of large institutions, and some observers have expressed serious reservations about economic outcomes in a highly concentrated banking system. In response to increasing concern about potential changes in market power, we extend our framework to study monopolistic bankers who have the exclusive right to offer banking services.

Throughout the analysis below, we first assume that the monopolist subject to instability is active in period 0. Thus, in the following subsection, we characterize the best responses of the banking authority and the monopolistic bank. Specifically, we begin by focusing on optimal resolution should a crisis occur. We then use the backwards induction approach to analyze the behavior of a monopolistic bank. In particular, we derive what payments the monopolist would choose to make in normal times and then how it allocates resources to depositors and makes the portfolio choice before the sunspot state is identified. Prior to concluding the analysis of fragility, we clarify the role of the resolution mechanism for rent-seeking, strategic behavior of monopolists with limited commitment. Again, as all intermediaries face the same decision problem, we omit the indexation j for each bank.

5.1 The best responses of the banking authority and the monopolist

5.1.1 Resolution in the panic state

After making the early payment c_1 to each of the first fraction π of depositors, the banker infers the sunspot signal. If the panic state is revealed, the bank is taken over by the banking authority. Notably, as the regulator has an objective to maximize depositors' welfare, the optimization problem of the resolution authority would be the same as competitive banks upon the revelation of the sunspot state. Specifically, the authority can calculate the fraction of remaining impatient depositors based on the strategy profile (2) and will choose payments conditional on this information. That is, the authority will offer a common amount $c_{1\beta}$ to each impatient depositor in period 1 and a common amount $c_{2\beta}$ to each remaining patient depositor in period 2 in order to maximize ex-post welfare V_β specified in (3) subject to the resource constraints (6) and (7). In solving this problem, we again divide the above two constraints along with possible outcomes in normal times into three distinct scenarios summarized in Table 1. Furthermore, the conditions characterizing the optimal choice of $c_{1\beta}$ and $c_{2\beta}$ still follow those in (8) as they are determined by the regulatory authority.

In seeking to summarize the sum of depositors' expected utilities in state β, first suppose that the monopolist finds it optimal to make its portfolio choice such that no excess liquid assets are

held, i.e., $1 - x = \pi c_1$, as described by Scenario I. In these situations, the authority must resort to liquidation to meet the extra demand for period-1 consumption should the panic state be inferred, implying that the constraint (6) does not bind and thus $\mu_{1\beta} = 0$. In this vein, the relationship between $c_{1\beta}$ and $c_{2\beta}$ is characterized by:

$$u'(c_{1\beta}) = \frac{R}{r} u'(c_{2\beta}). \tag{16}$$

Combining the resource constraint (7) and the above optimality condition, we can write ex-post welfare as follows:

$$V_\beta = \left((1-\pi) \left[\pi + (1-\pi) \left(\frac{R}{r} \right)^{\frac{1-\gamma}{\gamma}} \right] \right)^\gamma u(r(1 - \pi c_1)). \tag{17}$$

Yet, it is also possible that the monopolistic bank holds excess liquidity against runs, i.e., $1 - x > \pi c_1$, and the banking authority liquidates long-term assets, as in Scenario II. Under this case, the constraint (6) again does not bind and thus the relationship between $c_{1\beta}$ and $c_{2\beta}$ is still delineated by (16). Similarly, using the resource constraint (7) and the ex-post optimal risk-sharing condition in equation (16), we can express the expected utility of depositors in state β as follows:

$$V_\beta = \left((1-\pi) \left[\pi + (1-\pi) \left(\frac{R}{r} \right)^{\frac{1-\gamma}{\gamma}} \right] \right)^\gamma u((1 - x - \pi c_1) + rx). \tag{18}$$

It is clear that the only difference between the value functions (17) and (18) is whether liquid assets are used for additional period-1 consumption beyond the first π withdrawals.

Lastly, as suggested by Scenario III, the banker holds a sufficient amount of precautionary liquid assets so that costly liquidation never occurs even though a positive measure of patient consumers run on the bank. That is, the constraint (6) binds and therefore $\mu_{1\beta} > 0$. Then, the welfare level in the panic state can be summarized as:

$$V_\beta = [(1-\pi)\pi]^\gamma u(1 - x - \pi c_1) + (1-\pi)^{2\gamma} u(Rx), \tag{19}$$

reflecting that excess liquidity and matured investment are paid to remaining impatient and patient agents respectively.

Importantly, conditions (17) to (19) also state that once the portfolio choice and the early payment c_1 are made to the first π consumers, there would be no uncertainty related to the expected utility in resolution. Alternatively, the value function V_β would be deterministic as the monopolistic bank anticipates that the banking authority will reallocate remaining resources according to (8).

5.1.2 Bank payments in normal times

We now turn to study how much the monopolist will pay to each withdrawing depositor if the good aggregate state is revealed. First, from the strategy profile (2), the monopolistic bank knows that no one will further request the period-1 payment after the first fraction π of depositors has been served. Next, let $\mathbb{I}$ denote the indicator function: $\mathbb{I}_{(1-x=\pi c_1)} = 1$ and $\mathbb{I}_{(1-x>\pi c_1)} = 0$ if no excess liquidity is held; otherwise, $\mathbb{I}_{(1-x=\pi c_1)} = 0$ and $\mathbb{I}_{(1-x>\pi c_1)} = 1$. Note also that the banker derives utility by consuming profits denoted η_α, which is the difference between the bank's remaining resources and total payments to patient agents in period 2. As a result, the common amount contingent on state α, $c_{2\alpha}$, is chosen to maximize:

$$\eta_\alpha \equiv \max_{\{c_{2\alpha}\}} R\left(1-\pi c_1\right)\mathbb{I}_{(1-x=\pi c_1)} + \left(1-x-\pi c_1+Rx\right)\mathbb{I}_{(1-x>\pi c_1)} - \left(1-\pi\right)c_{2\alpha}$$

subject to the agents' participation constraint:

$$\underbrace{\pi u\left(c_1\right) + \left(1-q\right)\left(1-\pi\right)u\left(c_{2\alpha}\right) + qV_\beta}_{=W} \geq W^A, \tag{20}$$

where the expression for V_β is given by (17), (18), or (19). That is, the participation constraint states that depositors must not derive lower expected utility over both sunspot states than autarkic welfare. Moreover, since the monopolistic bank can extract all of the gains from consumers, the constraint (20) holds with equality.

It is also worth noting that although the crisis state did not occur, this constraint is still relevant when the bank chooses $c_{2\alpha}$ ex-post. To see why this is the case, recall that all uncertainty about the expected utility in resolution has been resolved once the portfolio choice and the consumption level c_1 to the first fraction π of depositors are determined. For this reason, the monopolist has already considered the effects of what the resolution authority would have done should a crisis occur when issuing short-term liabilities before the sunspot state is inferred. That is, from the monopolist's point of view, the last term on the left-hand-side of the participation constraint, qV_β, is "fixed." Yet, unlike payments in resolution that are not determined by the monopolist, the bank payoff in normal times $c_{2\alpha}$ is chosen to maximize monopoly rents, which in turn affects the early return on deposits c_1.

Letting μ_d denote the multiplier on the participation constraint of depositors, the solution regarding the optimal choice of the bank payment in period 2 requires:

$$\left(1-q\right)u'\left(c_{2\alpha}\right) = \frac{1}{\mu_d}. \tag{21}$$

The above risk-sharing condition implies that when the shadow value μ_d is lower where the par-

ticipation constraint is relaxed, the remaining depositors' expected consumption in normal times would be relatively lower.

5.1.3 Early payments

Moving backwards, the monopolist must decide its early payment to the first fraction π of depositors, recognizing that ex-post payoffs in resolution will be offered according to (8) and it would adopt the appropriate state-contingent consumption level in normal times according to (21). As the bank has no information as to the realization of the aggregate state, it chooses the same amount c_1 to each withdrawing, risk-averse depositor. Note also that this early payment scheme is optimal for the monopolist in the sense that any other feasible allocations where depositors consume different amounts are dominated. As a result, the bank can offer lower common payments in state α in order to maximize profits. In this manner, the value of c_1 is chosen to solve:

$$\Pi \equiv \max_{\{c_1\}} (1-q)\,\eta_\alpha + \underbrace{(q \times 0)}_{\textit{zero profits in resolution}} \tag{22}$$

subject to (20) due to the uncertainty about the realized aggregate state at this point in time.

The solution characterizing the optimal choice of c_1 requires $\eta'_\alpha = 0$. Specifically, under Scenario I, if no precautionary liquid assets are held, the early payment is determined by:

$$u'(c_1) - qr\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} u'\left(r\left(1-\pi c_1\right)\right) = \frac{R}{\mu_d}. \tag{23}$$

Under Scenario II where the bank holds excess liquidity and costly liquidation occurs in state β, c_1 is chosen such that:

$$u'(c_1) - q\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} u'\left((1-x-\pi c_1) + rx\right) = \frac{1}{\mu_d}. \tag{24}$$

Furthermore, under Scenario III, no long-term investment is liquidated in the event of a run and thus the early return on deposits satisfies:

$$u'(c_1) - q\left[(1-\pi)\,\pi\right]^{\gamma} u'\left(1-x-\pi c_1\right) = \frac{1}{\mu_d}. \tag{25}$$

Across all situations, if the participation constraint is tightened, the monopolist must increase its early payment so that depositors' welfare derived from banking services does not drop below their reservation level. Moreover, the above conditions also reveal that though the bank earns zero profits in resolution, it does not completely neglect the crisis state since depositors' expected utility in a

run enters through the participation constraint.

5.1.4 Portfolio choice

We proceed to discuss how the intermediary makes its portfolio choice in period 0. In particular, as suggested by Scenario I, portfolio choices are made such that the bank depletes all of its holdings of assets in the storage technology once the aggregate state is inferred:

$$x = 1 - \pi c_1. \tag{26}$$

Nevertheless, if it is optimal to possess excess liquidity, the monopolist would choose x to maximize expected profits (22) subject to (20) and there are two possible situations. To begin, under Scenario II, though a more liquid portfolio is chosen by the bank, the authority still needs to liquidate productive investment upon the revelation of the crisis state:

$$(1-r)\,q\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} u'\left((1-x-\pi c_1)+rx\right)=\frac{R-1}{\mu_d}. \tag{27}$$

In addition, the monopolist may hoard a sufficient amount of liquidity so that the resolution authority does not need to resort to costly liquidation for depositors' consumption contingent on the crisis state. That is, under Scenario III, the optimality condition regarding the portfolio choice is:

$$q\left(\left[(1-\pi)\,\pi\right]^{\gamma} u'\left(1-x-\pi c_1\right)-(1-\pi)^{2\gamma} R u'\left(Rx\right)\right)=\frac{R-1}{\mu_d}. \tag{28}$$

Notably, both conditions (27) and (28) indicate that once the participation constraint faced by the intermediary is relaxed, the bank places more deposits into the investment technology in order to realize higher expected rents.

5.2 Equilibrium fragility and allocations

The above analysis demonstrates the best responses of a banking authority and a monopolistic banker to the strategy profile (2). In particular, there are three distinct scenarios, and letting the superscripts I, II, and III designate the results under each scenario, we can express the best-response allocations as three different vectors as follows:

$$\hat{c}^{*} \equiv \left\{\hat{x}^{*}, \hat{c}_1^{*}, \hat{c}_{2\alpha}^{*}, \hat{c}_{1\beta}^{*}, \hat{c}_{2\beta}^{*}\right\} \quad \text{for } * = I, II, III.^{10}$$

[10] See the Appendix for the determination of the consumption levels.

Notably, in each vector, information about the monopolist's portfolio choice and consumption levels to depositors is contained. The components are determined by conditions (8), (23) - (28), the resource constraints (4), (6), (7), and the participation constraint (20). Moreover, let $\hat{W}^*$ represent the expected utility delivered by the monopolist.

As indicated by the incentive to run condition under competitive intermediaries, whether monopolistic banks are fragile also depends on if the condition $\hat{c}_1^* \geq \hat{c}_{2\beta}^*$ holds. In this vein, Scenario III is eliminated and the reasoning is similar to that under perfect competition. That is, when the monopolist holds a sufficiently liquid portfolio such that no costly liquidation is needed should a run be identified, $1 - \hat{x}^{III} - \pi\hat{c}_1^{III} = (1-\pi)\,\pi\hat{c}_{1\beta}^{III}$ and $(1-\pi)^2\,\hat{c}_{2\beta}^{III} = R\hat{x}^{III}$. Moreover, given that the bank generates positive profits, $\hat{c}_{2\alpha}^{III} < \frac{1-\hat{x}^{III}-\pi\hat{c}_1^{III}+R\hat{x}^{III}}{1-\pi} = \pi\hat{c}_{1\beta}^{III} + (1-\pi)\,\hat{c}_{2\beta}^{III}$. Thus, we obtain $\frac{\hat{c}_{2\alpha}^{III}}{\hat{c}_{2\beta}^{III}} = \pi\frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}} + (1-\pi) < 1$, as $\hat{c}_{1\beta}^{III} < \hat{c}_{2\beta}^{III}$. Combining $\hat{c}_{2\alpha}^{III} < \hat{c}_{2\beta}^{III}$ and $\hat{c}_1^{III} < \hat{c}_{2\alpha}^{III}$ indicates that $\hat{c}_1^{III} < \hat{c}_{2\beta}^{III}$ and the fragility condition $\hat{c}_1^{III} \geq \hat{c}_{2\beta}^{III}$ is violated. Furthermore, if long-term investment is completely illiquid, the solution to the bank's optimization problem lies under this scenario too. As a result, we confine our attention to Scenarios I and II and $r \in (0, 1]$ under a monopolistic banking system.

Lemma 2. *If the monopolistic banking system is fragile, the solution to the bank's optimization problem belongs to either Scenario I or II.*

In particular, if the probability of a crisis is relatively low, the monopolistic bank does not hold any excess liquid assets to meet additional period-1 withdrawals should the panic state occur. However, as long as the potential for run behavior is high enough, the monopolist expects to earn higher profits by holding a more liquid portfolio to mitigate liquidation costs if the aggregate state turns out to be bad.

5.3 The impact of a higher run probability on fragility

We now ask: How does the likelihood of a crisis affect the monopolistic bank's susceptibility to panics? In the following proposition, we show that the monopolistic bank's portfolio choice has a non-trivial effect on depositors' incentives to run on the banking system.

Proposition 5. *Suppose that run equilibria exist. Under this condition, if $q \leq \hat{q}^I$, where $\hat{q}^I$ is defined in the appendix, depositors' incentives to panic are stronger if the probability of a run is higher. However, if $q > \hat{q}^I$, depositors' inclination to run decreases when a crisis is more likely.*

We start by discussing cases where runs are less likely to occur, i.e., $q \leq \hat{q}^I$, and the monopolist only holds liquid assets for the first fraction π of depositors. Intuitively, if the prospect of a run is higher, the participation constraint faced by the intermediary is tightened, inducing the

monopolistic bank to respond by increasing early payments. As the bank has fewer resources left for the remaining consumption levels should a run occur, depositors' incentives to run on the bank are stronger. Notably, this pattern of instability contrasts sharply across the competitive structure of the banking system. That is, though both economies are susceptible to fragility, the parameter space where the run condition holds is larger under a monopolist if panics are more likely but the opposite occurs under perfect competition. Therefore, such findings unveil an additional vulnerability in the banking system: as long as there is the potential for depositor sentiment to deteriorate, a bank with market power will discount activity in resolution, leaving themselves more unstable.

Yet, over higher probabilities of a run, i.e., $q \in \left(\hat{q}^I, \hat{q}^{II}\right)$, where $\hat{q}^{II}$ is defined in the appendix, the bank holds a more liquid portfolio and depositors' incentives to panic decrease as crises are more likely.[11] In particular, there are two competing effects at work. First, similar to that when the bank does not hold excess liquidity, the early payment is raised due to the tighter depositors' participation constraint, which tends to make the monopolist less stable. Nevertheless, as a relatively more liquid portfolio is chosen, lower liquidation costs are incurred should a panic emerge and comparatively more illiquid assets are held to maturity to augment period-2 payoffs. Notably, this latter effect dominates over higher run probabilities whenever precautionary liquidity is held, alleviating the overall degree of financial fragility. This further suggests that there may exist a maximum probability of a run with which the monopolistic bank is unstable. That is, $\hat{c}_1^{II} \geq \hat{c}_{2\beta}^{II}$ if and only if:

$$q \leq \frac{r\left(R-1\right)}{R\left(R-r\right)} = \hat{q}_F^{II}, \tag{29}$$

where $\hat{q}_F^{II} < \hat{q}^{II}$ and it can be verified that $\hat{q}^I < \hat{q}_F^{II}$ if and only if $\gamma > \gamma^I$ and $\pi > \pi^I$. Further, $\hat{q}_F^{II} = q_F^{II}$, where q_F^{II} is given by (14), indicates that the monopolistic bank is as fragile as its competitive counterpart if both types of intermediaries hold precautionary liquid assets in their portfolios. This is the case because whenever excess liquidity is held and the utility function takes the constant-relative-risk-aversion form, instability depends only on exogenous parameters, i.e., r and R, but not on different levels of liquidity transformation endogenously implemented by banks.

In what follows, we turn to examine the existence of run equilibria in an active monopolistic bank so that the results in Proposition 5 hold. As a starting point, we focus on situations in which $q \leq \hat{q}^I$ and thus the monopolist would find it optimal not to hold excess liquidity against runs. It follows from the discussion below Proposition 5 that depositors' inclination to panic is stronger if the run probability is higher. For this reason, if $\gamma > \gamma^I$ and $\pi > \pi^I$ where depositors have incentives to panic over higher values of q, there may exist a minimum probability $\hat{q}_F^I < \hat{q}^I$ with which runs may occur. To demonstrate such a possibility, we further restrict our attention to higher degrees of liquidity risk, i.e., $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$, for the sake of

[11] Notably, $\hat{q}^{II}$ denotes the threshold probability above which costly liquidation is not needed in the event of a run.

tractability. In this vein, whenever the liquidation cost is low enough, i.e., $r \in \left(\hat{r}^I, 1\right)$ where $\hat{r}^I = \left(\frac{\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{\frac{1}{1-\gamma}} - \pi - (1-\pi)^2 R^{-1}}{(1-\pi)\pi R^{-\frac{1}{\gamma}}} \right)^{\frac{\gamma}{1-\gamma}}$, monopolists would not be susceptible to panics in economies with a low probability of state β, implying that $\hat{q}_F^I > 0$. By comparison, for any $r \leq \hat{r}^I$, the incentive to run condition holds even though the likelihood of a crisis is negligible and hence we have $\hat{q}_F^I = 0$ in these situations.

