The Asymmetric Effects of Monetary Policy on Housing Across the Level of Development*

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April 2014

Abstract

We study the effects of money growth in a neoclassical growth model with wealth effects. As the capital stock is the only component of wealth which contributes to an individual's utility, the model should be interpreted as a model of housing production and housing wealth since the capital stock affects utility. Consistent with empirical evidence on the relationship between residential investment and GDP across countries, there are significant non-linearities between housing market activity and aggregate income in our framework.

1 Introduction

The objective of this paper is to develop a model to study the effects of persistent monetary policy on housing market activity across countries. A starting point for thinking about the effects of policy across countries is to look at the effects of inflation on overall macroeconomic activity.

Available evidence indicates that the effects of policy are highly sensitive to the *level of economic activity.* For example, Ahmed and Rogers (2000) find evidence of a long-run Tobin effect on investment in the United States. In a study of fourteen different industrialized countries, Rapach (2003) observes a positive correlation between inflation and output. Unfortunately, policymaking in the developing world is complicated by distortions from low levels of income. In contrast to advanced countries, there is convincing evidence that inflation and output are negatively related in developing countries.[1]

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[1]Bae and Ratti (2000) conclude that output is negatively related to money growth in Argentina and Brazil. Other contributions study the impact of inflation on output growth – Fischer (1993) and Barro (1995) find that inflation is negatively related to growth.

How do the effects of monetary policy on housing conditions vary across countries? As classic examples, both Summers (1981) and Fama and Schwert (1977) contend that investment in housing is attractive relative to other types of assets in inflationary periods in the United States. However, there is limited work on the relationship between inflation and the housing stock in the developing world. Consequently, theoretical work can provide important guidance to policymakers seeking to expand access to housing in low income countries.

Understanding how housing investment responds to inflation across countries requires an understanding of how housing investment depends on the level of income across countries. In a study of nearly 40 countries from 1963-1970, Burns and Grebler (1976) find evidence of significant non-linearities from GDP to residential construction. At low levels of income, the share of housing to total income is low but increases as GDP is higher. The share of housing to GDP peaks at moderate levels of income and then declines with income in the richest countries. Fisher and Jaffe (2003) also find that the relationship between GDP per capita and homeownership is non-linear.[2]

The non-monotonic relationship between GDP and the housing stock observed in the data provides the foundation of our work. In particular, the non-monotonicity leads to important asymmetries in the response of housing to inflation across the stages of economic development. Consequently, our work demonstrates that policymakers in the developing world must acknowledge that the effects of monetary policy on housing conditions will not be the same as in advanced countries.

Developing models in which there are simultaneously different effects of policy on economic activity across countries is a challenge. Standard neoclassical growth models tend to produce unique steady-state equilibria in which the effects of policy are monotonic. For example, models that emphasize the store of value role of money generally conclude that inflation promotes capital accumulation and the effect is permanent. By comparison, models that emphasize the transactions role of money through a Stockman (1981) cash-in-advance constraint are associated with a reverse-Tobin effect.

Following numerous monetary growth models, we also motivate the transactions role of money through a standard Stockman (1981) cash-in-advance constraint. As put forward by numerous papers in urban economics, housing wealth is an additional argument along with consumption in individuals' utility functions.[3,4] From a development perspective, Burns and Grebler provide a wide array of arguments supporting housing for an improvement in the "human condition." Notably, access to housing promotes health conditions. It also generates stability and is a source of pride for residents. Dietz and Haurin (2003) provide an extensive survey of the consequences of homeownership in developed countries.

[2] Malpezzi and Mayo (1987a, 1987b) study the determinants of housing demand in developing countries using household data.

[3] Wheaton (1982) and Arnott et al. (1999) are prominent examples.

[4] Wang and Yip (1992) study the effects of monetary policy on capital accumulation in a monetary growth model with endogenous labor supply.

