Housing, Stochastic Liquidity Preference, and Monetary Policy*

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Abstract

In recent years, the connection between housing market activity and monetary policy has received a large amount of attention. For example, how does optimal monetary policy depend on housing market conditions? To address the importance of housing for wealth accumulation, we study a model in which housing is traded across generations of individuals. Following Diamond and Dybvig (1983), individuals face stochastic liquidity preference shocks which impede housing accumulation. Intermediaries arise to help insure individuals against such idiosyncratic risk. However, inflation limits the extent of risk-sharing. In this manner, the model follows recent contributions which emphasize that shocks to discount rates contribute to wealth inequality. Moreover, in contrast to one-sector monetary growth models, we demonstrate that it is important to disaggregate fixed investment between the residential and non-residential sectors to determine the effects of money growth. In particular, monetary policy will have asymmetric effects across the components of the overall capital stock. We conclude with policy experiments studying how optimal monetary policy depends on housing market fundamentals. In response to adverse supply conditions in the housing sector, monetary policy should be more aggressive in order to promote residential investment and the housing stock. However, monetary policy should be conservative in order to limit exposure to risk if fundamentals favor housing demand.

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1 Introduction

The connection between housing market activity and monetary policy has received a large amount of attention in recent years. How does monetary policy affect activity in

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the housing market? What role does housing play in overall macroeconomic activity? Moreover, how does optimal monetary policy depend on conditions in the housing sector?

Conventional wisdom views housing as a significant component of personal wealth in most countries. According to the Survey of Consumer Finances, U.S. primary residences made up approximately 30% of total household wealth in 2010. In addition to the bequest motive for accumulating housing wealth, saving through homeownership provides individuals with a mechanism for modifying consumption behavior. That is, a wide array of evidence demonstrates that housing wealth promotes consumption. For example, Carroll (2004) finds that the long-run marginal propensity to consume out of housing wealth to be around 9 cents per dollar. Furthermore, investment in residential structures and housing services is an important component of GDP. Based upon data from the Bureau of Economic Analysis, these two activities contributed to roughly 15.5% of GDP in the United States in 2012.

As housing is such a large component of economic activity, it is clearly important to understand the monetary transmission mechanism to the housing sector. Both Summers (1981) and Piazessi and Schneider (2012) conclude that housing investment becomes more attractive relative to corporate capital during inflationary periods. In addition, Fama and Schwert (1977) find that housing investment is an effective way of avoiding the effects of inflation on real returns. Therefore, the available evidence indicates that it is important to disaggregate investment activity between the residential and non-residential sectors of the economy in formulating models of the macroeconomy. Yet, standard monetary growth models instead focus on the overall capital stock.

The objective of this paper is to develop a framework to study the transmission of monetary policy to the housing sector in a rigorous general equilibrium framework. To address the importance of housing for wealth accumulation, we develop a model in which housing is traded across generations of individuals. Moreover, as housing wealth plays a role in household savings, it is a significant factor in intertemporal consumption behavior.

A key motivating observation for our work is that monetary policy transmits to conditions in the housing sector through impacting the incentives of intermediaries in the banking system. Therefore, it is important than the model be based upon a framework with well-defined roles for financial intermediaries. In other words, as argued by Smith (2003), 'intermediation should be taken seriously.' Towards this goal, we consider that financial intermediaries perform important risk-pooling services on behalf of their depositors as articulated by Diamond and Dybvig (1983).

Following Bencivenga and Smith (1991), we extend the Diamond and Dybvig framework to an infinite-horizon three period overlapping generations framework. However, in contrast to their approach, there are multiple production sectors in our model: a standard (non-residential) capital goods sector as in the neoclassical growth model and a residential sector which produces housing capital. As in Diamond and Dybvig and Bencivenga and Smith, individuals face stochastic liquidity preference shocks. From an alternative perspective, depositors are subject to shocks to their

rate of time preference. In contrast to their work, the liquidity preference shocks also impede housing accumulation. In this manner, the model follows recent contributions such as Krusell and Smith (1998) which emphasize that shocks to discount rates contribute to wealth inequality.

Further, money is also the only asset to meet short-term liquidity needs. Consequently, our approach is distinct from recent approaches to studying housing market activity such as Iacoviello and Pavan (2013) which omit financial intermediation and money. Notably, the transmission of monetary policy to the housing sector in our framework with risk-pooling intermediaries impacts the incentives of banks to provide opportunities to share idiosyncratic liquidity risk. In particular, inflation limits the ability of intermediaries to provide risk-sharing. In this manner, the model articulates a transmission mechanism to the housing market through a rigorously-structured liquidity risk channel.