On the other hand, we must also characterize conditions where the bank generates positive monopoly rents. In particular, the level of expected profits Π^I is a strictly decreasing function of q:

$$\frac{d\Pi^I}{dq} = -\eta_\alpha^I - (1-q)\left[R\pi \frac{d\hat{c}_1^I(q)}{dq} + (1-\pi)\frac{d\hat{c}_{2\alpha}^I(q)}{dq}\right] < 0,$$

as $\frac{d\hat{c}_1^I(q)}{dq} > 0$ and $\frac{d\hat{c}_{2\alpha}^I(q)}{dq} > 0$. Further recall that q_A^I defined in (13) represents the maximum probability of a crisis with which a competitive bank not holding excess liquidity is able to deliver expected utility as high as financial autarky. This in turn implies that as the run probability approaches q_A^I, the participation constraint (20) will be tightened such that the portfolio choice and the payment scheme by a monopolist mirrors that by a competitive bank. In this vein, bank profits would necessarily be negative if the likelihood of a panic is beyond the cutoff q_A^I. Therefore, for any $\gamma > \gamma^I$ and $\pi > \pi^I$, run equilibria under a monopolist that does not hold excess liquidity exist $\forall q \in \left(\hat{q}_F^I, \min\left\{q_A^I, \hat{q}^I\right\}\right]$, where $\hat{q}_F^I \in \left[0, \hat{q}^I\right)$.

Next, to derive important results analytically rather than relying on numerical examples solely, we first confine to cases where agents are highly exposed to liquidity risk, i.e., $\pi > \frac{R-1}{R-r}$. As a consequence, banks not holding excess liquidity are always active, indicating that $\hat{q}^I < q_A^I$:

Proposition 6. *Suppose that $\gamma > \gamma^I$ and $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$ such that $\hat{q}_F^I < \hat{q}^I < q_A^I$. Under these conditions, if $r \in \left(\hat{r}^I, 1\right)$, run equilibria exist $\forall q \in \left(\hat{q}_F^I, \hat{q}^I\right]$, where $\hat{q}_F^I > 0$. If instead $r \in \left(0, \hat{r}^I\right]$, run equilibria exist $\forall q \in \left(0, \hat{q}^I\right]$.*

As highlighted above, in stark contrast to those associated with a competitive bank, run equilibria under a monopoly may not exist when the probability of a crisis is sufficiently low. That is, due to its ability to exploit market power to obtain rents, the monopolistic bank tends to issue a relatively lower early payment such that it may not be prone to instability.

To proceed, in order to gain some degree of tractability when characterizing run equilibria under a somewhat weaker parameter restriction on π, we consider a case where $r = 1$. Specifically, when the liquidation value of long-term assets is close to their face value, autarkic welfare is given by $W^A = \pi u(1) + (1-\pi)u(R)$ and $\hat{q}^I = 1$. Importantly, it can also be verified that for any $\gamma > 2$, $\pi > \frac{1}{\gamma - 1}$, and $R < \underline{R}^I$, where $\underline{R}^I$ is defined in case ii of Proposition 3, the fragility

condition $\hat{c}_1^I \geq \hat{c}_{2\beta}^I$ holds in a more crisis-prone economy and $q_A^I < \hat{q}^I$. Recall that there may exist a minimum probability $\hat{q}_F^I$ with which depositors have inclination to run on the monopolist, we then offer the following proposition.

Proposition 7. *Suppose that* $r = 1$. *Further suppose that* $\gamma \in (2, \infty)$, $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$, *and* $R \in \left(1, \underline{R}^I\right)$. *Under these conditions,* $\hat{q}_F^I < q_A^I < \hat{q}^I$ *and run equilibria exist* $\forall q \in \left(\hat{q}_F^I, q_A^I\right]$, *where* $\hat{q}_F^I \in \left[0, q_A^I\right)$.

Notably, in a setting absent costly liquidation, the asset return can be shown to play an essential role for the stability of a monopolistic bank. In particular, as long as the value of R remains at a sufficiently high level, monopolists are immune to fragility regardless of agents' degrees of risk aversion or the proportion of impatient depositors in the economy. In this regard, such a finding contrasts sharply to that revealed in Proposition 3 where competitive banks may still be vulnerable to a crisis over relatively lower run probabilities. On the other hand, if the asset return is low enough, highly risk-averse depositors may still run on the monopolist though liquidity demand is not particularly large. That is, since the period-2 payoff in resolution is expected to be so low, the fragility condition holds over both lower and higher values of q. Nevertheless, runs are only observed before expected monopoly rents reduce to zero.

To shed light on cases where $r < 1$ while the value of π is relatively low, we present Figure 4 using the same parameter set as in Figure 2. Under the parameter values considered, $\hat{q}_F^I = 0$ and $\Pi^I \geq 0$ if and only if $q \leq q_A^I$, where $q_A^I < \hat{q}^I$, implying that run equilibria exist $\forall q \in \left(0, q_A^I\right]$.

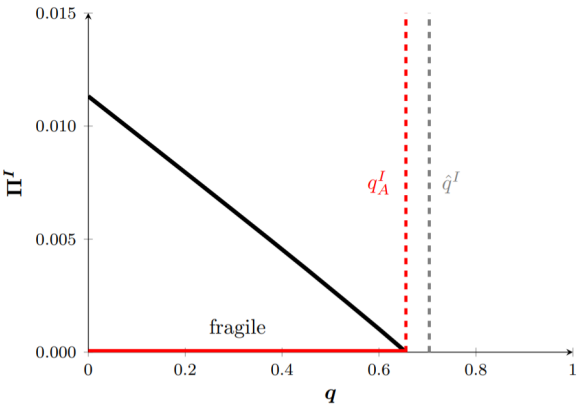

Figure 4: The existence of run equilibria under Scenario I of monopolistic banks

We now move on to examine run equilibria if $q \in \left(\hat{q}^I, \hat{q}^{II}\right)$ where it would be optimal for monopolists to hold precautionary liquid assets against crises, as implied by Proposition 5. To refrain from repeating the discussion above, the fragility condition $\hat{c}_1^{II} \geq \hat{c}_{2\beta}^{II}$ holds if $q \leq \hat{q}_F^{II} = q_F^{II}$. Note that $\hat{q}_F^{II} < \hat{q}^{II}$ owing to the fact that if $q \geq \hat{q}^{II}$, the solution to the bank's optimization

problem lies under Scenario III where runs are off the equilibrium path of play. Note also that for any $\gamma > \gamma^I$ and $\pi > \pi^I$, $\hat{q}^I < \hat{q}_F^{II}$.

In addition, in order to examine whether the monopolistic banking system is active, it can be shown that the level of expected bank profits Π^{II} is a strictly decreasing function of q:

$$\frac{d\Pi^{II}}{dq} = -\eta_\alpha^{II} + (1-q)\left[(R-1)\frac{d\hat{x}^{II}(q)}{dq} - \pi\frac{d\hat{c}_1^{II}(q)}{dq} - (1-\pi)\frac{d\hat{c}_{2\alpha}^{II}(q)}{dq}\right] < 0,$$

as $\frac{d\hat{x}^{II}(q)}{dq} < 0$, $\frac{d\hat{c}_1^{II}(q)}{dq} > 0$, and $\frac{d\hat{c}_{2\alpha}^{II}(q)}{dq} > 0$. Recall that q_A^{II} defined in (15) denotes the upper bound probability with which a monopolist holding excess liquidity earns positive expected rents. In this manner, run equilibria exist $\forall q \in \left(\hat{q}^I, \min\left\{\hat{q}_F^{II}, q_A^{II}\right\}\right]$. For tractability, we next concentrate on higher degrees of liquidity risk, i.e., $\pi > \frac{R-1}{R-r}$, so that $\hat{q}_F^{II} < q_A^{II}$.

Proposition 8. *If $\gamma > \gamma^I$ and $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$, $\hat{q}^I < \hat{q}_F^{II} < q_A^{II}$ and run equilibria exist $\forall q \in \left(\hat{q}^I, \hat{q}_F^{II}\right]$.*

Unlike monopolists not hoarding excess liquid assets over lower values of q, banks holding more liquid portfolios are not subject to fragility when the run probability is beyond the cutoff $\hat{q}_F^{II}$.

Lastly, due to the complexity of characterizing run equilibria when the degree of liquidity risk is relatively low, we resort to numerical simulation. In particular, to underline the impact of consumers' participation decisions on run equilibria when precautionary liquidity is held by the monopolist, we illustrate Figure 5 below using the same set of parameters for Figure 3. Under the parameter values considered, the monopolist possessing excess liquidity emerges when the likelihood of a crisis is higher, which is absent in Figure 4. Notably, intermediaries are only active $\forall q < q_A^{II}$, where $\hat{q}^I < q_A^{II} < \hat{q}_F^{II}$, indicating that if extra liquid assets are held, runs occur as an equilibrium if and only if $q \in \left(\hat{q}^I, q_A^{II}\right]$.

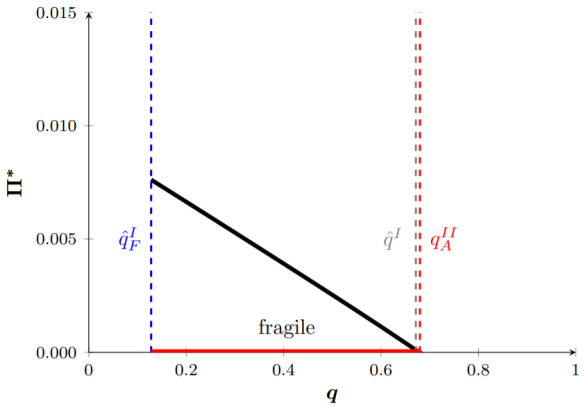

Figure 5: The existence of run equilibria under Scenarios I and II of monopolistic banks

5.4 The role of bank resolution for rent-seeking, strategic behavior

Our previous analysis assumes that whenever a run is identified, a banking authority takes over the intermediary and uses the bank's resources to maximize the remaining depositors' welfare. Such a process in the event of a crisis imposed by the authority resembles resolution for most distressed depository institutions in practice. More importantly, bank resolution in this setting needs to be incorporated for rent-seeking, strategic behavior of monopolists with limited commitment to arise in equilibrium. To see this precisely, suppose that monopolistic banks are able to choose the payment schedule in both sunspot states, where the only constraint is to deliver the expected utility amounting to autarkic welfare to depositors. However, should the good aggregate state not occur, the participation constraint involving payments in both normal and crisis times would be undefined. That is, in the absence of a banking regulator, payoffs in the inferred panic state are indeterminate as there is nothing to constrain the bank's actions. From this perspective, an equilibrium with a monopolist would not exist as, ex-ante, the bank is not be able to attract any deposits.

In this regard, in an environment free of interventions by the regulator, one may need to introduce a type of commitment or another constraint so that the participation condition is relevant ex-post and ex-ante. The monopolist, for example, may further promise that it would earn zero profits during a crisis and, by doing so, its expected rents would be maximized over relatively lower run probabilities. Nonetheless, as we mentioned previously, if commitment is allowed in this setting, then runs can always be prevented by an off-equilibrium threat, as suggested by Diamond and Dybvig (1983). We then summarize our arguments below.

Proposition 9. *Rent-seeking, strategic behavior of a monopolist with limited commitment emerges if and only if bank resolution is present.*

5.5 Discussion

In what follows, we discuss our different results in relation to the literature. In particular, one might infer from Keister (2016) that a transfer of resources would be necessary to induce banks to behave aggressively as intermediaries completely ignore the bad sunspot state. However, this insight is based upon a setting where banks are perfectly competitive and do not engage in rent-seeking, strategic behavior. Instead, we show in Proposition 5 that the similar aggressive response by the bank to increase short-term liabilities can arise without a fiscal backstop if the bank maximizes its own profits. Yet, as the monopolist needs to keep depositors indifferent to financial autarky and consumers' expected utility in resolution enters through the participation constraint, the bank only discounts the crisis state.

Notably, identifying such a novel aspect of vulnerability in the banking system sheds light

on the effectiveness of imposing certain capital requirements to improve financial stability and/or welfare. To be specific, though banks in the model have no resources initially, a policy to limit their total payments before the aggregate state is identified can be thought to represent a measure to preserve the bank's capital position. In this manner, our findings suggest that if there is an explicit "cap," payouts are more likely to be effectively restricted when runs are unlikely to occur in highly competitive environments. However, if the banking sector is highly concentrated where intermediaries aim to maximize monopoly rents, then setting an upper bound on returns on deposits may only apply at higher probabilities of a crisis. It is also worth noting that if this policy decision is made under discretion, the maximum early payments requirement is not a binding constraint on competitive banks' behavior as there are no incentive distortions. Furthermore, the policymaker is not incentivized to choose such a limit on earliest demandable deposits issued by a monopolist as agents' expected utility would not be improved.

6 Relative fragility and welfare in competitive versus monopolistic banks

The preceding analysis demonstrates the role of financial sector competition for banking activity and instability. We now address another question of interest: Which banking sector is more vulnerable to a financial crisis? The existing research such as Chang and Velasco (2001) and Matsuoka (2013) demonstrates that when the prospect of a run is zero, the monopolist issues lower earliest demandable deposits and thus it is always better for stability than its competitive counterpart. In our setting with limited commitment and a positive probability of a run, we again show that the monopolist remains relatively less illiquid as it delivers lower welfare than the competitive bank. Yet, the monopolistic bank is only less susceptible to a panic over lower values of q while it is as fragile as the competitive bank in a more crisis-prone economy. Our findings then can be summarized as follows:

Proposition 10. *Suppose that both banking economies prone to a run are active. Under this condition, if $q \leq \hat{q}^I$, the scope for instability of a monopolist is strictly contained in that of a competitive bank. If instead $q > \hat{q}^I$, the degree of fragility is the same across the competitive structure of the banking system. Yet, agents' expected utility under perfect competition dominates that under monopolistic banks regardless of the probability of a run.*

Over lower probabilities of a crisis, i.e., $q \leq \hat{q}^I$, both banking structures do not hold excess liquidity to meet unexpected withdrawals should a panic occur. Owing to the fact that the monopolistic bank remains relatively less illiquid, it is able to offer higher ex-post payoffs on deposits,

leaving itself less susceptible to a crisis than the competitive one. In this manner, lower payments to the first fraction π of depositors at the cost of reduced risk-sharing between impatient and patient consumers make the banking system more stable.

Nevertheless, is a relatively less illiquid banking sector always associated with weaker degrees of instability? No. Since both economies possessing extra liquid assets admit a same maximum probability with which a run can occur, the degree of fragility is the same when the likelihood of a panic is higher, i.e., $q > \hat{q}^I$. In other words, reduced risk-sharing between impatient and patient depositors does not necessarily promote the relative stability of the monopolist if there is a sufficiently high potential for consumer sentiment to deteriorate. This interesting result is new to the literature.

7 An economy with government bailouts

In this section, we proceed to extend our framework by incorporating government bailouts à la Keister (2016). In particular, in the absence of a fiscal backstop, the effects of moral hazard on banking behavior and relative welfare and fragility across the competitive structure of the banking system are ignored. Thus, the primary purpose of this section is to examine whether our previous insights are robust to the introduction of bailouts.

Once fiscal/bailout policy is included, a slight change in the agent's utility function is needed as consumers also derive utility from public goods provided by the regulator. In this manner, agents' preferences are expressed by:

$$u\left(c_1, c_2; \omega_i\right) + v\left(g\right) = \frac{\left(c_1 + \omega_i c_2\right)^{1-\gamma}}{1-\gamma} + \delta \frac{g^{1-\gamma}}{1-\gamma},$$

where g denotes the level of public goods and δ measures the relative importance of public consumption in overall utility.

We proceed by discussing how fiscal/bailout policy is conducted. In particular, there is a banking authority who levies taxes in period 1 for the provision of public goods. Moreover, tax revenues are put into a technology which transforms units of the private good one-for-one into units of the public good. To be specific, consider an autarkic economy where an agent holds a portfolio consisting of $1 - x$ units and x units of liquid and illiquid assets respectively. Thus, the authority will collect the amount of taxes τ at the start of period 1, which is equal to the level of public goods, to maximize:

$$W = \max_{\{x,\tau\}} \pi u\left(1 - x - \tau + rx\right) + \left(1 - \pi\right) u\left(1 - x - \tau + Rx\right) + v\left(g\right)$$

subject to:

$$\tau \leq 1 - x \quad \text{and} \quad g = \tau.$$

In particular, the first constraint stating that the tax payment made comes from liquid assets holds with equality if individuals do not possess excess liquidity.