2 The Model

The representative agent solves the following problem of choosing a mix of consumption $c(t)$, and housing accumulation $h(t)$ over time t:

$$\max_{c(t),h(t)} \quad \int_0^\infty e^{-\rho t}\left[\phi\frac{c(t)^{1-\alpha}}{1-\alpha}+(1-\phi)\frac{h(t)^{1-\alpha}}{1-\alpha}\right]dt$$

subject to:

$$\dot{m}(t)+\dot{h}(t)=Ah^{\theta}-c(t)-\delta h(t)+v(t)-\pi m(t)$$

and is limited by the following cash-in-advance constraint:

$$\Gamma[c(t)+\dot{h}(t)]\leq m(t) \tag{1}$$

where ρ is the discount factor, α the coefficient of relative risk aversion, and ϕ is the fraction of utility derived from the consumption good. Aside from housing capital, real cash balances $m(t)$ comprise the other asset in the economy. The monetary authority injects a lump sum transfer of money at time t, $v(t)$. There exists a housing technology Ah^{θ} for which $0<\theta<1$ and A describes its productivity parameter. The depreciation rate is δ and π represents the inflation rate. The cash-in-advance constraint (1) applies to both consumption and investment where Γ denotes the fraction Γ of expenditures in the economy requiring cash-financing.

We apply Pontryagin's Maximum Principle to solve the agent's problem. Letting $z(t)=\dot{h}(t)$, the corresponding current-valued Hamiltonian is:

$$\begin{aligned}\mathcal{H}\left[c(t),h(t),\lambda_1(t),\lambda_2(t),\lambda_3(t)\right]&=\phi\frac{c(t)^{1-\alpha}}{1-\alpha}\;+\;(1-\phi)\frac{h(t)^{1-\alpha}}{1-\alpha}\;+\\&\lambda_1(t)z(t)\;+\;\lambda_2(t)\left[Ah(t)^{\theta}-c(t)-\delta h(t)+v(t)-\pi m(t)-z(t)\right]+\\&\lambda_3(t)\left[m(t)-\Gamma(c(t)+z(t))\right].\end{aligned}$$

The optimal choices of the control variables are:

$$\frac{\partial\mathcal{H}(\cdot)}{\partial c(t)}=\phi c(t)^{-\alpha}-\lambda_2(t)-\lambda_3(t)\Gamma=0 \tag{2}$$

$$\frac{\partial\mathcal{H}(\cdot)}{\partial z(t)}=\lambda_1(t)-\lambda_2(t)-\lambda_3(t)\Gamma=0 \tag{3}$$

The Euler equation for the housing stock is:

$$\dot{\lambda_1}(t) = \rho\lambda_1(t) - \lambda_2(t)[A\theta h(t)^{-(1-\theta)} - \delta] - (1-\phi)h(t)^{-\alpha} \tag{4}$$

By comparison, the Euler equation for money balances can be expressed as:

$$\dot{\lambda_2}(t) = \rho\lambda_2(t) - [\lambda_3(t) - \pi\lambda_2(t)] \tag{5}$$

Imposing steady-state on the system yields consumption as a function of the housing stock:

$$c(h) = \left[\left(\frac{\phi}{1-\phi}\right)\left(\rho + \frac{\delta - A\theta h^{-(1-\theta)}}{1+(\rho+\pi)\Gamma}\right)\right]^{\frac{1}{\alpha}} h. \tag{6}$$

Equation (6) is the analogue to the standard modified golden rule equation in our model. Further, the budget constraint each period must be balanced:

$$c(h) = Ah^{\theta} - \delta h \tag{7}$$

In terms of describing steady-state activity, we begin by discussing the interpretation behind (6). As mentioned, this equation would be the standard modified golden rule modified by the constraint that a fraction Γ of transactions must be financed with cash. That is, in a standard cash-in-advance model we would have:

$$A\theta h^{-(1-\theta)} = \rho + \delta + \rho(\rho+\pi)\Gamma$$

However, in our framework, the optimal level of housing accumulation is based upon the fact that housing directly provides utility to individuals because of the services housing provides. That is, (6) indicates that the optimal amount of consumption expenditure involves a trade-off between the marginal utility of consumption and the marginal utility from housing wealth.

Lemma 1. (Consumption-Housing Allocation) *The consumption-housing allocation of (6) may be written as follows:*

$$c_a(h) = \frac{\Phi}{\Psi(\pi)^{\frac{1}{\alpha}}}\left[(\rho\Psi(\pi)+\delta)\,h^{\alpha} - A\theta h^{-(1-\theta-\alpha)}\right]^{\frac{1}{\alpha}}$$

where $\Phi \equiv \left(\frac{\phi}{1-\phi}\right)^{\frac{1}{\alpha}}$, $\Psi(\pi) \equiv [1+(\rho+\pi)\Gamma]$ and $c(h) \equiv c_a(h)$.