The overlapping generations structure is important for our framework as we want to incorporate that housing is a reproducible asset that is traded over time, across generations of individuals. In the first period, young individuals work and save. In the second period, middle-aged individuals who are not exposed to liquidity shocks purchase homes and consume. In the final period, old homeowners finance their consumption based upon proceeds from the sale of their homes. As previously mentioned, the model economy consists of two production sectors: the residential sector and non-residential sector which resembles the production sector in the standard neoclassical growth model. Thus, there are two types of capital: physical (non-residential) capital and residential capital.

While previous empirical work has shown that inflation stimulates housing sector activity, our research develops a rich framework to show that there are important asymmetries resulting from monetary stimulus. As in a large volume of work on housing, we show that the durability of housing as an asset plays a huge role. Moreover, we study how optimal policy intervention depends on conditions in the housing sector. Interestingly, we find that optimal money growth is higher if productivity in the residential sector is low. In this manner, we argue that policy should be designed according to supply conditions in the housing sector. By comparison, inflation should be lower when conditions favor housing demand.

The remainder of the paper is as follows: Section 2 describes the environment of the benchmark economy and the banks behavior. In Section 3 we modify the benchmark model to include government transfers. Section 4 examines an economy with government debt as an asset choice in addition to physical capital, residential capital, and cash. Section 5 discusses what optimal monetary policy looks like in this environment. The final section is the conclusion.

2 Environment in the Benchmark Economy

We consider an economy with an infinite sequence of three-period lived overlapping generations. At the beginning of each period, a continuum of workers are born with population mass equal to 1. Individuals born at date t are considered to be

'*young*', at date *t+1* they are '*middle-aged*' and at date *t+2* they become '*old*.' In the initial period, there is an initial young generation, middle-aged generation, and an old generation. Young agents are endowed with one unit of labor which they supply inelastically without any disutility from effort.

In each period, there are two separate sectors where production occurs: the residential sector and the non-residential sector. Homes are produced in the residential sector and physical (non-residential) capital is produced in the non-residential sector. As in the standard neoclassical growth model, physical capital is homogeneous with consumption. Production in the non-residential sector requires labor and physical capital as inputs. The production of housing only requires residential capital. The two different types of capital are not substitutable.

Young individuals work, but do not consume. Instead, individuals only desire to consume during middle and old-age. Based upon their accumulated savings, some individuals purchase homes in middle-age. Old individuals sell their homes to finance their consumption.

Using physical capital and the labor of young agents, a firm produces the consumption good using the production technology $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$ where L_t represents labor, K_t denotes the level of the non-residential capital stock, and A represents a technology parameter in the non-residential sector. The capital-labor ratio in the non-residential sector is $k_t = \frac{K_t}{L_t}$.

Home builders produce homes using a single input, residential capital. The production function in the housing sector is $H_t = BK_t^h$ where K_t^h refers to the residential capital stock. Production functions of this type with constant returns to scale in the housing sector are often used in the supply-side literature on housing. For example, Albouy and Ehrlich (2012) look at a two-factor model of housing production in which land and materials are the two primary inputs. As we are studying an economy with two sectors of production, there are two different stocks of capital: physical capital and residential capital. Thus, we simplify the presentation by aggregating land and materials together as one primary input – the residential capital stock. The parameter B reflects the level of productivity in the residential sector. That is, B is an important supply-side factor in the housing sector which is likely to vary over time due to regulatory changes, geographic constraints, and other factors such as construction costs.[1]

As emphasized in numerous papers in urban economics, housing is durable over time. For example, Glaeser and Gyourko (2005b) stress that "...old housing does not disappear quickly. Housing may be the quintessential durable good, since homes often are decades, if not a century, old." To emphasize the relative differences in durability between physical capital and housing, homes have depreciation rate δ while physical capital depreciates completely.[2]

In our baseline economy, there are four types of assets: money, physical capital,

[1] See Epple et al. (2010) for discussion on housing production functions. Glaeser et al. (2008) argue that limited housing supply is a key determinant in housing price appreciation that takes place in housing bubbles. Glaeser et al. (2005a,b) look at determinants of housing supply.