Let x^{A*} and τ^{A*} represent the optimal level of illiquid assets and taxes under financial autarky. Notably, over lower levels of liquidity risk, i.e., $\pi \in \left(0, \frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}\right]$, a consumer's portfolio choice is such that the agent does not hold excess liquidity and thus $x^{A*} = \frac{\left[\pi r^{1-\gamma}+(1-\pi)R^{1-\gamma}\right]^{\frac{1}{\gamma}}}{[\pi r^{1-\gamma}+(1-\pi)R^{1-\gamma}]^{\frac{1}{\gamma}}+\delta^{\frac{1}{\gamma}}}$ and $\tau^{A*} = \frac{\delta^{\frac{1}{\gamma}}}{[\pi r^{1-\gamma}+(1-\pi)R^{1-\gamma}]^{\frac{1}{\gamma}}+\delta^{\frac{1}{\gamma}}}$. Yet, when the degree of liquidity risk is intermediate, i.e., $\pi \in \left(\frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}, \frac{R-1}{R-r}\right)$, a more liquid portfolio is chosen to mitigate costly liquidation if the agent becomes impatient. In this vein, $x^{A*} = \frac{\left(\frac{1-\pi}{1-r}\right)^{\frac{1}{\gamma}}-\left(\frac{\pi}{R-1}\right)^{\frac{1}{\gamma}}}{(1-\pi)^{\frac{1}{\gamma}}(1-r)^{\frac{\gamma-1}{\gamma}}+\pi^{\frac{1}{\gamma}}(R-1)^{\frac{\gamma-1}{\gamma}}+(R-r)^{\frac{\gamma-1}{\gamma}}\delta^{\frac{1}{\gamma}}}$ and $\tau^{A*} = \frac{1-(1-r)x^{A*}}{\left(\frac{\pi(R-r)}{\delta(R-1)}\right)^{\frac{1}{\gamma}}+1}$. Once the probability of resorting to costly liquidation is sufficiently high, i.e., $\pi \in \left[\frac{R-1}{R-r}, 1\right)$, no illiquid assets will be held and thus $x^{A*} = 0$ and $\tau^{A*} = \frac{1}{\left(\frac{\pi(R-r)}{\delta(R-1)}\right)^{\frac{1}{\gamma}}+1}$. We then summarize autarkic utility W^{A*} as follows:

$$W^{A*} = \begin{cases} \frac{\left(\left[\pi r^{1-\gamma}+(1-\pi)R^{1-\gamma}\right]^{\frac{1}{\gamma}}+\delta^{\frac{1}{\gamma}}\right)^{\gamma}}{1-\gamma} & \pi \in \left(0, \frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}\right] \\ \frac{\left(\left[\pi^{\frac{1}{\gamma}}\left(\frac{R-1}{R-r}\right)^{\frac{\gamma-1}{\gamma}}+(1-\pi)^{\frac{1}{\gamma}}\left(\frac{1-r}{R-r}\right)^{\frac{\gamma-1}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)^{\gamma}}{1-\gamma} & \pi \in \left(\frac{R-1}{\left(\frac{R}{r}\right)^{\gamma}(1-r)+R-1}, \frac{R-1}{R-r}\right) \\ \frac{\pi^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}\left(\frac{R-1}{R-r}\right)^{\frac{1-\gamma}{\gamma}}}{(1-\gamma)\left[\pi^{\frac{1}{\gamma}}+\delta^{\frac{1}{\gamma}}\left(\frac{R-1}{R-r}\right)^{\frac{1}{\gamma}}\right]^{1-\gamma}} & \pi \in \left[\frac{R-1}{R-r}, 1\right) \end{cases}, \tag{30}$$

where $r \in (0, 1)$. It is also worth noting that if $r = 0$, it is never optimal to hold zero liquid assets while if $r = 1$, agents allocate as much resources into the productive technology as possible.

Nonetheless, if consumers form coalitions or deposit their funds in monopolistic banks, the regulator will collect taxes from each intermediary after the aggregate state is inferred. That is, we consider that taxes, τ_s for $s = \alpha, \beta$, are levied ex-post, as in Keister and Narasiman (2016), rather than ex-ante, as in Keister (2016).[12] Specifically, the same tax rate will be imposed on all banks in normal times whereas, in times of crisis, the regulator will lower the provision of public goods to augment the remaining private consumption levels. Consequently, the difference between taxes in

[12]Note that fiscal/bailout policy as in Keister and Narasiman (2016) generates the same type of bailout-related incentive distortions, if any, as in Keister (2016) and limits the authority's ability to tax short-term liabilities as a macroprudential policy. More importantly, this approach yields a relatively high degree of tractability that allows us to characterize results analytically as much as possible.

the respective sunspot states represents the size of bailouts.

7.1 Equilibrium under a competitive banking system

To begin, we examine the fragility of a competitive bank in which the approach follows that of Keister and Narasiman (2016) and Li (2020) but agents' financial participation is considered.[13] To be specific, we derive the best responses of bank j and the banking authority to the strategy profile (2) by working backwards. Note that the difference from the analysis under Section 4.1 is that upon the revelation of the sunspot state, the banking authority conducts fiscal/bailout policy and offers public goods before the remaining payments on deposits are rescheduled.

7.1.1 The best-response allocation

Ex-post allocation of consumption Ex-post, payments $c_{2\alpha}^j$, $c_{1\beta}^j$, and $c_{2\beta}^j$ will be chosen to maximize (3) subject to:

$$
\begin{aligned}
0 &\leq 1 - x^j - \pi c_1^j - \tau_\alpha^j && (31)\\
(1-\pi)\, c_{2\alpha}^j &= 1 - x^j - \pi c_1^j - \tau_\alpha^j + R x^j && (32)\\
1 - x^j - \pi c_1^j - \tau_\beta^j &\leq (1-\pi)\, \pi c_{1\beta}^j && (33)\\
(1-\pi)^2\, c_{2\beta}^j &= R\left\{ x^j - \frac{1}{r}\left[(1-\pi)\, \pi c_{1\beta}^j - \left(1 - x^j - \pi c_1^j - \tau_\beta^j\right)\right]\right\} && (34)
\end{aligned}
$$

Note that the interpretation for the above four constraints is similar to that for constraints (4) to (7) with the exception that the bank further pays taxes upon the revelation of the aggregate state. In solving the above optimization problem, we can divide the constraint set into four different scenarios. Specifically, the first three mirror those summarized in Table 1 and the additional one is that the bank neither holds excess liquidity nor liquidates long-term assets should a run occur as shown in Table 2 below. Notably, Scenario IV emerges here as the bank anticipates a lower tax rate in state β and can rely solely on fiscal transfers to meet any extra period-1 withdrawal demand. This would not be possible in settings where no bailouts are made since some agents would receive zero consumption in the event of a run, leading to negative infinite utility. Overall, similar to Li (2020) where the liquidation cost is introduced, we are able to capture potential different banking behavior under Scenarios II, III, and IV that are absent in Keister (2016).

	State α	State β
Scenario IV	No excess liquidity	No liquidation

[13] Yet, in comparison to Keister and Narasiman (2016) and Li (2020), we abstract from the issue of financial regulation so that we are able to concentrate on the impact of competition on fragility in the clearest way.

Table 2: The fourth scenario

Similarly, letting μ_1^j, $\mu_{2\alpha}^j$, $\mu_{1\beta}^j$, and $\mu_{2\beta}^j$ denote the multipliers on the resource constraints (31) to (34), the first-order conditions regarding the choice of the payments $c_{2\alpha}^j$, $c_{1\beta}^j$, and $c_{2\beta}^j$ are again given by equations in (8).

State-contingent taxes In inferred state s, the authority will choose a tax rate τ_s^j per unit of deposits for bank j to produce an efficient level of public goods and anticipates that each intermediary will allocate the remaining resources according to (8). Letting $\sigma(j)$ denote the distribution of depositors across banks and $g_s = \int \tau_s^j d\sigma(j)$ represent the total amount of taxes for public goods, then τ_s^j will be chosen to solve:

$$\max_{\{\tau_s^j\}} \int V_s^j d\sigma(j) + v(g_s) \tag{35}$$

subject to the resource constraints (31) to (34).

Thus, the first-order conditions characterizing state-contingent taxes are as follows:

$$v'(g_\alpha) = \int \left(\mu_1^j + \mu_{2\alpha}^j\right) d\sigma(j) \quad \text{and} \quad v'(g_\beta) = \int \left(\frac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j\right) d\sigma(j) \quad \text{for all } j. \tag{36}$$

Specifically, as no runs occur in normal times, the same amount of taxes will be collected from all intermediaries even if they might have made different levels of early payments. In this manner, τ_α^j is determined such that the marginal value of public goods equals that of private consumption across periods 1 and 2 averaged across banks. By comparison, the authority will levy a tax rate τ_β^j to equalize the marginal utility of public consumption to that of private goods at every individual bank. That is, regardless of banks' portfolio choices and their short-term liabilities, intermediaries have the same amount of liquid and illiquid assets per remaining depositor after taxes have been collected. To be precise, define $\bar{x} = \int x^j d\sigma(j)$ and $\bar{c}_1 = \int c_1^j d\sigma(j)$ to be the economy-wide average of illiquid assets and early payments respectively. Thus, for Scenarios I and II where it is desirable for banks to liquidate long-term assets should a run occur, the resource constraint in state β can be expressed as:

$$(1-\pi)\left[\pi c_{1\beta}^j + (1-\pi)\frac{r}{R}c_{2\beta}^j\right] = 1 - (1-r)\bar{x} - \pi\bar{c}_1 - \tau_\beta^j. \tag{37}$$

Yet, for Scenarios III and IV where no costly liquidation in the event of a run is efficient, the resource constraints become:

$$(1-\pi)\pi c_{1\beta}^j = 1 - \bar{x} - \pi\bar{c}_1 - \tau_\beta^j \quad \text{and} \quad (1-\pi)^2 c_{2\beta}^j = R\bar{x}. \tag{38}$$

Generally, the amount of resources available to a bank for ex-post payoffs on deposits in state β only depends on aggregate economic conditions, i.e., $\bar{x}$ and $\bar{c}_1$, and not on the bank's own decisions, i.e., x^j and c_1^j.

Early payments Moving backwards, bank j needs to decide how much consumption to give to each depositor with $i \leq \pi$. In particular, it would choose to offer the same payment c_1^j to maximize (9) subject to the resource constraints (31) to (34). The first-order condition for this problem is:

$$u'\left(c_1^j\right) = (1-q)\left(\mu_1^j + \mu_{2\alpha}^j\right). \tag{39}$$

In stark contrast to equation (10), the bank optimally ignores the value of resources in state β as the regulator is expected to lower the tax rate should runs occur to augment private consumption.

Portfolio choice We now turn to study intermediaries' portfolio choices. To be specific, x^j is chosen to maximize (9) subject to the resource constraints (31) to (34). The condition characterizing the optimal portfolio choice is given by:

$$\mu_1^j + \mu_{2\alpha}^j = R\mu_{2\alpha}^j. \tag{40}$$

The above condition highlights bailout-related incentive distortions on the asset side of bank balance sheet. That is, compared to condition (11) in an economy where fiscal transfers are not available, the bank does not take into account the effects of a panic on its portfolio choice.

7.1.2 Equilibrium fragility and allocations

The above analysis derives the best responses of competitive banks to the strategy profile (2). Letting the superscript bI, bII, ..., bIV designate the results under each scenario, we can summarize the best-response allocations as four different vectors as follows:

$$c^{b*} \equiv \left\{x^{b*}, c_1^{b*}, c_{2\alpha}^{b*}, g_\alpha^{b*}, c_{1\beta}^{b*}, c_{2\beta}^{b*}, g_\beta^{b*}\right\} \quad \text{for } * = I, II, ..., IV,^{14}$$

where information about the bank's portfolio choice and both private and public consumption levels is contained. In particular, these components are determined by conditions (8), (36), (39), (40), and the resource constraints (31) to (34). In addition, let W^{b*} denote the sum of depositors' expected utilities delivered by the bank.

Similar to that demonstrated in the previous two sections with competitive and monopolistic banks without bailouts, banks are prone to instability if and only if $c_1^{b*} \geq c_{2\beta}^{b*}$. Following such an

[14] See the Appendix for the calculated private and public consumption levels.

incentive to run condition, we rule out Scenarios III and IV where the bank does not liquidate long-term assets should a panic be identified. As the intuition closely follows that for the elimination of Scenario III under competitive banks in the absence of fiscal transfers, it is omitted here. Note also that condition (40) suggests that $\mu_1^{b*} > 0$. In other words, no excess liquidity will be held as a precaution against runs due to the moral hazard associated with government bailouts. Thus, the solution to the bank's optimization problem also will not lie under Scenario II.

Lemma 3. *If the competitive banking system with bailouts is fragile, the solution to the bank's optimization problem belongs to Scenario I only.*

In particular, it can be shown that if the proportion of impatient consumers is large, i.e., $\pi > \pi^{bI} = \frac{1+\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}} - \sqrt{\left[1+\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}}\right]^2 - 4\left[\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}} - \delta^{\frac{1}{\gamma}}\right]}}{2}$ where $\pi^{bI} \in (0,1)$, banks find it optimal not to hold excess liquidity but resort to costly liquidation during a panic. Moreover, $c_{2\alpha}^{bI} > c_1^{bI}$ if and only if $q < 1 - \frac{1}{R} = \bar{q}^{bI}$. Thus, to focus on pure strategy Nash equilibria throughout the analysis of competitive banks in the presence of bailouts, we limit our attention to lower crisis probabilities and hence patient agents never run on the bank in normal times.

7.1.3 The impact of a higher run probability on fragility

We proceed to study the impact of a higher run probability on financial fragility. As implied by Lemma 3, we focus on situations in which $\pi > \pi^{bI}$ and $q < \bar{q}^{bI}$ so that it would be efficient for intermediaries to implement allocation c^{bI}. We then offer the following proposition:

Proposition 11. *Suppose that run equilibria exist. Under this condition, depositors' incentives to run on a competitive bank are stronger if the probability of a crisis is higher.*

When the potential for a panic is more likely, the bank makes the portfolio choice and issues liabilities in an aggressive manner. That is, due to the moral hazard associated with government bailouts, competitive banks tend to overinvest in illiquid assets from a social point of view while paying too much short-term liabilities. As a result, this imprudent behavior strengthens depositors' incentive to run on the financial system. Notably, such findings, which are in sharp contrast to those in Proposition 1, indicate that the way that competitive intermediaries react revolves around whether they have access to fiscal backstops.

Next, to characterize a precise parameter region of q where the results in Proposition 11 hold, we seek to study the existence of run equilibria in an active competitive banking system with bailouts. To our knowledge, there has not been previous work that offers such a characterization in a two-asset version of Keister (2016). Notably, this analysis in non-trivial as it allows us to shed

light on critical issues such as the relationship between economic conditions and financial fragility, which is limited in the existing literature.

To start, as implied by Proposition 11, intermediaries behave in a more aggressive manner as q increases, implying that there may exist a minimum run probability with which panics occur in equilibrium. To be specific, $c_1^{bI} \geq c_{2\beta}^{bI}$ if and only if:

$$q \geq 1 - \left(\frac{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}}{\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\left[r\left(1-\pi\right)R^{\frac{1-\gamma}{\gamma}} + \delta^{\frac{1}{\gamma}}\right]} \right)^{\gamma} = q_F^{bI}. \tag{41}$$

Note that if $\delta < \delta_1^{bI} = \left[\frac{1}{4}\left(\frac{1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}}{\left(\frac{1}{r}\right)^{\frac{1}{\gamma}}-1}\right)\right]^{\gamma}$, $0 < \pi^{bI} < \pi_1^{bI} < \pi_2^{bI} < 1$, where π_1^{bI} and π_2^{bI} are defined in the appendix, and if $\pi \in \left(\pi_1^{bI}, \pi_2^{bI}\right)$, $q_F^{bI} < \bar{q}^{bI} = 1 - \frac{1}{R}$. That is, if the size of the public sector is small enough and the degree of liquidity risk is intermediate, intermediaries may be susceptible to fragility $\forall q \in \left(q_F^{bI}, \bar{q}^{bI}\right)$. The intuition is as follows.

First, the authority tends to make an abundance of fiscal transfers during a panic when the public sector is large, eliminating depositors' incentives to run on the bank and vice versa. Yet, over lower values of δ, whether banks are vulnerable to crises also hinges on the total fundamental demand for early consumption. For instance, over a somewhat lower range of liquidity risk $\pi \in \left(\pi^{bI}, \pi_1^{bI}\right]$, there are more resources left for the remaining private consumption once the bank identifies the sunspot state. As a result, the incentive to run condition never holds if the run probability is lower than $\bar{q}^{bI}$.

The same is true if there is a large fraction of depositors who have an immediate demand to consume early, i.e., $\pi \in \left[\pi_2^{bI}, 1\right)$. To see this, recognizing that they infer the sunspot state late, intermediaries behave conservatively by offering lower short-term obligations. Consequently, the fragility condition is violated if the incentive distortion is insignificant. By comparison, at intermediate values of liquidity risk, i.e., $\pi \in \left(\pi_1^{bI}, \pi_2^{bI}\right)$, banks behave in a less prudent manner and hence runs may occur in equilibrium in a more crisis-prone economy over lower values of q.

Note also that there may exist possibilities that the incentive to run condition holds $\forall q < \bar{q}^{bI}$. In particular, if $\delta < \left(\frac{\left(\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] - \left(R^{\frac{1}{\gamma}-1}\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)^2}{4\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left[\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}-1\right]} \right)^{\gamma} = \delta_2^{bI} < \delta_1^{bI}$, $0 < \pi^{bI} < \pi_1^{bI} < \pi_3^{bI} < \pi_4^{bI} < \pi_2^{bI} < 1$, where π_3^{bI} and π_4^{bI} are defined in the appendix, and if $\pi \in \left(\pi_3^{bI}, \pi_4^{bI}\right)$, $q_F^{bI} = 0$ implying that the fragility condition holds $\forall q \in \left(0, \bar{q}^{bI}\right)$. That is, when the public sector is sufficiently small and the degree of liquidity risk is intermediate, low public insurance and banks' aggressive behavior leave the economy prone to instability though the incentive distortion is barely present.

Nevertheless, whether crises occur in equilibrium also depends on the existence of an active

banking system, which has been overlooked in the previous work. That is, in Keister (2016), the parameter space where banks are fragile may actually include equilibria that are infeasible from an ex-ante perspective. Thus, to guarantee the emergence of intermediaries, it is worth emphasizing that banks are able to offer higher welfare than autarky if and only if $q \leq q_A^{bI}$, where q_A^{bI} is defined by solving: $W^{bI} - W^{A*} = 0$. Given that $\frac{dW^{bI}}{dq} < 0$ and $\frac{dW^{A*}}{dq} = 0$, the solution to q_A^{bI} is unique. In this vein, run equilibria exist for any $q \in \left(q_F^{bI}, \min\left\{\bar{q}^{bI}, q_A^{bI}\right\}\right)$. In addition, it can be demonstrated that if agents place less weight on public consumption, i.e., $\delta < \delta_3^{bI}$ where δ_3^{bI} is defined in the appendix, then $\pi^{bI} < \pi_1^{bI} < \pi_3^{bI} < \frac{R-1}{R-r} < \pi_4^{bI} < \pi_2^{bI}$. Again, for tractability, we first examine whether depositors play the run strategy profile in equilibrium when they are highly exposed to liquidity risk, i.e., $\pi \geq \frac{R-1}{R-r}$, under which $\bar{q}^{bI} < q_A^{bI}$.