Corollary 1. *Assume that $\alpha + \theta = 1$. Also, let $\alpha = 1/(2n)$ for $n \in \mathbb{N}^+$. As a result, $c_a(h) \geq 0$ and is expressed as:*

$$c_a(h) = \frac{\Phi}{\Psi(\pi)^{\frac{1}{\alpha}}} \left[(\rho\Psi(\pi) + \delta) h^{\alpha} - A\theta \right]^{\frac{1}{\alpha}} \tag{8}$$

The locus from (8) behaves as follows. Let $h_a^* = \left(\frac{A\theta}{\rho\Psi(\pi)+\delta} \right)^{\frac{1}{\alpha}}$. *For any* $h \leq (>) h_a^*$, $c_a'(h) \leq (>) 0$.

The behavior of equation (8) is the centerpiece of our framework and is consistent with the observations of Burns and Grebler. At low levels of income, higher levels of income are associated with lower consumption as individuals begin to favor housing more. At $h_a^* = \left(\frac{A\theta}{\rho\Psi(\pi)+\delta} \right)^{\frac{1}{\alpha}}$, the share of residential investment peaks. Beyond h_a^*, residential activity and consumption expenditures move together.

The second steady-state equilibrium condition is:

$$c_b(h(t)) = Ah(t)^{\theta} - \delta h(t). \tag{9}$$

3 Steady-State Equilibrium Activity

Using the geometric properties of the two consumption equations (8) and (9):

Proposition 1. (Existence of Multiple Steady-States) *Assume that* $\alpha + \theta = 1$. *Also, let* $\alpha = 1/(2n)$ *for* $n \in \mathbb{N}^+$. *Under these conditions, there are two steady-state equilibria.*

The graphical description of steady-states is provided in Figure 1. In the low housing steady-state, there is little wealth accumulation as a large amount of income is instead used to finance consumption expenditures. By comparison, in the high housing steady-state, there is a lot of housing wealth which provides large amounts of service flows to residents.

We turn to the effects of monetary policy across countries. The effects of monetary policy on activity revolve around the behavior of (8):

Lemma 2. (The Effects of Monetary Policy on the Consumption-Housing Choice). *Assume the conditions in Proposition 1 hold. Then,* $dc_a/d\pi < 0$ *for* $h < \hat{h}$ *and* $dc/d\pi > 0$ *for* $\hat{h} < h < h_b^*$. *In addition,* $dc_a/d\pi \leq 0$ *when* $h \geq h_b^*$.

The graphical illustration of the effects of policy is shown in Figure 2. Over a majority of levels of the housing stock, the higher inflation rate raises the tax on consumption and residential investment. As a result:

Proposition 2. (The Effects of Monetary Policy on Steady-State Equilibrium Activity). *Assume the conditions in Proposition 1 hold. In the low capital steady-state, an increase in the inflation rate is associated with a lower housing stock. In the high capital steady-state, the housing stock increases with inflation.*

The effect of inflation on economic activity is shown by the downward movement of the locus associated with (8) from steady-states A and B in the figure. At low levels of the housing stock, an increase in the inflation rate by increasing the tax on residential investment distorts wealth accumulation further and leads to a lower steady-state stock of housing. However, in the advanced economy steady-state the tax on consumption promotes the accumulation of housing wealth. In this manner, our results are consistent with the available evidence pointing to an increase in housing market activity in response to higher inflation rates.

One might be skeptical regarding the assumptions that $\alpha + \theta = 1$ and $\alpha = 1/(2n)$ *for* $n \in \mathbb{N}^+$. However, their only role is to provide a degree of tractability so that the existence of multiple steady-states and the effects of monetary policy may be shown analytically as portrayed in Figures 1 and 2. To illustrate that our results are robust, we consider some numerical examples which relax the assumptions in Lemma 1 and the Propositions. As one example, consider the following set of parameters: $A = 1.2$, $\delta = 0.25$, $\rho = 0.025$, $\phi = 0.8$, $\alpha = 0.5$, $\theta = 0.7$, and $\Gamma = 0.3$.

Table 1: Monetary Policy in the Low Development Steady-State

π	0.01	0.04	0.07	0.10	0.13	0.15
h	9.556	9.390	9.226	9.065	8.907	8.803
c	3.437	3.407	3.378	3.349	3.320	3.300

Table 2: Monetary Policy in the High Development Steady-State

π	0.01	0.04	0.07	0.10	0.13	0.15
h	94.035	94.293	94.551	94.807	95.062	95.231
c	5.364	5.355	5.346	5.336	5.327	5.321

In contrast to the benchmark assumptions used in the Lemma and Propositions, the high degree of non-linearity of the system always includes a degenerate steady-state in which there is no housing accumulation or consumption. However, the other two non-degenerate steady-states follow the behavior in Proposition 2.

4 Dynamics

The dynamical system follows:

$$\dot{c}(t) = \left(\frac{c(t)^{1+\alpha}}{\alpha\phi}\right)\left\{(1-\phi)h(t)^{-\alpha} + \lambda_2(t)\left[A\theta h(t)^{-(1-\theta)} - \delta\right]\right\} - \left(\frac{\rho}{\alpha}\right)c(t)$$

$$\dot{h}(t) = Ah(t)^{\theta} - \delta h(t) - c(t)$$

$$\dot{\lambda_2}(t) = \lambda_2(t)\left[\rho - (\mu - \pi) + \frac{1}{\Gamma}\right] - \left(\frac{\phi}{\Gamma}\right)c(t)^{-\alpha}$$

The Jacobian is:

$$J = \begin{pmatrix} \frac{\partial \dot{c}(t)}{\partial c(t)} & \frac{\partial \dot{c}(t)}{\partial h(t)} & \frac{\partial \dot{c}(t)}{\partial \lambda_2(t)} \\ \frac{\partial \dot{h}(t)}{\partial c(t)} & \frac{\partial \dot{h}(t)}{\partial h(t)} & \frac{\partial \dot{h}(t)}{\partial \lambda_2(t)} \\ \frac{\partial \dot{\lambda_2}(t)}{\partial c(t)} & \frac{\partial \dot{\lambda_2}(t)}{\partial h(t)} & \frac{\partial \dot{\lambda_2}(t)}{\partial \lambda_2(t)} \end{pmatrix}_{SS}$$

As there are multiple steady-state equilibria, we use numerical examples to illustrate the local stability properties of the system in the neighborhood of each steady-state. Let γ_i represent the eigenvalue associated with variable i. We begin with a set of parameters that satisfy the assumptions imposed in Proposition 1: $A = 3$, $\delta = .15$, $\rho = .025$, $\phi = .7$, $\alpha = .5$, $\theta = .5$, $\Gamma = .4$:

Table 3: Stability of a Low Development Steady-State in an Example of Proposition 2

π	0.01	0.04	0.07	0.10	0.13	0.15
h	5.572	5.404	5.242	5.089	4.935	4.837
c	6.246	6.164	6.083	6.002	5.924	5.872
γ_1	3.953	3.994	4.0359	4.078	4.121	4.1502
γ_2	-0.038	-0.037	-0.036	-0.035	-0.034	-0.034
γ_3	1.849	1.857	1.866	1.874	1.882	1.888

Table 4: Stability of a High Development Steady-State in an Example of Proposition 2

π	0.01	0.04	0.07	0.10	0.13	0.15
h	260.323	261.463	262.587	263.696	264.790	265.511
c	9.355	9.290	9.225	9.161	9.098	9.057
γ_1	-0.093	-0.0932	-0.09328	-0.09331	-0.09333	-0.09335
γ_2	0.119	0.117	0.115	0.113	0.111	0.109
γ_3	2.467	2.466	2.4662	2.4660	2.4659	2.4658

As each steady-state is associated with one negative eigenvalue, each is saddle-path stable.[5] The steady-states in Tables 1 and 2 also exhibit saddle-path stability:

[5] The eigenvalue associated with the behavior of consumption in the advanced steady-state is relatively sluggish in response to the inflation rate. This takes place because consumption in the advanced steady-state is much larger relative to the poor steady-state.

Table 5: Stability of the Low Development Steady-State in Table 1

π	0.01	0.04	0.07	0.10	0.13	0.15
h	9.556	9.390	9.226	9.065	8.907	8.803
c	3.437	3.407	3.378	3.349	3.320	3.300
λ_1	-0.013	-0.012	-0.011	-0.010	-0.009	-0.009
λ_2	0.891	0.898	0.904	0.911	0.917	0.922
λ_3	3.584	3.587	3.590	3.594	3.597	3.600

Table 6: Stability of the High Development Steady-State in Table 2

π	0.01	0.04	0.07	0.10	0.13	0.15
h	94.035	94.293	94.551	94.807	95.062	95.231
c	5.364	5.355	5.346	5.336	5.327	5.321
λ_1	-0.102	-0.102	-0.102	-0.102	-0.102	-0.102
λ_2	0.127	0.126	0.125	0.124	0.124	0.123
λ_3	3.323	3.323	3.323	3.322	3.322	3.322

As both steady-states are approachable in each set of examples, the varying effects of monetary policy are informative about the relationship between inflation and housing accumulation across the stages of economic development.