Proposition 12. *Suppose that $\delta < \delta_3^{bI}$. Under this condition, if $\pi \in \left[\frac{R-1}{R-r}, \pi_4^{bI}\right)$, $q_F^{bI} = 0 < \bar{q}^{bI} < q_A^{bI}$ and run equilibria exist $\forall q \in \left(0, \bar{q}^{bI}\right)$. If instead $\pi \in \left[\pi_4^{bI}, \pi_2^{bI}\right)$, $0 < q_F^{bI} < \bar{q}^{bI} < q_A^{bI}$ and run equilibria exist $\forall q \in \left(q_F^{bI}, \bar{q}^{bI}\right)$.*

In comparison to competitive intermediaries in setups where the fiscal backstop is not available, banks under the bailout regime may only be fragile when the run probability is high enough.

However, since it is complex to proceed analytically over lower levels of π regardless of the value of r, we construct a set of numerical examples below. Specifically, in the left panel of Figure 6, we first examine a case absent the liquidation cost by using parameters $(\pi, \gamma, R, r, \delta) = (0.5, 3, 1.5, 1, 0.01)$ under which $W^{A*} = \frac{\left(\left[\pi + (1-\pi)R^{1-\gamma}\right]^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}\right)^{\gamma}}{1-\gamma}$. In contrast to that in Keister (2016), though agents' inclination to run is satisfied for any $q \in \left[q_F^{bI}, \bar{q}^{bI}\right)$, run behavior will only be observed if $q \in \left[q_F^{bI}, q_A^{bI}\right]$ as no deposits are made once the run probability is beyond q_A^{bI}.

In addition, in the right panel of Figure 6, we adopt parameters $(\pi, \gamma, R, r, \delta) = (0.5, 3, 1.5, 0.985, 0.01)$ under which $\pi < \frac{R-1}{R-r}$. Again, we highlight how consumers' participation decisions matter for the parameter space in which bank runs are on the equilibrium path of play. Similarly, once the probability of a run is high enough, intermediaries are not able to offer a desirable level of services to attract funds and hence runs only occur if $q \in \left[q_F^{bI}, q_A^{bI}\right]$. Upon comparing the two graphs, we see that the degree of fragility appears to be higher if the long-term asset is more illiquid. However, one should note that over relatively higher run probabilities in environments without the liquidation cost, intermediaries are not active due to a higher value of consumers' outside options.

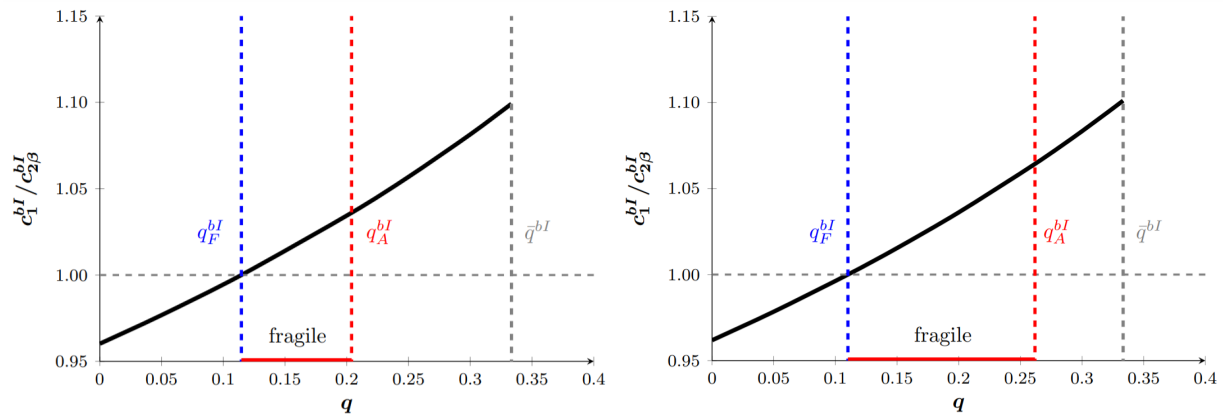

Figure 6: The existence of run equilibria under competitive banks with bailouts

7.2 Equilibrium under a monopolistic banking system

We now turn to augment our baseline model of monopolistic banks presented in the last section to include government bailouts. Notably, unlike that in the benchmark framework, the extended analysis features an active regulator for the provision of public goods. In this regard, the authority is prone to intervening in the crisis state by rescheduling the remaining private consumption levels so that fiscal policy can be conducted properly once a run is identified. In what follows, we derive the best responses of the regulatory authority and then the monopolist.

7.2.1 The best responses of the banking authority and the monopolist

Resolution and taxes in the panic state To begin, we discuss the payment scheme on deposits dictated by the banking authority in state β after public goods have been provided. Notably, the regulator will adjust payoffs to maximize V_β^j specified in (3) subject to the resource constraints (33) and (34). In solving this problem, we divide the aforementioned two constraints along with possible outcomes in normal times into four different scenarios summarized in Tables 1 and 2. Again, the conditions characterizing the optimal choice of $c_{1\beta}^j$ and $c_{2\beta}^j$ follow those in (8).

We subsequently consider the optimal tax rate upon the revelation of the crisis state. In particular, anticipating that all resources left would be used to augment private consumption, the regulator chooses the tax rate to maximize (35) subject to the constraints (33) and (34). The optimality condition regarding the choice of τ_β^j is still characterized by the second equation in (36).

To summarize state-contingent welfare, first suppose that it is profit-maximizing for the monopolist not to hold any excess liquidity and the authority resorts to costly liquidation in the event of a

crisis. Then, $\mu_{1\beta}^{j}=0$ and the efficient plan for $c_{1\beta}^{j}$ and $c_{2\beta}^{j}$ is set according to (16). Consequently, the ex-post welfare level contingent on the panic state can be written as:

$$V_{\beta}^{j}+v\left(g_{\beta}\right)=\left(\left(1-\pi\right)\left[\pi+\left(1-\pi\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)^{\gamma}u\left(\tau_{\alpha}^{j}+r\left(1-\pi c_{1}^{j}-\tau_{\alpha}^{j}\right)\right). \quad (42)$$

Nevertheless, the monopolistic bank may hold excess liquid assets and the authority liquidates long-term assets prematurely. Under this scenario, the relationship between $c_{1\beta}^{j}$ and $c_{2\beta}^{j}$ still follows (16) and combining this ex-post risk-sharing condition, the optimal condition for the tax rate (36), and the resource constraint (34) yields:

$$V_{\beta}^{j}+v\left(g_{\beta}\right)=\left(\left(1-\pi\right)\left[\pi+\left(1-\pi\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)^{\gamma}u\left(\left(1-x^{j}-\pi c_{1}^{j}\right)+rx^{j}\right). \quad (43)$$

Furthermore, as indicated by Scenario III, a sufficient amount of precautionary liquidity is held so that liquidation of productive assets is not needed though runs on the bank occur. That is, the constraint (33) binds and thus $\mu_{1\beta}^{j}>0$. We then summarize the expected utility in the crisis state as follows:

$$V_{\beta}^{j}+v\left(g_{\beta}\right)=\left[\left(1-\pi\right)\pi+\delta^{\frac{1}{\gamma}}\right]^{\gamma}u\left(1-x^{j}-\pi c_{1}^{j}\right)+\left(1-\pi\right)^{2\gamma}u\left(Rx^{j}\right). \quad (44)$$

Finally, if neither excess liquidity is held nor illiquid assets are recouped in period 1, ex-post welfare of depositors in state β can be expressed by:

$$V_{\beta}^{j}+v\left(g_{\beta}\right)=\left[\left(1-\pi\right)\pi+\delta^{\frac{1}{\gamma}}\right]^{\gamma}u\left(\tau_{\alpha}^{j}\right)+\left(1-\pi\right)^{2\gamma}u\left(R\left(1-\pi c_{1}^{j}-\tau_{\alpha}^{j}\right)\right). \quad (45)$$

Notably, conditions (42) to (45) imply that once the portfolio choice, the early payment c_{1}^{j}, and the tax rate in normal times are pinned down, ex-post welfare $V_{\beta}^{j}+v\left(g_{\beta}\right)$ is deterministic.

Bank payments and taxes in normal times We now examine how much the monopolist will pay to each withdrawing depositor if the good aggregate state is revealed and also the tax rate in normal times. To this end, let indicator functions $\mathbb{I}_{\left(1-x^{j}=\pi c_{1}^{j}+\tau_{\alpha}^{j}\right)}=1$ and $\mathbb{I}_{\left(1-x^{j}>\pi c_{1}^{j}+\tau_{\alpha}^{j}\right)}=0$ if bank j does not hold any precautionary liquid assets after taxes in state α are collected; otherwise, if excess liquidity is held, $\mathbb{I}_{\left(1-x^{j}=\pi c_{1}^{j}+\tau_{\alpha}^{j}\right)}=0$ and $\mathbb{I}_{\left(1-x^{j}>\pi c_{1}^{j}+\tau_{\alpha}^{j}\right)}=1$. Thus, since all remaining depositors will wait until period 2 to withdraw, the monopolistic bank chooses $c_{2\alpha}^{j}$ to maximize:

$$\eta_{\alpha}^{j}\equiv\max_{\{c_{2\alpha}\}}R\left(1-\pi c_{1}^{j}-\tau_{\alpha}^{j}\right)\mathbb{I}_{\left(1-x^{j}=\pi c_{1}^{j}+\tau_{\alpha}^{j}\right)}+\left(1-x^{j}-\pi c_{1}^{j}-\tau_{\alpha}^{j}+Rx^{j}\right)\mathbb{I}_{\left(1-x^{j}>\pi c_{1}^{j}+\tau_{\alpha}^{j}\right)}-\left(1-\pi\right)c_{2\alpha}^{j}$$

subject to the agents' participation constraint:

$$\underbrace{\pi u\left(c_1^j\right)+(1-q)\left[V_\alpha^j+v\left(g_\alpha\right)\right]+q\left[V_\beta^j+v\left(g_\beta\right)\right]}_{=W}=W^{A*}, \tag{46}$$

where, at this point in time, there is no uncertainty related to ex-post welfare $\left[V_\beta^j+v\left(g_\beta\right)\right]$ given by (42), (43), (44), or (45). As before, letting μ_d^j denote the multiplier on the above constraint, the first-order condition for the problem is again given by (21).

To proceed, we move backwards to consider the tax rate contingent on the good aggregate state. In general, the regulator aims to efficiently divide the economy's remaining resources offset by the bank's potential profits between public and private consumption. Nevertheless, the regulator's optimization problem in state α specified in (35) is undefined and the reason is as follows. When issuing short-term liabilities, the bank has considered the effects of state β, implying that once the first π payments are made, the first and third terms $\pi u\left(c_1^j\right)$ and $q\left[V_\beta^j+v\left(g_\beta\right)\right]$ on the left-hand-side of the participation constraint are deterministic. This further indicates that ex-post welfare in normal times $\left[V_\alpha^j+v\left(g_\alpha\right)\right]$ will be "fixed" and cannot be improved regardless of the regulator's choice of the tax rate. In this vein, as simple benchmarks, one may assume that τ_α^j takes the value of τ^{A*} or other feasible one as long as it is higher than the expected tax rate in state β.

Early payments In deciding the early payment, the monopolist chooses c_1^j to solve (22) subject to (46). Similarly, the solution to the choice of c_1^j requires $\frac{d\eta_\alpha^j}{dc_1^j}=0$. To be specific, under Scenario I where the bank does not hold excess liquid assets but the regulator resorts to liquidation of investment, c_1^j is determined by:

$$u'\left(c_1^j\right)-rq\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)^\gamma u'\left(\tau_\alpha^j+r\left(1-\pi c_1^j-\tau_\alpha^j\right)\right)=\frac{R}{\mu_d^j}. \tag{47}$$

Yet, under Scenario II in which excess liquidity is held while costly liquidation still occurs in the event of a crisis, the early payment will be set as follows:

$$u'\left(c_1^j\right)-q\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)^\gamma u'\left(\left(1-x^j-\pi c_1^j\right)+rx^j\right)=\frac{1}{\mu_d^j}. \tag{48}$$

Moreover, if a sufficiently liquid portfolio is chosen such that no illiquid assets are called prematurely as in Scenario III, the earliest demandable deposit is chosen to satisfy:

$$u'\left(c_1^j\right)-q\left[(1-\pi)\,\pi+\delta^{\frac{1}{\gamma}}\right]^\gamma u'\left(1-x^j-\pi c_1^j\right)=\frac{1}{\mu_d^j}. \tag{49}$$

Lastly, under Scenario IV where there is neither excess liquidity nor costly liquidation, the bank chooses the early return on deposits in the following way:

$$u'\left(c_1^j\right) - q\left(1-\pi\right)^{2\gamma} Ru'\left(R\left(1-\pi c_1^j - \tau_\alpha^j\right)\right) = \frac{R}{\mu_d^j}. \tag{50}$$

Notably, across all situations, if the shadow value μ_d^j is higher implying that the participation constraint is tightened, the monopolist responds by increasing the early payment c_1^j.

Portfolio choice In period 0 after consumers deposit their endowments, monopolistic bank j makes its portfolio choice. Specifically, for Scenarios I and IV where all holdings of assets in the storage technology are exhausted after taxes in state α have been collected, x^j is determined by:

$$x^j = 1 - \pi c_1^j - \tau_\alpha^j. \tag{51}$$

However, the banker may find it profit-maximizing to hoard excess liquidity and x^j is chosen to maximize (22) subject to (46). Note that under Scenario II, a relatively liquid portfolio is held to mitigate the negative effects of liquidation should a run occur:

$$\left(1-r\right) q\left(\left(1-\pi\right)\left[\pi + \left(1-\pi\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}\right)^{\gamma} u'\left(\left(1-x^j-\pi c_1^j\right) + rx^j\right) = \frac{R-1}{\mu_d^j}. \tag{52}$$

In addition, it is also possible that the bank holds a sufficiently liquid portfolio to eliminate the possibility of liquidation of long-term investment:

$$q\left(\left[\left(1-\pi\right)\pi + \delta^{\frac{1}{\gamma}}\right]^{\gamma} u'\left(1-x^j-\pi c_1^j\right) - \left(1-\pi\right)^{2\gamma} Ru'\left(Rx^j\right)\right) = \frac{R-1}{\mu_d^j}. \tag{53}$$

It is also worth noting that conditions (52) and (53) suggest that when facing a tighter participation constraint, the bank places less funds into the investment technology to keep agents indifferent to autarky.

7.2.2 Equilibrium fragility and allocations

The preceding analysis summarizes the best responses of a regulator and the monopolistic bank to the strategy profile (2) and allocations of resources can be characterized by vectors as follows:

$$\hat{c}^{b*} \equiv \left\{\hat{x}^{b*}, \hat{c}_1^{b*}, \hat{c}_{2\alpha}^{b*}, \hat{g}_\alpha^{b*}, \hat{c}_{1\beta}^{b*}, \hat{c}_{2\beta}^{b*}, \hat{g}_\beta^{b*}\right\} \quad \text{for } * = I, II, ..., IV.^{15}$$

[15] See the Appendix for the determination of the private and public consumption levels.

Notably, the intermediary's portfolio choices, payments on deposits, and the provision of public goods are defined by conditions (8), (47) to (53), the resource constraints (31), (33), (34), and the participation constraint (46). Furthermore, let $\hat{W}^{b*}$ denote aggregate welfare delivered by a monopolistic bank and the authority.

As in previous sections, the best response allocation is run prone if and only if $\hat{c}_1^{b*} \geq \hat{c}_{2\beta}^{b*}$. Consequently, we rule out Scenarios III and IV since the monopolist has sufficient resources left for the authority to offer period-2 payments should the crisis state be inferred and thus it is immune to instability.

Lemma 4. *If the monopolistic banking system with bailouts is fragile, the solution to the bank's optimization problem belongs to either Scenario I or II.*

In sharp contrast to that under perfect competition with bailouts, run equilibria under a monopolist may still exist when precautionary liquid assets are held against bank panics.

7.2.3 The impact of a higher run probability on fragility

To proceed, we examine how depositors' incentives to run change across the probabilities of a crisis. Note that as we discussed previously, the tax rate $\hat{\tau}_\alpha^{b*}$ can take any possible values such as τ^{A*} or it may vary significantly over the run probabilities as long as the basic criterion $\hat{\tau}_\alpha^{b*} > \hat{\tau}_\beta^{b*}$ is satisfied. In this manner, there could exist various patterns of agents' inclination to panic and thus the existence of run equilibria. However, what remains robust is that runs occur in equilibrium if and only if the incentive to run condition $\hat{c}_1^{b*} \geq \hat{c}_{2\beta}^{b*}$ holds and the bank extracts positive expected profits. Moreover, when the potential for run behavior is higher, bailouts affect a competitive and a monopolistic bank's responses through different channels. In particular, unlike competitive intermediaries whose incentives are distorted due to the provision of bailouts, monopolists do not completely ignore but discount possible activity in periods of distress. This necessarily happens as depositors' expected utility in state β enters through the participation constraint and bailouts only tighten or relax such a constraint.