5 Conclusion

The objective of this paper is to provide insights into the effects of inflation on housing market activity across countries. A key hypothesis in our work is that there are significant non-linearities in the relationship between residential investment and GDP. Interestingly, the presence of the non-linearity generates multiple steady-states. Moreover, the effects of inflation on housing market activity vary in systematic ways across steady-states demonstrating that the effects of monetary policy on housing should vary across the stages of economic development.

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Appendix.

Proof of Proposition 1. It is easily shown that $c_b(0) < c_a(0)$. In addition, $c_a(h_a^*) < c_b(h_a^*)$. Thus, there is one steady-state on the interval [0,h_a^*]. Further, there is an $h_0 \epsilon (h_a^*, \infty)$ where $c_b(h_0) = 0$. However, $c_a(h_0) > 0$. Consequently, there is a second steady-state on the interval (h_a^*, ∞).

Proof of Proposition 2. By taking the first order conditions of consumption with respect to the inflation rate we have:

$$\frac{dc_a}{d\pi} = -\frac{\Phi}{\alpha}\left[\frac{[\rho\Psi(\pi)+\delta]h^\alpha - A\theta}{\Psi(\pi)}\right]^{\frac{1-\alpha}{\alpha}}\left[\frac{(\delta h^\alpha - A\theta)\Gamma}{\Psi(\pi)^2}\right]$$

We also have:

$$\frac{dh_a^*}{d\pi} = -\left[\frac{A\theta}{(\rho\Psi(\pi)+\delta)^{1+\alpha}}\right]^{\frac{1}{\alpha}}\frac{\Gamma\rho}{\alpha} < 0.$$

Let $\pi' > \pi$ and note that $(1-\alpha)/\alpha$ is odd. Then, $dc_a/d\pi < 0$ for $h < \hat{h}$ and $dc/d\pi > 0$ for $\hat{h} < h < h_b^*$, where

$$\hat{h} = \frac{A\theta[\Psi(\pi)+\Gamma]}{2\rho[\Psi(\pi)\Gamma] + \delta[\Psi(\pi)+\Gamma]}.$$

Similarly, $dc_a/d\pi \leq 0$ when $h \geq h_b^*$.