Discussion In what follows, we highlight the important implications from our extension. In practice, to ameliorate the impact of moral hazard and promote financial stability, there have been calls for restrictions on policymakers' discretion in the use of taxpayer funding in recent years.[16] Much as in the previous literature such as Keister (2016), our analysis suggests that under perfect competition, restricting bailouts may correct incentive distortions. Intuitively, if the size of fiscal transfers

[16] See Stern and Feldman (2004, p.128), Lacker (2009), and the Federal Reserve Bank of Richmond (2017), among others.

is small enough, competitive banks need to take into account the value of resources during a crisis. In this vein, drawing bailout boundaries explicitly may be desirable if the effects of (partially) corrected incentive distortion dominate the loss of public insurance. Importantly, by the virtue that allocations under discretion are exactly the same as those under a limited-bailouts regime if the upper bound on fiscal transfers is set to be sufficiently high, a limited-bailouts policy weakly dominates a discretionary one. This implication is similar in spirit to the literature on monetary policy of inflation targeting, i.e., constrained discretion by Bernanke and Mishkin (1997). That is, it is plausible that, in practice, bailout policy should impose a limit to discipline policymakers so as to correct the incentive distortion of depositors without ruling out all flexibility ascribed to discretion.

In contrast, restricting public insurance always introduces a greater scope for fragility under a monopolistic bank, especially when panics are highly likely to occur. To understand why, note the anticipation of smaller bailouts tightens the participation constraint facing the monopolist and, in response, the bank must offer larger early deposit returns. Given a smaller amount of bailouts and thus the resources should a panic occur, the bank with market power becomes more susceptible to fragility. Moreover, as the depositors' expected utility is kept to be the same under both policy regimes, the limited-bailouts one is always inferior under a monopolistic banking system.

7.3 Relative fragility and welfare in competitive versus monopolistic banks in the presence of bailouts

Lastly, we compare the degree of fragility and welfare across these two banking economies. To begin, it is similar to that in Proposition 10 that due to monopolistic banks' strategic behavior, depositors' welfare is strictly dominated by that under perfectly competitive banks. Yet, in the presence of public insurance, competitive banks may be more fragile (stable) than monopolists if bailouts are large (small). To demonstrate these possibilities regarding the relative fragility across banking structures, we next establish a set of numerical examples.

For instance, consider the following parameters $(R, \pi, \gamma, q, r, \delta) = (0.35, 8, 1.05, 1\%, 0.985, 10)$. Further suppose that $\hat{\tau}_{\alpha}^{bI} = \tau^{A*} = 0.576395$. Under the parameter values considered, $\frac{c_1^{bI}}{c_{2\beta}^{bI}} > 1 > \frac{\hat{c}_1^{bI}}{\hat{c}_{2\beta}^{bI}}$ in which the monopolistic bank remains stable whereas its competitive counterpart is prone to a crisis. Nevertheless, suppose instead that $\hat{\tau}_{\alpha}^{bI} = 0.574250 < \tau^{A*}$. Under these conditions, though both economies are vulnerable to a crisis, $\frac{\hat{c}_1^{bI}}{\hat{c}_{2\beta}^{bI}} > \frac{c_1^{bI}}{c_{2\beta}^{bI}}$, implying that depositors have more incentives to run on the monopolist than on the competitive bank. The intuition is as follows. When the regulator is expected to levy a relatively higher tax rate in normal times, bailouts tend to be larger during a run and therefore the monopolist is immune to instability. In this vein, it is trivial to argue that competitive banks are more fragile. If instead the monopolistic bank antici-

pates a comparatively lower tax rate in the absence of runs, fiscal transfers in the event a panic would be smaller. Among these scenarios, the monopolist needs to augment short-term liabilities to satisfy the consumers' participation constraint, leading itself to become more illiquid and thus more unstable than the competitive one.

8 Conclusion

In recent years, the presence of "Too-Big-To-Fail" intermediaries in advanced economies has generated a heated debate about the effects of increasing concentration on banking activity and stability. In particular, there exists a great amount of concern that large-scale financial intermediaries may be able to exert their market power and thus concentration is likely associated with less competitive outcomes. In light of these observations, we build on a version of Diamond and Dybvig (1983) with limited commitment to study the role of financial sector competition for fragility.

In particular, as the probability of a run is higher, competitive banks behave more prudently when issuing liabilities regardless of their portfolio choices, improving the degree of financial stability. In contrast, if there is greater potential for consumer sentiment to deteriorate, depositors' participation constraints facing a monopolist are tightened, which induces the bank to respond more aggressively prior to inferring the aggregate state. Notably, over initially lower probabilities of a run, the bank holds a more illiquid portfolio such that depositors' incentives to panic are stronger. Yet, when runs are more likely to occur where the monopolist finds it optimal to hold precautionary liquid assets against crises, depositors' incentives to panic are weaker as the propensity to run is higher. Interestingly, the scope for instability under a monopolist is weakly contained in that under perfect competition, regardless of run probabilities. Nevertheless, once government bailouts are available, a higher run probability induces competitive banks to behave imprudently due to the incentive distortion. By comparison, agents' inclination to run on a monopolistic bank can either increase or decrease, depending on the structure of financial assistance. In addition, competitive intermediaries are more fragile if bailouts are large but this does not occur if fiscal transfers are small.

There are many important extensions to consider for future work. For example, one possible extension would be to study how liquidity creation and fragility depend on the dual effects from increasing returns to scale and the industrial organization of the banking system. In the current framework, we abstract from the important issue of scale economies, which may be suggested to be a significant factor in banking consolidation. Thus, by incorporating economies of scale, it would be interesting to explore how the relative stability across the competitive structure of the banking system changes and the trade-offs between liquidity insurance and fragility.

9 Technical appendix

The best-response allocation of a competitive bank

The best-response allocation of a competitive bank is determined by conditions (8), (10), (11), and the resource constraints (4) to (7).

Scenario I: $1-x-\pi c_1=0$ and $1-x-\pi c_1<(1-\pi)\pi c_{1\beta}$

If $q\in\left(0,q^I\right]$, where $q^I=\frac{1}{1+\left(\frac{1-r}{R-1}\right)\left[\pi\left(\frac{R}{r}\right)+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\right]^{\gamma}}$, the bank does not hold excess liquidity and resorts to costly liquidation should a run occur. Thus, the vector $c^I=\left\{x^I,c_1^I,c_{2\alpha}^I,c_{1\beta}^I,c_{2\beta}^I\right\}$ is given by:

$$\begin{aligned}
x^I &= 1-\pi c_1^I,\\
c_1^I &= \frac{1}{\pi+(1-\pi)\left(\left(1-q\right)R^{1-\gamma}+q\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^{\gamma}r^{1-\gamma}\right)^{\frac{1}{\gamma}}},\\
c_{2\alpha}^I &= \frac{R}{1-\pi}\left(1-\pi c_1^I\right),\\
c_{1\beta}^I &= \frac{1}{(1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1}{r}\right)}\left(1-\pi c_1^I\right),\\
c_{2\beta}^I &= \frac{\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}}{(1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1}{r}\right)}\left(1-\pi c_1^I\right),
\end{aligned}$$

where $\frac{c_{2\beta}^I}{c_{1\beta}^I}=\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}>1$ and $\frac{c_{2\alpha}^I}{c_1^I}=\left(\left(1-q\right)R+qR\left[\pi\left(\frac{R}{r}\right)^{\frac{\gamma-1}{\gamma}}+(1-\pi)\right]^{\gamma}\right)^{\frac{1}{\gamma}}>1$.

Scenario II: $1-x-\pi c_1>0$ and $1-x-\pi c_1<(1-\pi)\pi c_{1\beta}$

If $q\in\left(q^I,q^{II}\right)$, where $q^{II}=\frac{1}{1+\left(\frac{1-r}{R-1}\right)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\right]^{\gamma}}$, the bank holds excess liquid assets and resorts to costly liquidation should a run occur. Thus, the vector $c^{II}=\left\{x^{II},c_1^{II},c_{2\alpha}^{II},c_{1\beta}^{II},c_{2\beta}^{II}\right\}$ is

given by:

$$\begin{aligned}
x^{II} &= \frac{\pi c_1^{II} + (1-q)^{\frac{1}{\gamma}}(1-\pi)\left(\frac{R-r}{1-r}\right)^{\frac{1}{\gamma}} c_1^{II} - 1}{R-1}, \\
c_1^{II} &= \frac{1}{\pi + (1-q)^{\frac{1}{\gamma}}(1-\pi)\left(\frac{R-r}{1-r}\right)^{\frac{1-\gamma}{\gamma}} + q^{\frac{1}{\gamma}}(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{R-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}}, \\
c_{2\alpha}^{II} &= \left(\frac{(1-q)(R-r)}{1-r}\right)^{\frac{1}{\gamma}} c_1^{II}, \\
c_{1\beta}^{II} &= \left(\frac{q(R-r)}{R-1}\right)^{\frac{1}{\gamma}} c_1^{II}, \\
c_{2\beta}^{II} &= \left(\frac{qR(R-r)}{r(R-1)}\right)^{\frac{1}{\gamma}} c_1^{II},
\end{aligned}$$

where $\frac{c_{2\beta}^{II}}{c_{1\beta}^{II}} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} > 1$ and $\frac{c_{2\alpha}^{II}}{c_1^{II}} = \left(\frac{(1-q)(R-r)}{1-r}\right)^{\frac{1}{\gamma}} > 1$.

Scenario III: $1 - x - \pi c_1 > 0$ and $1 - x - \pi c_1 = (1-\pi)\pi c_{1\beta}$

If $q \geq q^{II}$, the bank holds excess liquid assets but does not resort to costly liquidation should a run occur. First note that $\frac{c_{1\beta}}{c_{2\beta}}$ is determined by:

$$(1-q)(R-1)\left[\pi\frac{c_{1\beta}}{c_{2\beta}} + (1-\pi)\right]^{-\gamma} - q\left[\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{-\gamma} - R\right] = 0,$$

where $\frac{c_{2\beta}^{III}}{c_{1\beta}^{III}} > 1$ holds. Then, the vector $c^{III} = \left\{x^{III}, c_1^{III}, c_{2\alpha}^{III}, c_{1\beta}^{III}, c_{2\beta}^{III}\right\}$ is computed by substituting $\frac{c_{1\beta}^{III}}{c_{2\beta}^{III}}$ along with:

$$\begin{aligned}
x^{III} &= \frac{(1-\pi)^2}{R} c_{2\beta}^{III}, \\
c_1^{III} &= \left((1-q)R\left[\pi\frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} + (1-\pi)\right]^{-\gamma} + qR\right)^{-\frac{1}{\gamma}} c_{2\beta}^{III}, \\
c_{2\alpha}^{III} &= \left[\pi\frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} + (1-\pi)\right] c_{2\beta}^{III}, \\
c_{1\beta}^{III} &= \frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} c_{2\beta}^{III}, \\
c_{2\beta}^{III} &= \frac{1}{(1-\pi)\pi\frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} + \frac{(1-\pi)^2}{R} + \pi\left((1-q)R\left[\pi\frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} + (1-\pi)\right]^{-\gamma} + qR\right)^{-\frac{1}{\gamma}}},
\end{aligned}$$

where $\frac{c_{2\alpha}^{III}}{c_1^{III}} = \left((1-q)R + qR\left[\pi\frac{c_{1\beta}^{III}}{c_{2\beta}^{III}} + (1-\pi)\right]^{\gamma}\right)^{\frac{1}{\gamma}} > 1$. Yet, as discussed in the main text, $\frac{c_1^{III}}{c_{2\beta}^{III}} < 1$.

■

Proof of Lemma 1

Note that if $q \leq q^I$, where $q^I = \frac{1}{1+\left(\frac{1-r}{R-1}\right)\left[\pi\left(\frac{R}{r}\right)+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\right]^{\gamma}}$, $\mu_1^I > 0$ and $\mu_{1\beta}^I = 0$. That is, the bank does not hold excess liquid assets but resorts to costly liquidation should a run occur. Yet, if $q^I < q < q^{II}$, where $q^{II} = \frac{1}{1+\left(\frac{1-r}{R-1}\right)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\right]^{\gamma}}$, $\pi c_1^{II} < 1 - x^{II}$ and $1 - x^{II} - \pi c_1^{II} < (1-\pi)\pi c_{1\beta}^{II}$ hold. In this vein, it would be optimal for the bank to hold excess liquid assets and to resort to costly liquidation in the panic state. Lastly, $\frac{c_1^{III}}{c_{2\beta}^{III}} < 1$ suggests that run equilibria will never exist under Scenario III.

■

Proof of Proposition 1

It can be easily shown that both $\frac{c_1^I}{c_{2\beta}^I}$ and $\frac{c_1^{II}}{c_{2\beta}^{II}}$ are monotonically decreasing in q:

$$\begin{aligned}
\frac{d\frac{c_1^I}{c_{2\beta}^I}}{dq} &= \frac{r^{\gamma-1}R^{2-\gamma}\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^{-\gamma} - R}{\left((1-q)\,r^{\gamma-1}R^{2-\gamma}\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^{-\gamma} + qR\right)^{\frac{2}{\gamma}}} < 0 \quad \text{and} \\
\frac{d\frac{c_1^{II}}{c_{2\beta}^{II}}}{dq} &= -\frac{1}{\gamma}\left(\frac{R\,(R-r)}{r\,(R-1)}\right)^{-\frac{1}{\gamma}} q^{-\frac{1}{\gamma}-1} < 0.
\end{aligned}$$

■

Proof of Proposition 2

First, note that if the asset return R is close to 1, the autarkic welfare level is characterized by $W^A = \frac{1}{1-\gamma}$ for any π, which is given by (1), and q^I is arbitrarily close to zero. Given that the bank is able to deliver the expected utility higher than financial autarky when the run probability is negligible, we must have $W^I\left(q^I\right) > W^A = \frac{1}{1-\gamma}$. This further suggests that if $W^A = \frac{1}{1-\gamma}$, $q^I < q_A^I$ for any (π, γ, R, r). In what follows, we turn to the proof of cases i and ii.

i. Suppose that $\gamma \in \left(1, \min\left\{2, \gamma^I\right\}\right]$ and $R < r^{\frac{1-\gamma}{2-\gamma}}$ or $\gamma \in \left(2, \gamma^I\right]$ such that $0 < \pi_F^I < 1 < \pi^I$, where $\gamma^I = \frac{\ln\left(\frac{R^2-2rR+r}{(1-r)R}\right)}{\ln\left(\frac{R}{r}\right)} + 1$, $\pi_F^I = \frac{R^{\frac{1}{\gamma}}-1}{\left(\frac{R}{r}\right)^{\frac{\gamma-1}{\gamma}}-1}$, and $\pi^I = \frac{\left(\frac{R^2-2rR+r}{(1-r)R}\right)^{\frac{1}{\gamma}}-1}{\left(\frac{R}{r}\right)^{\frac{\gamma-1}{\gamma}}-1}$. Further suppose that

$\pi > \max\left\{\pi_F^I, \frac{R-1}{R-r}\right\}$. Under these conditions, $0 < q_F^I < q^I$. Importantly, as $W^I\left(q^I\right) > W^A$, $q_F^I < q^I < q_A^I$ and therefore run equilibria exist $\forall q \in \left(0, q_F^I\right]$.

ii. If $\gamma > \gamma^I$ and $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$, $0 < q^I < q_F^I$. Using the analogous reasoning demonstrated above, we have $W^I\left(q^I\right) > W^A$ and thus $q^I < q_A^I$. Consequently, run equilibria exist $\forall q \in \left(0, q^I\right]$.

■

Proof of Proposition 3

As a start, suppose that $r = 1$. Further suppose that $\gamma > 2$ and $\pi > \frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}$. Under these conditions, $q_F^I \in (0,1)$ and the incentive to run condition holds $\forall q \in \left(0, q_F^I\right]$. In addition, $W^I \geq W^A$ if and only if $q \leq q_A^I$.

To proceed, we examine the existence of run equilibria. Note that:

$$\lim_{R\to 1} W^I\left(q_F^I\right) = \lim_{R\to 1} W^A = \frac{1}{1-\gamma} \quad \text{and} \quad \lim_{R\to\infty} W^I\left(q_F^I\right) = \frac{\pi^\gamma}{1-\gamma} > \lim_{R\to\infty} W^A = \frac{\pi}{1-\gamma}.$$

Furthermore, it is helpful to derive the following properties of W^A and $W^I\left(q_F^I\right)$:

$$\frac{dW^A}{dR} > 0, \quad \frac{dW^A}{d^2R} < 0, \quad \frac{dW^I\left(q_F^I\right)}{dR} > 0, \quad \text{and} \quad \frac{dW^I\left(q_F^I\right)}{d^2R} < 0.$$

That is, both W^A and $W^I\left(q_F^I\right)$ are strictly concave. Moreover, $\left.\frac{dW^I\left(q_F^I\right)}{dR}\right|_{R=1} \geq (<) \left.\frac{dW^A}{dR}\right|_{R=1}$ if and only if $\pi \leq (>) \frac{1}{\gamma-1}$.

Next, we demonstrate that $\frac{1}{\gamma-1} > \frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}$ holds for any $\gamma > 2$ and $R > 1$. To this end, we define:

$$\Delta\pi^I \equiv \frac{1}{\gamma-1} - \frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}} = \frac{\Lambda^I}{(\gamma-1)\left(R^{\frac{\gamma-1}{\gamma}} - 1\right)},$$

where $\Lambda^I \equiv \left(R^{\frac{\gamma-1}{\gamma}} - 1\right) - (\gamma-1)\left(R^{\frac{1}{\gamma}} - 1\right)$. That is, the sign of $\Delta\pi^I$ depends on the sign of Λ^I. To proceed, differentiating Λ^I with respect to R yields:

$$\frac{d\Lambda^I}{dR} = \left(\frac{\gamma-1}{\gamma}\right)\left(1 - R^{\frac{2-\gamma}{\gamma}}\right) R^{-\frac{1}{\gamma}} > 0,$$

for any $\gamma > 2$ and $R > 1$. By $\lim_{R\to 1} \Lambda^I = 0$ and $\frac{d\Lambda^I}{dR} > 0$, $\Delta\pi^I > 0$ and hence $\frac{1}{\gamma-1} > \frac{R^{\frac{2-\gamma}{\gamma}} - R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}$.