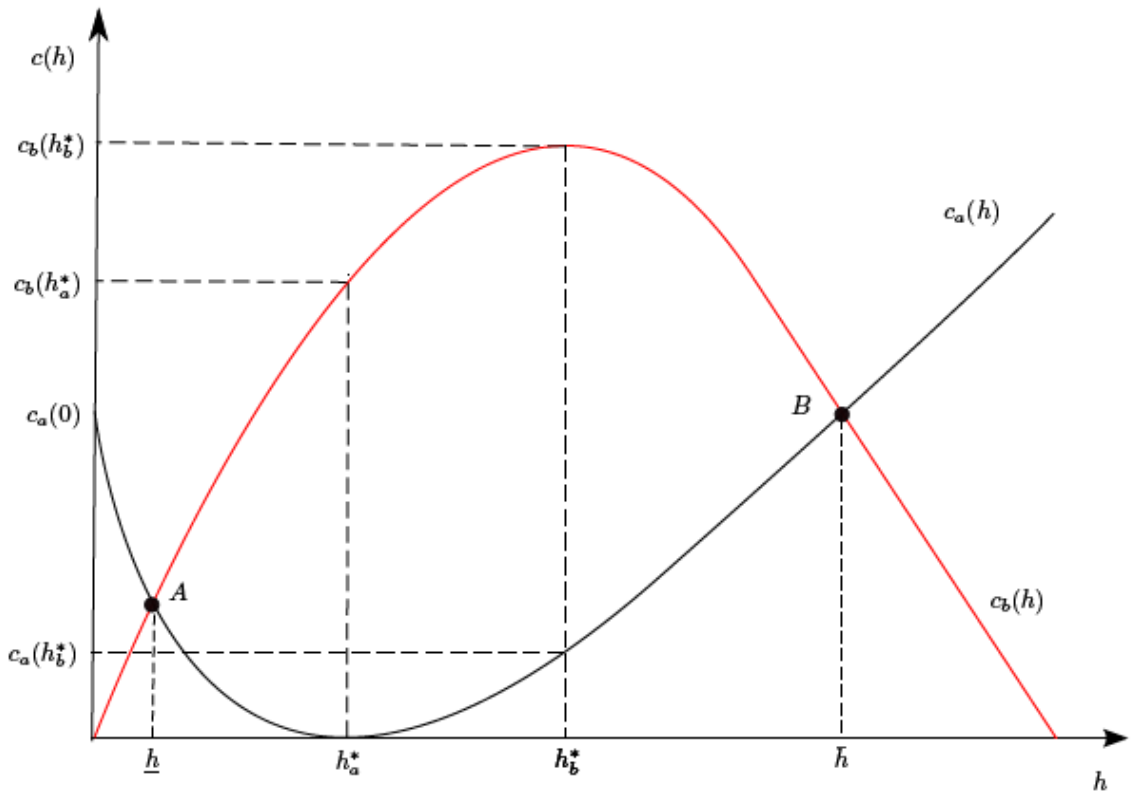

Figure 1: Existence of Multiple Steady-States

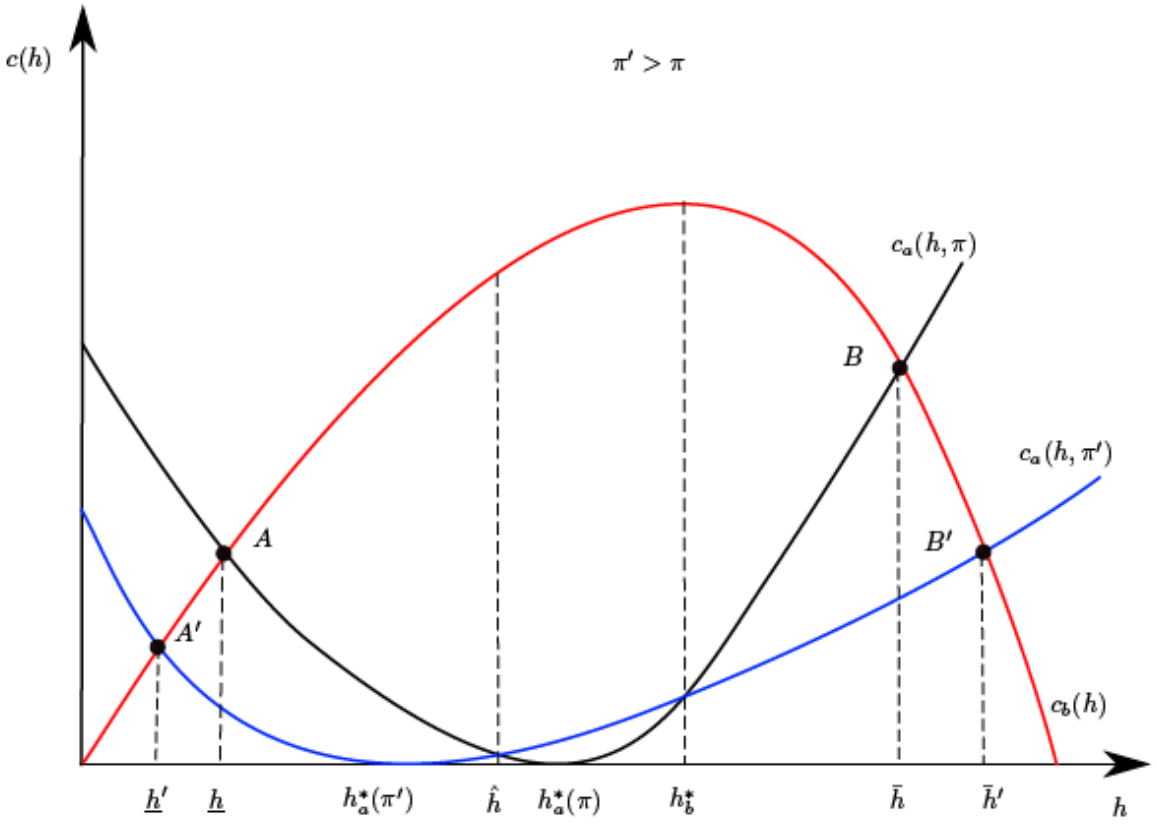

Figure 2: The Effects of Monetary Policy