To summarize, suppose that $\gamma \in (2,\infty)$. Further suppose that $\pi \in \left(\frac{R^{\frac{2-\gamma}{\gamma}}-R^{\frac{1-\gamma}{\gamma}}}{1-R^{\frac{1-\gamma}{\gamma}}}, \frac{1}{\gamma-1}\right]$. Under these conditions, $W^I\left(q_F^I\right) > W^A$, implying that $q_F^I < q_A^I$ and thus run equilibria exist $\forall q \in \left(0, q_F^I\right]$. However, suppose that $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$. Under this condition, there exists a threshold $\underline{R}^I \in (1,\infty)$ such that if $R \in \left(1, \underline{R}^I\right)$, $W^I\left(q_F^I\right) < W^A$, suggesting that $q_A^I < q_F^I$ and hence run equilibria exist $\forall q \in \left(0, q_A^I\right]$. If instead $R \in \left(\underline{R}^I, \infty\right)$, $W^I\left(q_F^I\right) > W^A$, implying that $q_F^I < q_A^I$ and therefore run equilibria exist $\forall q \in \left(0, q_F^I\right]$.

■

Proof of Proposition 4

Suppose that $\gamma > \gamma^I$ and $\pi > \pi^I$. Under these conditions, $0 < q^I < q_F^{II} < q^{II}$. Note also that q_F^{II} is arbitrarily close to zero if R is close to 1. Thus, using the similar reasoning demonstrated in the proof of Proposition 2, for any $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$, $W^{II}\left(q_F^{II}\right) > W^A = \frac{1}{1-\gamma}$, implying that $q_F^{II} < q_A^{II}$. Consequently, run equilibria exist for any $q \in \left(0, q_F^{II}\right]$.

■

The best-response allocation of the banking authority and the monopolistic bank

The best-response allocation in a monopolistic banking system is determined by conditions (8), (23) to (28), the resource constraints (4), (6), (7), and the participation constraint (20).

Scenario I: $1 - x - \pi c_1 = 0$ and $1 - x - \pi c_1 < (1-\pi)\pi c_{1\beta}$

If $q \in \left(0, \hat{q}^I\right]$, where $\hat{q}^I$ is defined in the proof of Lemma 2 below, no excess liquid assets are held while costly liquidation occurs should a run be identified. First, note that c_1 is determined by:

$$\left\{ \begin{array}{c} \pi\left(\hat{c}_1^I\right)^{1-\gamma} + (1-q)(1-\pi)\left(\frac{(1-q)R}{\left(\hat{c}_1^I\right)^{-\gamma} - qr^{1-\gamma}\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma}\left(1-\pi\hat{c}_1^I\right)^{-\gamma}}\right)^{\frac{1-\gamma}{\gamma}} \\ +qr^{1-\gamma}\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma}\left(1-\pi\hat{c}_1^I\right)^{1-\gamma} - (1-\gamma)W^A \end{array} \right\} = 0.$$

Once the solution for $\hat{c}_1^I$ is obtained numerically, the full allocation summarized by the vector

$\hat{c}^I = \left\{\hat{x}^I, \hat{c}_1^I, \hat{c}_{2\alpha}^I, \hat{c}_{1\beta}^I, \hat{c}_{2\beta}^I\right\}$ is calculated by substituting $\hat{c}_1^I$ along with $\hat{x}^I$, $\hat{c}_{2\alpha}^I$, $\hat{c}_{1\beta}^I$, and $\hat{c}_{2\beta}^I$:

$$\begin{aligned}
\hat{x}^I &= 1 - \pi\hat{c}_1^I, \\
\hat{c}_{2\alpha}^I &= \left(\frac{(1-q)\,R}{\left(\hat{c}_1^I\right)^{-\gamma} - qr^{1-\gamma}\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma}\left(1-\pi\hat{c}_1^I\right)^{-\gamma}}\right)^{\frac{1}{\gamma}}, \\
\hat{c}_{1\beta}^I &= \frac{1}{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1}{r}\right)}\left(1-\pi\hat{c}_1^I\right), \\
\hat{c}_{2\beta}^I &= \frac{\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}}{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1}{r}\right)}\left(1-\pi\hat{c}_1^I\right),
\end{aligned}$$

where $\frac{\hat{c}_{2\beta}^I}{\hat{c}_{1\beta}^I} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} > 1$ and it can be verified that there is room for the condition $\frac{\hat{c}_{2\alpha}^I}{\hat{c}_1^I} > 1$ to hold.

Scenario II: $1 - x - \pi c_1 > 0$ and $1 - x - \pi c_1 < (1-\pi)\,\pi c_{1\beta}$

If $q \in \left(\hat{q}^I, \hat{q}^{II}\right)$, where $\hat{q}^{II}$ is defined in the proof of Lemma 2 below, excess liquid assets are held and costly liquidation occurs should a run be identified. Thus, the vector $\hat{c}^{II} = \left\{\hat{x}^{II}, \hat{c}_1^{II}, \hat{c}_{2\alpha}^{II}, \hat{c}_{1\beta}^{II}, \hat{c}_{2\beta}^{II}\right\}$ is given by:

$$\begin{aligned}
\hat{x}^{II} &= \frac{1 - \pi\hat{c}_1^{II} - q^{\frac{1}{\gamma}}(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{R-r}{R-1}\right)^{\frac{1}{\gamma}}\hat{c}_1^{II}}{1-r}, \\
\hat{c}_1^{II} &= \left(\frac{(1-\gamma)\,W^A}{\pi + (1-q)^{\frac{1}{\gamma}}(1-\pi)\left(\frac{R-r}{1-r}\right)^{\frac{1-\gamma}{\gamma}} + q^{\frac{1}{\gamma}}(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{R-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}}\right)^{\frac{1}{1-\gamma}}, \\
\hat{c}_{2\alpha}^{II} &= \left(\frac{(1-q)\,(R-r)}{1-r}\right)^{\frac{1}{\gamma}}\hat{c}_1^{II}, \\
\hat{c}_{1\beta}^{II} &= \left(\frac{q\,(R-r)}{R-1}\right)^{\frac{1}{\gamma}}\hat{c}_1^{II}, \\
\hat{c}_{2\beta}^{II} &= \left(\frac{qR\,(R-r)}{r\,(R-1)}\right)^{\frac{1}{\gamma}}\hat{c}_1^{II},
\end{aligned}$$

where $\frac{\hat{c}_{2\beta}^{II}}{\hat{c}_{1\beta}^{II}} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} > 1$ and $\frac{\hat{c}_{2\alpha}^{II}}{\hat{c}_1^{II}} = \left(\frac{(1-q)(R-r)}{1-r}\right)^{\frac{1}{\gamma}} > 1$.

Scenario III: $1 - x - \pi c_1 > 0$ and $1 - x - \pi c_1 = (1-\pi)\,\pi c_{1\beta}$

If $q \geq \hat{q}^{II}$, excess liquid assets are held and no costly liquidation occurs should a run be

identified. Note that $\frac{c_{1\beta}}{c_{2\beta}}$ and $c_{2\beta}$ are jointly determined by:

$$\begin{cases} \frac{1}{c_{2\beta}} - \frac{(1-\pi)^2}{R} - \pi\left(\frac{qR}{R-1}\left[\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{-\gamma}-1\right]\right)^{-\frac{1}{\gamma}} - (1-\pi)\,\pi\frac{c_{1\beta}}{c_{2\beta}} = 0 \\ \\ c_{2\beta} = \dfrac{\left[(1-\gamma)W^A\right]^{\frac{1}{1-\gamma}}}{\left[\pi\left(\left(\frac{qR}{R-1}\right)\left[\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{-\gamma}-1\right]\right)^{\frac{\gamma-1}{\gamma}} + (1-\pi)\left((1-q)\left(\left(\frac{q}{(1-q)(R-1)}\right)\left[\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{-\gamma}-R\right]\right)^{\frac{\gamma-1}{\gamma}} + q\left[\pi\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{1-\gamma}+(1-\pi)\right]\right)\right]^{\frac{1}{1-\gamma}}}. \end{cases}$$

Then, the vector $\hat{c}^{III} = \left\{\hat{x}^{III}, \hat{c}_1^{III}, \hat{c}_{2\alpha}^{III}, \hat{c}_{1\beta}^{III}, \hat{c}_{2\beta}^{III}\right\}$ is computed by substituting $\frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}}$ and $\hat{c}_{2\beta}^{III}$ along with:

$$\begin{aligned} \hat{x}^{III} &= \frac{(1-\pi)^2}{R}\hat{c}_{2\beta}^{III}, \\ \hat{c}_1^{III} &= \left(\frac{qR}{R-1}\left[\left(\frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}}\right)^{-\gamma}-1\right]\right)^{-\frac{1}{\gamma}}\hat{c}_{2\beta}^{III}, \\ \hat{c}_{2\alpha}^{III} &= \left(\left(\frac{q}{(1-q)(R-1)}\right)\left[\left(\frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}}\right)^{-\gamma}-R\right]\right)^{-\frac{1}{\gamma}}\hat{c}_{2\beta}^{III}, \\ \hat{c}_{1\beta}^{III} &= \frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}}\hat{c}_{2\beta}^{III}, \end{aligned}$$

where $\frac{\hat{c}_{2\alpha}^{III}}{\hat{c}_1^{III}} = \left(\left(\frac{1}{(1-q)R}\right)\frac{\left(\frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}}\right)^{-\gamma}-R}{\left(\frac{\hat{c}_{1\beta}^{III}}{\hat{c}_{2\beta}^{III}}\right)^{-\gamma}-1}\right)^{-\frac{1}{\gamma}} > 1$. Yet, as discussed in the main text, $\frac{\hat{c}_1^{III}}{\hat{c}_{2\beta}^{III}} < 1$.

■

Proof of Lemma 2

To begin, we establish that $\frac{d\hat{c}_1^*}{dq} > 0$ for $* = I, II$. To this end, first note that $\hat{c}_{2\alpha}^*$ is a strictly increasing function of $\hat{c}_1^*$:

$$\begin{aligned} \frac{d\hat{c}_{2\alpha}^I}{d\hat{c}_1^I} &= \frac{1}{\gamma}\left(\frac{(1-q)\,R}{\left(\hat{c}_1^I\right)^{-\gamma} - qr^{1-\gamma}\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma}\left(1-\pi\hat{c}_1^I\right)^{-\gamma}}\right)^{\frac{1-\gamma}{\gamma}} \\ &\times\left(\frac{(1-q)\,R\left[\gamma\left(\hat{c}_1^I\right)^{-\gamma-1} + \pi\gamma qr^{1-\gamma}\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma}\left(1-\pi\hat{c}_1^I\right)^{-\gamma-1}\right]}{\left[\left(\hat{c}_1^I\right)^{-\gamma} - qr^{1-\gamma}\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma}\left(1-\pi\hat{c}_1^I\right)^{-\gamma}\right]^2}\right) > 0 \end{aligned}$$

and

$$\frac{d\hat{c}_{2\alpha}^{II}}{d\hat{c}_{1}^{II}} = \left(\frac{(1-q)(R-r)}{1-r}\right)^{\frac{1}{\gamma}} > 0.$$

Note also that in equilibrium, $\hat{W}^* = W^A$ and $(1-\pi)\,u\left(\hat{c}_{2\alpha}^{*}\right) > \hat{V}_{\beta}^{*}$, where $\hat{V}_{\beta}^{*}$ represents equilibrium ex-post welfare in state β and is given by (17) or (18). The proof below is by contradiction. Suppose that $\frac{d\hat{c}_{1}^{*}}{dq} \leq 0$ such that $\frac{d\hat{c}_{2\alpha}^{*}}{dq} \leq 0$. Then, the derivative $\frac{d\hat{W}^*}{dq}$ is as follows:

$$\begin{aligned}
\frac{d\hat{W}^{I}}{dq} &= \underbrace{\pi\left[u'\left(\hat{c}_{1}^{I}\right) - qr\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} u'\left(r\left(1-\pi\hat{c}_{1}^{I}\right)\right)\right]\frac{d\hat{c}_{1}^{I}}{dq}}_{\leq 0} \\
&\quad + \underbrace{(1-\pi)(1-q)\,u'\left(\hat{c}_{2\alpha}^{I}\right)\frac{d\hat{c}_{2\alpha}^{I}}{dq}}_{\leq 0} \\
&\quad + \underbrace{\left[\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} u\left(r\left(1-\pi\hat{c}_{1}^{I}\right)\right) - (1-\pi)\,u\left(\hat{c}_{2\alpha}^{I}\right)\right]}_{<0} \quad \text{and}
\end{aligned}$$

$$\begin{aligned}
\frac{d\hat{W}^{II}}{dq} &= \underbrace{\left[\pi + \left(\frac{R-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)q^{\frac{1}{\gamma}}\right]u'\left(\hat{c}_{1}^{II}\right)\frac{d\hat{c}_{1}^{II}}{dq}}_{\leq 0} \\
&\quad + \underbrace{(1-\pi)(1-q)\,u'\left(\hat{c}_{2\alpha}^{II}\right)\frac{d\hat{c}_{2\alpha}^{II}}{dq}}_{\leq 0} \\
&\quad + \underbrace{\frac{1}{\gamma}\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^{\gamma} u\left(\left(1-\hat{x}^{II}-\pi\hat{c}_{1}^{II}\right)+r\hat{x}^{II}\right) - (1-\pi)\,u\left(\hat{c}_{2\alpha}^{II}\right)}_{<0}
\end{aligned}$$

Nevertheless, the result that $\frac{d\hat{W}^*}{dq} < 0$ contradicts the fact that $\frac{d\hat{W}^*}{dq} = \frac{dW^A}{dq} = 0$. Consequently, $\frac{d\hat{c}_{1}^{*}}{dq} > 0$ and thus $\frac{d\hat{c}_{2\alpha}^{*}}{dq} > 0$.

To proceed, we demonstrate that $\frac{d\left(1-\hat{x}^{II}-\pi\hat{c}_{1}^{II}\right)}{dq} > 0$ and $\frac{d\left[(1-\pi)\pi\hat{c}_{1\beta}^{II}-\left(1-\hat{x}^{II}-\pi\hat{c}_{1}^{II}\right)\right]}{dq} < 0$. As a starting point, differentiating $\hat{x}^{II}$ with respect to q yields:

$$\frac{d\hat{x}^{II}}{dq} = -\left(\frac{1}{1-r}\right)\left[\pi\frac{d\hat{c}_{1}^{II}}{dq} + \left(\frac{R-r}{R-1}\right)^{\frac{1}{\gamma}}(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1}{\gamma}q^{\frac{1-\gamma}{\gamma}}\hat{c}_{1}^{II} + q^{\frac{1}{\gamma}}\frac{d\hat{c}_{1}^{II}}{dq}\right)\right] < 0.$$

Moreover, the derivative $\frac{d\left[R\hat{x}^{II}+\frac{R}{r}\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)\right]}{dq}$ is as follows:

$$R\frac{d\hat{x}^{II}}{dq}+\frac{R}{r}\frac{d\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)}{dq}=\left(\frac{R\left(R-r\right)}{r\left(R-1\right)}\right)^{\frac{1}{\gamma}}\left(1-\pi\right)\left[\pi\left(\frac{R}{r}\right)^{\frac{\gamma-1}{\gamma}}+\left(1-\pi\right)\right]\left(\frac{1}{\gamma}q^{\frac{1-\gamma}{\gamma}}\hat{c}_1^{II}+q^{\frac{1}{\gamma}}\frac{d\hat{c}_1^{II}}{dq}\right)>0.$$

Given that $\frac{d\hat{x}^{II}}{dq}<0$, we must have $\frac{d\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)}{dq}>0$.

In addition, note that:

$$\frac{d\hat{c}_{2\beta}^{II}}{dq}=\left(\frac{R\left(R-r\right)}{r\left(R-1\right)}\right)^{\frac{1}{\gamma}}\left(\frac{1}{\gamma}q^{\frac{1-\gamma}{\gamma}}\hat{c}_1^{II}+q^{\frac{1}{\gamma}}\frac{d\hat{c}_1^{II}}{dq}\right)>0.$$

Then, differentiating the resource constraint (7) with respect to q yields:

$$\underbrace{\left(1-\pi\right)^2\frac{d\hat{c}_{2\beta}^{II}}{dq}}_{>0}=\underbrace{R\frac{d\hat{x}^{II}}{dq}}_{<0}-\frac{R}{r}\frac{d\left[\left(1-\pi\right)\pi\hat{c}_{1\beta}^{II}-\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)\right]}{dq}.$$

Since $\frac{d\hat{c}_{2\beta}^{II}}{dq}>0$ and $\frac{d\hat{x}^{II}}{dq}<0$, we must have $\frac{d\left[(1-\pi)\pi\hat{c}_{1\beta}^{II}-\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)\right]}{dq}<0$.

To proceed, as terms $\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)$ and $\left[\left(1-\pi\right)\pi\hat{c}_{1\beta}^{II}-\left(1-\hat{x}^{II}-\pi\hat{c}_1^{II}\right)\right]$ are monotone functions of q, then it can be shown that $\pi\hat{c}_1^{II}<1-\hat{x}^{II}$ and $1-\hat{x}^{II}-\pi\hat{c}_1^{II}<\left(1-\pi\right)\pi\hat{c}_{1\beta}^{II}$ hold if and only if $q\in\left(\hat{q}^I,\hat{q}^{II}\right)$, where $\hat{q}^I$ and $\hat{q}^{II}$ are determined by solving:

$$\begin{cases}\dfrac{\left[(1-\gamma)W^A\right]^{\frac{1}{1-\gamma}}}{\left[\pi+(1-\pi)\left(\frac{R-r}{1-r}\right)^{\frac{1-\gamma}{\gamma}}\left((1-\hat{q}^I)^{\frac{1}{\gamma}}+(\hat{q}^I)^{\frac{1}{\gamma}}\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}\right)\right]^{\frac{1}{1-\gamma}}}-\dfrac{r}{\left(\frac{R-r}{R-1}\right)^{\frac{1}{\gamma}}(\hat{q}^I)^{\frac{1}{\gamma}}(1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+r\pi}=0\\ \dfrac{\left[(1-\gamma)W^A\right]^{\frac{1}{1-\gamma}}}{\left[\pi+(1-\pi)\left(\frac{R-r}{1-r}\right)^{\frac{1-\gamma}{\gamma}}\left((1-\hat{q}^{II})^{\frac{1}{\gamma}}+(\hat{q}^{II})^{\frac{1}{\gamma}}\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}\right)\right]^{\frac{1}{1-\gamma}}}\\ \qquad\qquad-\dfrac{r}{r\pi-\left((1-r)\pi-\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)\left(\frac{R-r}{R-1}\right)^{\frac{1}{\gamma}}(\hat{q}^{II})^{\frac{1}{\gamma}}(1-\pi)}=0.\end{cases}$$

To summarize, if $q\leq\hat{q}^I$, it would be optimal for the bank not to hold any excess liquid assets while costly liquidation occurs should the crisis state be inferred. Yet, if $\hat{q}^I<q<\hat{q}^{II}$, $\pi\hat{c}_1^{II}<1-\hat{x}^{II}$ and $1-\hat{x}^{II}-\pi\hat{c}_1^{II}<\left(1-\pi\right)\pi\hat{c}_{1\beta}^{II}$ hold. In this vein, it would be optimal for the bank to hold excess liquidity and costly liquidation occurs in the panic state.

■

Proof of Proposition 5

It follows from the proof of Lemma 2 that $\frac{d\hat{c}_1^I}{dq} > 0$ and thus:

$$\begin{aligned}
\frac{d\frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I}}{dq} &= \frac{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\frac{1}{r}\right)}{\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}} \frac{\frac{d\hat{c}_1^I}{dq}}{\left(1-\pi\hat{c}_1^I\right)^2} > 0 \\
\frac{d\frac{\hat{c}_1^{II}}{\hat{c}_{2\beta}^{II}}}{dq} &= -\frac{1}{\gamma}\left(\frac{R(R-r)}{r(R-1)}\right)^{-\frac{1}{\gamma}} q^{-\frac{1}{\gamma}-1} < 0.
\end{aligned}$$

Given that the equilibrium implements allocation $\hat{c}^I$ if $q \leq \hat{q}^I$ and $\hat{c}^{II}$ if $\hat{q}^I < q < \hat{q}^{II}$, the results stated in the proposition are established.

■

Proof of Propositions 6

Recall that if both types of intermediaries hold precautionary liquid assets in their portfolios, the monopolistic bank is as fragile as its competitive counterpart, i.e., $\hat{q}_F^{II} = q_F^{II}$. Thus, it follows from the proof of Proposition 4 that if $\gamma > \gamma^I$ and $\pi > \pi^I$, the fragility condition under a monopolist possessing excess liquidity must hold $\forall q \in \left(\hat{q}^I, \hat{q}_F^{II}\right]$. This further implies that as q goes to $\hat{q}^I$, depositors have incentives to run on the monopolistic bank.

To verify whether the minimum cutoff $\hat{q}_F^I$ can take the value of zero, we further assume that $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$ such that $W^A = \frac{1}{1-\gamma}$ and $\hat{q}_F^I < \hat{q}^I < q_A^I$. In this vein, $\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} \geq 1$ if and only if $r \leq \hat{r}^I = \left(\frac{\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{\frac{1}{1-\gamma}} - \pi - (1-\pi)^2 R^{-1}}{(1-\pi)\pi R^{-\frac{1}{\gamma}}}\right)^{\frac{\gamma}{1-\gamma}}$.

To summarize, suppose that $\gamma > \gamma^I$ and $\pi > \max\left\{\pi^I, \frac{R-1}{R-r}\right\}$. Under these conditions, if $r \in \left(\hat{r}^I, 1\right)$, $\hat{q}_F^I > 0$ and run equilibria exist $\forall q \in \left(\hat{q}_F^I, \hat{q}^I\right]$; if instead $r \in \left(0, \hat{r}^I\right]$, $\hat{q}_F^I = 0$ and therefore run equilibria exist $\forall q \in \left(0, \hat{q}^I\right]$.

■

Proof of Proposition 7

As a start, suppose that $\gamma = 1$ and thus the portfolio choice is trivial where all resources are placed into the investment technology. Note that as q goes to q_A^I, the following sets of payments converge:

$$\lim_{q\to q_A^I} \left\{c_1^I, c_{2\alpha}^I, c_{1\beta}^I, c_{2\beta}^I\right\} = \lim_{q\to q_A^I} \left\{\hat{c}_1^I, \hat{c}_{2\alpha}^I, \hat{c}_{1\beta}^I, \hat{c}_{2\beta}^I\right\}.$$

Thus, as implied by Proposition 3, $\lim_{q\to q_A^I} \frac{c_1^I}{c_{2\beta}^I} = \lim_{q\to q_A^I} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} \geq 1$ if $\gamma \in (2,\infty)$, $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$ and $R \in \left(1, \underline{R}^I\right)$.

In what follows, we examine when the monopolistic banking system is prone to fragility as q goes to zero. Specifically, note that the bank is unstable if and only if:

$$\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} = \left(\frac{\pi + (1-\pi) R^{1-\gamma}}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}}\right)^{\frac{1}{1-\gamma}} \left(\frac{(1-\pi)\left[\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}\right] + \pi R^{\frac{1}{\gamma}}}{R^{\frac{1}{\gamma}}}\right) \geq 1,$$

and it can be shown that as R approaches 1, $\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} = 1$. Thus, the possibilities that the monopolist is unstable when runs are unlikely are established if we can show that depositors have incentives to run as R increases. To this end, differentiating $\lim_{q\to 0} \frac{\hat{c}_1^*}{\hat{c}_{2\beta}^*}$ with respect to R and evaluating this partial derivative at $R = 1$ yield:

$$\frac{d}{dR}\left(\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I}\right)\Bigg|_{R=1} = \left(\frac{1-\pi}{\gamma}\right)\left[\gamma + (1-\gamma)(1-\pi) - 2\right].$$

In particular, $\frac{d}{dR}\left(\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I}\right)\Big|_{R=1} > 0$ if and only if $\pi > \frac{1}{\gamma-1}$. Thus, suppose that $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$. By $\lim_{q\to 0, R\to 1} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} = 1$ and $\frac{d}{dR}\left(\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I}\right)\Big|_{R=1} > 0$, $\lim_{q\to 0} \frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} \geq 1$ must hold for the asset return slightly higher than 1.

To summarize, suppose that $\gamma \in (2,\infty)$, $\pi \in \left(\frac{1}{\gamma-1}, 1\right)$, and $R \in \left(1, \underline{R}^I\right)$. Under these conditions, run equilibria under a monopolistic bank exist $\forall q \in \left(\hat{q}_F^I, q_A^I\right]$, where $\hat{q}_F^I \in \left[0, q_A^I\right)$.

■

Proof of Proposition 10

To begin, $\frac{d\frac{\hat{c}_1^*}{\hat{c}_{2\beta}^*}}{dq} > (<)\, 0$ if $q \leq (>)\, \hat{q}^I$, $\frac{d\frac{c_1^I}{c_{2\beta}^I}}{dq} < 0$, and $\frac{\hat{c}_1^{II}}{\hat{c}_{2\beta}^{II}} = \frac{c_1^{II}}{c_{2\beta}^{II}}\ \forall q > \hat{q}^I$, implying that $\frac{\hat{c}_1^I}{\hat{c}_{2\beta}^I} < \frac{c_1^I}{c_{2\beta}^I}$ $\forall q \leq \hat{q}^I$.

In addition, it is straightforward that competitive banks always deliver higher welfare than ones with market power as depositors are kept to be indifferent to financial autarky under monopolists.

■

The best-response allocation in a competitive banking economy with government bailouts

The best-response allocation is defined by conditions (8), (36), (39), (40), and the resource constraints (31) to (34). As discussed in the text, $\mu_1^{b*} > 0$, implying that the solution to the bank's optimization problem will not lie under Scenarios II and III.

Scenario I: $1-x-\pi c_1-\tau_\alpha=0$ and $1-x-\pi c_1-\tau_\beta<(1-\pi)\pi c_{1\beta}$

If $\pi\in\left(\pi^{bI},1\right)$, where $\pi^{bI}=\frac{1+\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}}-\sqrt{\left(\left[1+\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}}\right]^2-4\left[\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}}-\delta^{\frac{1}{\gamma}}\right]\right)}}{2}$, the bank does not hold excess liquid assets but resorts to costly liquidation should a run occur. Thus, the vector $c^{bI}=\left\{x^{bI},c_1^{bI},c_{2\alpha}^{bI},g_\alpha^{bI},c_{1\beta}^{bI},c_{2\beta}^{bI},g_\beta^{bI}\right\}$ is given by:

$$
\begin{aligned}
x^{bI} &= 1-\left(\pi+\left[(1-q)\,\delta\right]^{\frac{1}{\gamma}}\right)c_1^{bI},\\
c_1^{bI} &= \frac{1}{(1-q)^{\frac{1}{\gamma}}\left[(1-\pi)\,R^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}\right]+\pi},\\
c_{2\alpha}^{bI} &= \frac{R-R\left(\pi+\left[(1-q)\,\delta\right]^{\frac{1}{\gamma}}\right)c_1^{bI}}{1-\pi},\\
c_{1\beta}^{bI} &= \frac{1+\left[\left(\frac{1-r}{r}\right)\left[(1-q)\,\delta\right]^{\frac{1}{\gamma}}-\pi\right]c_1^{bI}}{\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)\left(\frac{1}{r}\right)},\\
c_{2\beta}^{bI} &= \left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\frac{1+\left[\left(\frac{1-r}{r}\right)\left[(1-q)\,\delta\right]^{\frac{1}{\gamma}}-\pi\right]c_1^{bI}}{\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)\left(\frac{1}{r}\right)},\\
g_\alpha^{bI} &= \tau_\alpha^{bI}=\left[(1-q)\,\delta\right]^{\frac{1}{\gamma}}c_1^{bI},\\
g_\beta^{bI} &= \tau_\beta^{bI}=\delta^{\frac{1}{\gamma}}\frac{1+\left[\left(\frac{1-r}{r}\right)\left[(1-q)\,\delta\right]^{\frac{1}{\gamma}}-\pi\right]c_1^{bI}}{\left((1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}\right)\left(\frac{1}{r}\right)},
\end{aligned}
$$

where $\frac{c_{2\beta}^{bI}}{c_{1\beta}^{bI}}=\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}>1$, $\frac{c_{2\alpha}^{bI}}{c_1^{bI}}=\left[(1-q)\,R\right]^{\frac{1}{\gamma}}>1\,\forall q<1-\frac{1}{R}$, and $\frac{\tau_\alpha^{bI}}{\tau_\beta^{bI}}=\frac{(1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}}{(1-\pi)rR^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}}>1$.

Scenario IV: $1-x-\pi c_1-\tau_\alpha=0$ and $1-x-\pi c_1-\tau_\beta=(1-\pi)\pi c_{1\beta}$

If $\pi\in\left(0,\pi^{bI}\right]$, the bank does not hold excess liquid assets and no costly liquidation takes place should the a run occur. Thus, the vector $c^{bIV}=\left\{x^{bIV},c_1^{bIV},c_{2\alpha}^{bIV},g_\alpha^{bIV},c_{1\beta}^{bIV},c_{2\beta}^{bIV},g_\beta^{bIV}\right\}$ is given

by:

$$\begin{aligned}
x^{bIV} &= 1-\left(\pi+(1-q)^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}\right)c_1^{bIV},\\
c_1^{bIV} &= \frac{1}{\pi+(1-\pi)(1-q)^{\frac{1}{\gamma}}R^{\frac{1-\gamma}{\gamma}}+(1-q)^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}},\\
c_{2\alpha}^{bIV} &= \frac{R\left[1-\left(\pi+(1-q)^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}\right)c_1^{bIV}\right]}{1-\pi},\\
c_{1\beta}^{bIV} &= \frac{(1-q)^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}}{(1-\pi)\pi+\delta^{\frac{1}{\gamma}}}c_1^{bIV},\\
c_{2\beta}^{bIV} &= \frac{R\left[1-\left(\pi+(1-q)^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}\right)c_1^{bIV}\right]}{(1-\pi)^2},\\
g_{\alpha}^{bIV} &= \tau_{\alpha}^{bIV}=(1-q)^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}c_1^{bIV},\\
g_{\beta}^{bIV} &= \tau_{\beta}^{bIV}=\frac{(1-q)^{\frac{1}{\gamma}}\delta^{\frac{2}{\gamma}}}{(1-\pi)\pi+\delta^{\frac{1}{\gamma}}}c_1^{bIV},
\end{aligned}$$

where $\frac{\tau_{\alpha}^{bIV}}{\tau_{\beta}^{bIV}}=\frac{(1-\pi)\pi+\delta^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}}}>1$, $\frac{c_{2\beta}^{bIV}}{c_{2\alpha}^{bIV}}=\frac{1}{1-\pi}>1$, $\frac{c_{2\alpha}^{bIV}}{c_1^{bIV}}=\left[(1-q)R\right]^{\frac{1}{\gamma}}>1\ \forall q<1-\frac{1}{R}$, and $\frac{c_1^{bIV}}{c_{1\beta}^{bIV}}=(1-q)^{-\frac{1}{\gamma}}\frac{\tau_{\alpha}^{bIV}}{\tau_{\beta}^{bIV}}>1$.

■

Proof of Lemma 3

Note that if $\pi>\pi^{bI}$, $\mu_1^{bI}>0$ and $\mu_{1\beta}^{bI}=0$. That is, the bank does not hold excess liquid assets but resorts to costly liquidation should a run occur. Note also that $\mu_1^{b*}>0$ implies that the bank will not hold excess liquid assets as a precaution against runs and therefore the solution to the bank's optimization problem will not lie under both Scenarios II and III. Furthermore, $\frac{c_1^{bIV}}{c_{2\beta}^{bIV}}<1$ indicates that run equilibria do not exist under Scenario IV.

■

Proof of Proposition 11

It can be easily derived that:

$$\frac{d\frac{c_1^{bI}}{c_{2\beta}^{bI}}}{dq}=\frac{(1-\pi)\left[\pi+(1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]+\delta^{\frac{1}{\gamma}}}{\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}\left[r(1-\pi)R^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}\right]}\frac{1}{\gamma}(1-q)^{-1-\frac{1}{\gamma}}>0.$$

In addition, note that $q_F^{bI} < \bar{q}^{bI} = 1 - \frac{1}{R}$ if and only if $\pi \in \left(\pi_1^{bI}, \pi_2^{bI}\right)$, where π_1^{bI} and π_2^{bI} are given by:

$$\pi_{1,2}^{bI} = \frac{1 \pm \sqrt{1 - 4\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}} \frac{1 - r^{\frac{1}{\gamma}}}{1 - r^{\frac{\gamma-1}{\gamma}} R^{\frac{1-\gamma}{\gamma}}}}}{2}.$$

Note also that if $\delta \geq \delta_1^{bI} = \left[\frac{1}{4}\left(\frac{1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}}{\left(\frac{1}{r}\right)^{\frac{1}{\gamma}}-1}\right)\right]^{\gamma}$, then the term $\left[1 - 4\left(\frac{\delta}{r}\right)^{\frac{1}{\gamma}} \frac{1 - r^{\frac{1}{\gamma}}}{1 - r^{\frac{\gamma-1}{\gamma}} R^{\frac{1-\gamma}{\gamma}}}\right] \leq 0$, implying that π_1^{bI} and π_2^{bI} do not exist. Otherwise, $0 < \pi_1^{bI} < \pi_2^{bI} < 1$.

Suppose that $\delta < \delta_1^{bI}$ and recall that π^{bI} is defined in the derivation of vectors summarized by c^{b*}. To show that $\pi^{bI} < \pi_1^{bI}$, we construct the proof below by contradiction. Suppose that $\pi_1^{bI} \leq \pi^{bI}$. Under this condition and given that $\lim_{\pi \to \pi^{bI}} c^{bI} = \lim_{\pi \to \pi^{bI}} c^{bIV}$, there should be room for the incentive to run condition under Scenario IV, $c_1^{bIV} \geq c_{2\beta}^{bIV}$, to hold for some $q < \bar{q}^{bI}$. However, this contradicts the results established in Lemma 3 that if the bank is prone to a run, the solution to the bank's optimization problem belongs to Scenario I only. Consequently, we must have $0 < \pi^{bI} < \pi_1^{bI} < \pi_2^{bI} < 1$.

Lastly, $q_F^{bI} > 0$ if and only if $\pi < \pi_3^{bI}$ and $\pi > \pi_4^{bI}$ where

$$\pi_{3,4}^{bI} = \frac{1+\left(R^{\frac{1}{\gamma}}-2\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}} \pm \sqrt{\left[1+\left(R^{\frac{1}{\gamma}}-2\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^2 - 4\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left(\left(R^{\frac{1}{\gamma}}-1\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}} + \left[\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}-1\right]\delta^{\frac{1}{\gamma}}\right)}}{2\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]}.$$

Note that if $\delta \geq \delta_2^{bI} = \left(\frac{\left(\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] - \left(R^{\frac{1}{\gamma}}-1\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)^2}{4\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left[\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}-1\right]}\right)^{\gamma}$, both π_3^{bI} and π_4^{bI} do not exist; otherwise, $0 < \pi^{bI} < \pi_1^{bI} < \pi_3^{bI} < \pi_4^{bI} < \pi_2^{bI} < 1$.

■

Proof of Proposition 12

The proof of the proposition is similar to that of Proposition 2. Note that if R is close to 1, the autarkic welfare level is characterized by $W^{A*} = \lim_{R \to 1} \frac{\pi^{\frac{1-\gamma}{\gamma}} + \delta^{\frac{1}{\gamma}}\left(\frac{R-1}{R-r}\right)^{\frac{1-\gamma}{\gamma}}}{(1-\gamma)\left[\pi^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}\left(\frac{R-1}{R-r}\right)^{\frac{1}{\gamma}}\right]^{1-\gamma}} = \frac{1}{1-\gamma}$ for any π, which is given by (30), and $\bar{q}^{bI}$ is arbitrarily close to zero. Given that the bank is able to deliver the expected utility higher than financial autarky when the probability of a run is negligible, we must have $W^{bI}\left(\bar{q}^{bI}\right) > W^{A*} = \frac{1}{1-\gamma}$.

Recall from Proposition 11 that if $\delta < \delta_2^{bI}$, $\pi^{bI} < \pi_1^{bI} < \pi_3^{bI} < \pi_4^{bI} < \pi_2^{bI}$. Next, we prove that if $\delta < \delta_3^{bI} = \left(\frac{\left[1+\left(R^{\frac{1}{\gamma}}-2\right)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]^2 - 4\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left[R^{\frac{1}{\gamma}}-1\right]\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}} - \left(\left[R^{\frac{1}{\gamma}}-1\right]\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}} - \left[\frac{2(R-1)}{R-r}-1\right]\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\right)^2}{4\left[1-\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right]\left[\left(\frac{R}{r}\right)^{\frac{1}{\gamma}}-1\right]}\right)^{\gamma}$,

$\pi_3^{bI} < \frac{R-1}{R-r} < \pi_4^{bI}$ holds and the proof below is by contradiction. Suppose that $\frac{R-1}{R-r} \leq \pi_3^{bI}$, which requires that $\delta \geq \delta_3^{bI}$. Given that $\pi_3^{bI} < \pi_4^{bI}$, $\frac{R-1}{R-r} < \pi_4^{bI}$ must hold. Yet, $\frac{R-1}{R-r} < \pi_4^{bI}$ requires that $\delta < \delta_3^{bI}$, which contradicts the assumption that for any $\delta \geq \delta_3^{bI}$, $\frac{R-1}{R-r} < \pi_3^{bI} < \pi_4^{bI}$. As a result, we must have $0 < \pi^{bI} < \pi_1^{bI} < \pi_3^{bI} < \frac{R-1}{R-r} < \pi_4^{bI} < \pi_2^{bI} < 1$ if $\delta < \delta_3^{bI}$.

To summarize, if $\delta < \delta_3^{bI}$ and $\pi \in \left[\frac{R-1}{R-r}, \pi_2^{bI}\right)$ such that the bank subject to fragility remains active $\forall q < \bar{q}^{bI}$, run equilibria exist $\forall q \in \left(q_F^{bI}, \bar{q}^{bI}\right)$, where $q_F^{bI} \in \left[0, \bar{q}^{bI}\right)$.

■

The best-response allocation in a monopolistic banking economy with government bailouts

The best-response allocation is determined by conditions (8), (47) to (53), the resource constraints (31), (33), (34), and the participation constraint (46).

Scenario I: $1 - x - \pi c_1 - \tau_\alpha = 0$ and $1 - x - \pi c_1 - \tau_\beta < (1-\pi)\, \pi c_{1\beta}$

Under this scenario, the bank does not hold excess liquid assets but resorts to costly liquidation should a run occur. Note that for a given $\hat{\tau}_\alpha^{bI}$, c_1 is determined by:

$$\left\{ \begin{array}{c} \pi \left(\hat{c}_1^{bI}\right)^{1-\gamma} + (1-q)\left[(1-\pi)\left(\hat{c}_{2\alpha}^{bI}\right)^{1-\gamma} + \delta\left(\hat{\tau}_\alpha^{bI}\right)^{1-\gamma}\right] \\ +q\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}\right)^{\gamma}\left(r\left(1-\pi\hat{c}_1^{bI} - \hat{\tau}_\alpha^{bI}\right) + \hat{\tau}_\alpha^{bI}\right)^{1-\gamma} - (1-\gamma)\, W^{A*} \end{array} \right\} = 0.$$

Once the solution for $\hat{c}_1^{bI}$ is obtained numerically, the full allocation summarized by the vector $\hat{c}^{bI} = \left\{\hat{x}^{bI}, \hat{c}_1^{bI}, \hat{c}_{2\alpha}^{bI}, \hat{g}_\alpha^{bI}, \hat{c}_{1\beta}^{bI}, \hat{c}_{2\beta}^{bI}, \hat{g}_\beta^{bI}\right\}$ is calculated by substituting $\hat{\tau}_\alpha^{bI}$ and $\hat{c}_1^{bI}$ along with $\hat{x}^{bI}$, $\hat{c}_{2\alpha}^{bI}$, $\hat{c}_{1\beta}^{bI}$, $\hat{c}_{2\beta}^{bI}$, and $\hat{g}_\beta^{bI}$:

$$\begin{aligned}
\hat{x}^{bI} &= 1 - \pi\hat{c}_1^{bI} - \hat{\tau}_\alpha^{bI} \\
\hat{c}_{2\alpha}^{bI} &= \left(\frac{R(1-q)}{\left(\hat{c}_1^{bI}\right)^{-\gamma} - rq\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}\right)^{\gamma}\left(r\left(1-\pi\hat{c}_1^{bI} - \hat{\tau}_\alpha^{bI}\right) + \hat{\tau}_\alpha^{bI}\right)^{-\gamma}}\right)^{\frac{1}{\gamma}} \\
\hat{c}_{1\beta}^{bI} &= \frac{r\left(1-\pi\hat{c}_1^{bI} - \hat{\tau}_\alpha^{bI}\right) + \hat{\tau}_\alpha^{bI}}{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}} \\
\hat{c}_{2\beta}^{bI} &= \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} \frac{r\left(1-\pi\hat{c}_1^{bI} - \hat{\tau}_\alpha^{bI}\right) + \hat{\tau}_\alpha^{bI}}{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}} \\
\hat{g}_\beta^{bI} &= \hat{\tau}_\beta^{bI} = \delta^{\frac{1}{\gamma}} \frac{r\left(1-\pi\hat{c}_1^{bI} - \hat{\tau}_\alpha^{bI}\right) + \hat{\tau}_\alpha^{bI}}{(1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}},
\end{aligned}$$

where $\frac{\hat{c}_{2\beta}^{bI}}{\hat{c}_{1\beta}^{bI}} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} > 1$. Note also that $\hat{g}_{\alpha}^{bI} = \hat{\tau}_{\alpha}^{bI}$ can take different values as long as $\hat{g}_{\alpha}^{bI} = \hat{\tau}_{\alpha}^{bI} > \hat{g}_{\beta}^{bI} = \hat{\tau}_{\beta}^{bI}$ and the resulting allocation entails that $\hat{c}_{2\alpha}^{bI} > \hat{c}_{1}^{bI} \geq \hat{c}_{2\beta}^{bI} > 0$.

Scenario II: $1 - x - \pi c_1 - \tau_\alpha > 0$ and $1 - x - \pi c_1 - \tau_\beta < (1-\pi)\pi c_{1\beta}$

Under this scenario, the bank holds excess liquid assets and also resorts to costly liquidation should a run occur. Thus, for a given $\hat{\tau}_{\alpha}^{bII}$, the vector $\hat{c}^{bII} = \left\{\hat{x}^{bII}, \hat{c}_1^{bII}, \hat{c}_{2\alpha}^{bII}, \hat{g}_{\alpha}^{bII}, \hat{c}_{1\beta}^{bII}, \hat{c}_{2\beta}^{bII}, \hat{g}_{\beta}^{bII}\right\}$ is given by:

$$\begin{aligned}
\hat{x}^{bII} &= \frac{1 - \pi\hat{c}_1^{bII} - q^{\frac{1}{\gamma}}\left(\frac{R-r}{R-1}\right)^{\frac{1}{\gamma}}\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}\right)\hat{c}_1^{bII}}{1-r}, \\
\hat{c}_1^{bII} &= \left(\frac{(1-\gamma)W^{A*} - (1-q)\delta\left(\hat{\tau}_{\alpha}^{bII}\right)^{1-\gamma}}{\pi + (1-q)^{\frac{1}{\gamma}}(1-\pi)\left(\frac{R-r}{1-r}\right)^{\frac{1-\gamma}{\gamma}} + q^{\frac{1}{\gamma}}\left((1-\pi)\left[\pi + (1-\pi)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right] + \delta^{\frac{1}{\gamma}}\right)\left(\frac{R-r}{R-1}\right)^{\frac{1-\gamma}{\gamma}}}\right)^{\frac{1}{1-\gamma}}, \\
\hat{c}_{2\alpha}^{bII} &= \left(\frac{(1-q)(R-r)}{1-r}\right)^{\frac{1}{\gamma}}\hat{c}_1^{bII}, \\
\hat{c}_{1\beta}^{bII} &= \left(\frac{q(R-r)}{R-1}\right)^{\frac{1}{\gamma}}\hat{c}_1^{bII}, \\
\hat{c}_{2\beta}^{bII} &= \left(\frac{qR(R-r)}{r(R-1)}\right)^{\frac{1}{\gamma}}\hat{c}_1^{bII}, \\
\hat{g}_{\beta}^{bII} &= \hat{\tau}_{\beta}^{bII} = \left(\frac{\delta q(R-r)}{R-1}\right)^{\frac{1}{\gamma}}\hat{c}_1^{bII},
\end{aligned}$$

where $\frac{\hat{c}_{2\beta}^{bII}}{\hat{c}_{1\beta}^{bII}} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} > 1$ and $\frac{\hat{c}_{2\alpha}^{bII}}{\hat{c}_{1}^{bII}} = \left(\frac{(1-q)(R-r)}{1-r}\right)^{\frac{1}{\gamma}} > 1$. Note that $\hat{g}_{\alpha}^{bII} = \hat{\tau}_{\alpha}^{bII}$ can take different values as long as $\hat{g}_{\alpha}^{bII} = \hat{\tau}_{\alpha}^{bII} > \hat{g}_{\beta}^{bII} = \hat{\tau}_{\beta}^{bII}$ and the resulting allocation entails that $\hat{c}_{2\alpha}^{bII} > \hat{c}_{1}^{bII} \geq \hat{c}_{2\beta}^{bII} > 0$.

Scenario III: $1 - x - \pi c_1 - \tau_\alpha > 0$ and $1 - x - \pi c_1 - \tau_\beta = (1-\pi)\pi c_{1\beta}$

Under this scenario, the bank holds excess liquid assets but no costly liquidation takes place should a run occur. First note that for a given $\hat{\tau}_{\alpha}^{bIII}$, $\frac{c_{1\beta}}{c_{2\beta}}$ and $c_{2\beta}$ are jointly determined by equations:

$$R\left(\frac{1}{c_{2\beta}} - \left[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}\right]\frac{c_{1\beta}}{c_{2\beta}}\right) = (1-\pi)^2 + R\pi\left(\left(\frac{qR}{R-1}\right)\left[\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{-\gamma} - 1\right]\right)^{-\frac{1}{\gamma}} \quad \text{and}$$

$$c_{2\beta} = \frac{\left[(1-\gamma)W^{A*} - (1-q)\delta\left(\hat{\tau}_{\alpha}^{bIII}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}{\left[\pi\left(\frac{qR}{R-1}\left[\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{-\gamma} - 1\right]\right)^{\frac{\gamma-1}{\gamma}} + (1-q)(1-\pi)^{\gamma}\left(\left[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}\right]\frac{c_{1\beta}}{c_{2\beta}} - \hat{\tau}_{\alpha}^{bIII} + (1-\pi)^2\right)^{1-\gamma} + q\left(\left[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}\right]\left(\frac{c_{1\beta}}{c_{2\beta}}\right)^{1-\gamma} + (1-\pi)^2\right)\right]^{\frac{1}{\gamma-1}}}$$

Thus, the vector $\hat{c}^{bIII} = \left\{\hat{x}^{bIII}, \hat{c}_1^{bIII}, \hat{c}_{2\alpha}^{bIII}, \hat{g}_\alpha^{bIII}, \hat{c}_{1\beta}^{bIII}, \hat{c}_{2\beta}^{bIII}, \hat{g}_\beta^{bIII}\right\}$ is computed by substituting $\hat{\tau}_\alpha^{bIII}$, $\frac{\hat{c}_{1\beta}^{bIII}}{\hat{c}_{2\beta}^{bIII}}$, and $\hat{c}_{2\beta}^{bIII}$ along with:

$$\begin{aligned}
\hat{x}^{bIII} &= \frac{(1-\pi)^2}{R}\hat{c}_{2\beta}^{bIII}, \\
\hat{c}_1^{bIII} &= \left(\left(\frac{qR}{R-1}\right)\left[\left(\frac{\hat{c}_{1\beta}^{bIII}}{\hat{c}_{2\beta}^{bIII}}\right)^{-\gamma} - 1\right]\right)^{-\frac{1}{\gamma}}\hat{c}_{2\beta}^{bIII}, \\
\hat{c}_{2\alpha}^{bIII} &= \frac{\left[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}\right]\frac{\hat{c}_{1\beta}^{bIII}}{\hat{c}_{2\beta}^{bIII}} - \hat{\tau}_\alpha^{bIII} + (1-\pi)^2}{1-\pi}\hat{c}_{2\beta}^{bIII}, \\
\hat{c}_{1\beta}^{bIII} &= \frac{\hat{c}_{1\beta}^{bIII}}{\hat{c}_{2\beta}^{bIII}}\hat{c}_{2\beta}^{bIII}, \\
\hat{g}_\beta^{bIII} &= \hat{\tau}_\beta^{bIII} = 1 - \left[\frac{(1-\pi)^2}{R} + \pi\left(\left(\frac{qR}{R-1}\right)\left[\left(\frac{\hat{c}_{1\beta}^{bIII}}{\hat{c}_{2\beta}^{bIII}}\right)^{-\gamma} - 1\right]\right)^{-\frac{1}{\gamma}} + (1-\pi)\pi\frac{\hat{c}_{1\beta}^{bIII}}{\hat{c}_{2\beta}^{bIII}}\right]\hat{c}_{2\beta}^{bIII}.
\end{aligned}$$

Note also that $\hat{g}_\alpha^{bIII} = \hat{\tau}_\alpha^{bIII}$ can take different values as long as $\hat{g}_\alpha^{bIII} = \hat{\tau}_\alpha^{bIII} > \hat{g}_\beta^{bIII} = \hat{\tau}_\beta^{bIII}$. Yet, as discussed in the main text, $\frac{\hat{c}_1^{bIII}}{\hat{c}_{2\beta}^{bIII}} < 1$.

Scenario IV: $1 - x - \pi c_1 - \tau_\alpha = 0$ and $1 - x - \pi c_1 - \tau_\beta = (1-\pi)\pi c_{1\beta}$

Under this scenario, the bank does not hold excess liquid assets and no costly liquidation takes place should a run occur. Note that for a given $\hat{\tau}_\alpha^{bIV}$, c_1 is determined by:

$$\left\{\begin{array}{l}
\pi\left(\hat{c}_1^{bIV}\right)^{1-\gamma} + (1-q)\left[(1-\pi)^\gamma\left(R\left(1-\pi\hat{c}_1^{bIV} - \hat{\tau}_\alpha^{bIV}\right)\right)^{1-\gamma} + \delta\left(\hat{\tau}_\alpha^{bIV}\right)^{1-\gamma}\right] \\
+q\left(\left[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}\right]^\gamma\left(\hat{\tau}_\alpha^{bIV}\right)^{1-\gamma} + (1-\pi)^{2\gamma}\left(R\left(1-\pi\hat{c}_1^{bIV} - \hat{\tau}_\alpha^{bIV}\right)\right)^{1-\gamma}\right)
\end{array}\right\} = (1-\gamma)W^{A*}.$$

Once the solution for $\hat{c}_1^{bIV}$ is obtained numerically, the full allocation summarized by the vector $\hat{c}^{bIV} = \left\{\hat{x}^{bIV}, \hat{c}_1^{bIV}, \hat{c}_{2\alpha}^{bIV}, \hat{g}_\alpha^{bIV}, \hat{c}_{1\beta}^{bIV}, \hat{c}_{2\beta}^{bIV}, \hat{g}_\beta^{bIV}\right\}$ is calculated by substituting $\hat{\tau}_\alpha^{bIV}$ and $\hat{c}_1^{bIV}$ along

with $\hat{x}^{bIV}$, $\hat{c}_{2\alpha}^{bIV}$, $\hat{c}_{1\beta}^{bIV}$, $\hat{c}_{2\beta}^{bIV}$, and $\hat{g}_{\beta}^{bIV}$:

$$\begin{aligned}
\hat{x}^{bIV} &= 1-\pi\hat{c}_{1}^{bIV}-\hat{\tau}_{\alpha}^{bIV}, \\
\hat{c}_{2\alpha}^{bIV} &= \frac{R\left(1-\pi\hat{c}_{1}^{bIV}-\hat{\tau}_{\alpha}^{bIV}\right)}{1-\pi}, \\
\hat{c}_{1\beta}^{bIV} &= \frac{\hat{\tau}_{\alpha}^{bIV}}{\left(1-\pi\right)\pi+\delta^{\frac{1}{\gamma}}}, \\
\hat{c}_{2\beta}^{bIV} &= \frac{R\left(1-\pi\hat{c}_{1}^{bIV}-\hat{\tau}_{\alpha}^{bIV}\right)}{\left(1-\pi\right)^{2}}, \\
\hat{g}_{\beta}^{bIV} &= \hat{\tau}_{\beta}^{bIV}=\frac{\delta^{\frac{1}{\gamma}}\hat{\tau}_{\alpha}^{bIV}}{\left(1-\pi\right)\pi+\delta^{\frac{1}{\gamma}}}.
\end{aligned}$$

Note also that $\hat{g}_{\alpha}^{bIV}=\hat{\tau}_{\alpha}^{bIV}$ can take different values as long as $\hat{g}_{\alpha}^{bIV}=\hat{\tau}_{\alpha}^{bIV}>\hat{g}_{\beta}^{bIV}=\hat{\tau}_{\beta}^{bIV}$. Yet, as discussed in the main text, $\frac{\hat{c}_{1}^{bIV}}{\hat{c}_{2\beta}^{bIV}}<1$.

■

Proof of Lemma 4

Under Scenarios III and IV, $\frac{\hat{c}_{1}^{bIII}}{\hat{c}_{2\beta}^{bIII}}<1$ and $\frac{\hat{c}_{1}^{bIV}}{\hat{c}_{2\beta}^{bIV}}<1$, suggesting that run equilibria do not exist.

■

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