[2] Ghossoub and Reed (2013) demonstrate that multiple monetary steady-state equilibria are possible if capital is durable and traded over time in a one-sector neoclassical growth model.

residential capital, and the stock of housing which includes both the previously existing stock of housing and newly constructed housing. The money supply is denoted as M_t and P_t represents the price level at period t. Thus, $m_t \equiv \frac{M_t}{P_t}$ is the real per capita money supply and $\frac{P_t}{P_{t+1}}$ represents the gross real return on money balances. The monetary authority follows a standard money-growth rule in which σ represents the growth rate of the money stock:

$$M_{t+1} = (1 + \sigma)M_t. \tag{1}$$

At the end of their youth, individuals face the possibility of incurring liquidity shocks where they will be forced to liquidate their savings prior to the end of the period. Since money is the only short-term store of value in the economy, individuals subject to the liquidity shocks will liquidate their funds and acquire money balances from the intermediary. The probability of the liquidity shock is equal to π and is publicly known. Each young agent is subject to the same probability that they will need money balances.[3]

Since individuals differ in terms of the timing of liquidating their savings, we refer to the young individuals who have positive realizations of the liquidity risk as '*impatient*' individuals. Individuals who are able to wait until their middle-age to liquidate their funds are '*patient'* individuals.

Moreover, as we describe below, patient individuals will accumulate housing wealth which they use to finance additional consumption when old. Consequently, impatient individuals only consume in their middle-age while patient individuals consume in middle-age and when they are old. That is, individuals have different discount rates. Patient individuals value utility when old according to the rate of time preference, β. In stark contrast, the rate of time preference on old-age utility is equal to zero for the impatient agents. In this manner, the model follows recent contributions such as Krusell and Smith (1998) which emphasize that shocks to discount rates contribute to wealth inequality.

Thus, following Diamond and Dybvig (1983), individuals are exposed to exogenous liquidity risk.[4] As in their framework, perfectly competitive banks arise to provide risk pooling services to individuals as a result of the idiosyncratic risk that they may encounter. Banks accept deposits from the young and choose a portfolio

[3]In this manner, our framework is similar in spirit to numerous papers in which the demand for money is motivated by spatial separation as articulated by Schreft and Smith (1997, 1998). In those models, there is limited communication across islands so privately-issued liabilities do not circulate. Moreover, money is the only asset that can cross locations. Whereas Schreft and Smith (1997, 1998) view that the spatial separation is a metaphor for information between individuals, spatial separation might also be confused for distinct housing markets given our application. To avoid this confusion, we merely impose that individuals subject to liquidity shocks must acquire money balances. Individuals familiar with the 'random relocation' approach behind money demand in Schreft and Smith can understand that our framework could be embedded with a spatial structure to motivate the transactions demand for money balances.

[4]In contrast, Ghossoub and Reed (2010) develop a framework with endogenous liquidity risk to show that the effects of monetary policy vary across countries.

of physical capital, residential investment, and money balances on their behalf. As individuals experiencing liquidity shocks liquidate all of their savings at the end of their youth, the expected lifetime utility of an agent is:

$$U(c^0_{t+1}, c^1_{t+1}, h_{t+1}, c_{t+2}) = \pi \ln(c^0_{t+1}) + (1-\pi)\left[\phi \ln(h_{t+1}) + (1-\phi)\ln(c^1_{t+1}) + \beta \ln(c_{t+2})\right] \tag{2}$$

where c^0_{t+1} represents the middle aged consumption among individuals who liquidate their savings early (in period 0 of their lifecycle), c^1_{t+1} is the level of middle-aged consumption among those who do not liquidate their savings until period 1, and c_{t+2} is the amount of consumption of old homeowners.

The timing of the model is described as follows. In the beginning of the first period, firms hire workers, rent capital, and produce in each sector. Workers are paid their wage income which is deposited in the bank. Next, banks determine their portfolio allocation. Then, workers learn if they will need to acquire liquid assets. Those who receive such shocks liquidate their savings and acquire money balances from the financial intermediary.

In the beginning of the next period, individuals with accumulated savings will purchase a home. Individuals with money balances use their liquidity to consume the consumption good while the remaining middle-aged consume their net income after financing housing expenditures. In the following period, old homeowners sell their homes and use the proceeds to finance their final level of consumption.

2.1 Trade

2.1.1 Factor Markets

At the beginning of every period production takes place and input factors are paid. Factor markets are perfectly competitive so factors are paid their marginal products. Consumer good producers rent physical capital from banks and use the labor provided by young agents to produce. Wages are given by:

$$w_t = A(1-\alpha)k_t^{\alpha} \tag{3}$$

The rental rate for physical capital is ρ_t :

$$\rho_t = A\alpha k_t^{\alpha-1} \tag{4}$$

Home builders use a single input, residential capital, which it rents from the bank at the rental rate r_t :

$$r_t = P_{h,t}B \tag{5}$$

2.1.2 Housing Demand

As previously stated, patient individuals purchase homes based upon their accumulated savings in their middle-age. In contrast, individuals who experience liquidity shocks consume all of their savings during middle age, leaving no resources to consume while old. A patient agent's accumulated wealth in their middle-age is equal to the return on their savings $r_t^1 w_t$ in which r_t^1 denotes the return a patient person will earn from deposits in the amount w_t. Housing demand among the patient group solves the following problem:

$$\underset{h_{t+1}}{Max} \phi \ln(h_{t+1}) + (1-\phi)\ln(c_{t+1}^1) + \beta \ln(c_{t+2}) \tag{6}$$

subject to:

$$c_{t+1}^1 = r_t^1 w_t - P_{h,t+1} h_{t+1} \tag{7}$$

$$c_{t+2} = (1-\delta) P_{h,t+1} h_{t+1} \tag{8}$$

As housing is a form of wealth accumulation, it also represents a consumption-savings decision in middle-age. At this time, homeowners reap the utility gains from purchasing a residence. However, it leaves less income available for personal consumption expenditures. In old-age, patient agents use the proceeds from selling their home to finance consumption.

The solution to the individual's lifetime utility maximization problem generates the following demand for housing:

$$h_{t+1} = \left(\frac{(\phi+\beta)}{(1+\beta)}\right)\left(\frac{r_t^1 w_t}{P_{h,t+1}}\right) \tag{9}$$

As observed in (9), an individual's demand for housing depends on a number of factors. Of course, the amount of savings affects their home affordability. If individuals value homeownership more (as exhibited by higher values of ϕ), their demand for housing will also be greater. That is, ϕ represents the consumption value of homeownership. Moreover, housing is the principal form of savings among the middle-aged. At higher rates of time preference over old-age utility, the ability to save through housing is also an important driver of housing demand. Finally, an individual's housing demand function is decreasing in the price of the housing stock. In turn, the consumption of a middle-aged person will be:

$$c_{t+1}^1 = \frac{(1-\phi)}{(1+\beta)} r_t^n w_t \tag{10}$$

In particular, consumption of middle-aged individuals will be lower if they derive more utility from homeownership and housing expenditures will be relatively high.

By comparison, consumption of old-age individuals will be financed by their housing wealth:

$$c_{t+2} = \left(\frac{(1-\delta)(\phi+\beta)}{(1+\beta)} \right) r_t^1 w_t \tag{11}$$

As housing wealth depends on their accumulated savings from their youth, the consumption of old individuals will depend on the interest income earned as well as the utility from owning a home.

2.1.3 The bank's problem

Based upon the anticipated demand for housing of a patient person, a bank chooses a portfolio assets to acquire on behalf of the middle-aged prior to the realization of the liquidity preference shock. Workers who experience liquidity shocks must liquidate their deposit balances into money. This liquidity preference shock provides a role for financial intermediation by banks. As in Diamond and Dybvig (1983), banks insure individuals against liquidity shocks by pooling their idiosyncratic risks and choosing a portfolio of assets on their behalf.

After receiving wage deposits from every worker, the bank allocates deposits between three assets: fiat currency (m_t), physical capital (i_t), and residential capital (i_t^h). The allocation of assets is limited by a bank's deposit base. Hence, the bank's balance sheet is expressed as:

$$w_t \geq m_t + i_t + i_t^h \tag{12}$$

Banks promise returns to both impatient agents and patient agents. The returns to the impatient are denoted as r_t^0 while r_t^1 denotes the returns to be paid to the patient individuals. Given that impatient individuals experience liquidity shocks, the return to impatient people will depend on the amount of money that the bank acquires and the inflation rate:

$$\pi r_t^0 w_t \leq m_t \left(\frac{P_t}{P_{t+1}} \right) \tag{13}$$

Comparatively, the return paid to patient agents depends upon the return on the bank's investment in physical capital and residential capital. The return to physical capital is ρ, and the return to residential capital is denoted as r. This creates an additional constraint for the bank:

$$(1-\pi)r_t^1 w_t \leq i_t\rho + i_t^h r \tag{14}$$

Since banks are perfectly competitive, the objective of each bank is to maximize the expected lifetime utility among its depositors. However, choosing a portfolio to obtain this objective requires understanding how much patient people will want to consume in old-age along with their demand for housing.

Based upon the anticipated demand for housing, the bank chooses m_t, i_t, and i_t^h to solve:

$$\underset{m_t,i_t,i_t^h}{Max}\ \pi \ln(r_t^0 w_t) + (1-\pi)\left[\phi \ln\left(\frac{(\phi+\beta)}{(1+\beta)}\frac{r_t^1 w_t}{P_{h,t+1}}\right) + (1-\phi)\ln\left(\frac{(1-\phi)}{(1+\beta)}r_t^1 w_t\right)\right]$$
$$+ (1-\pi)\left[\beta \ln\left(\frac{(1-\delta)(\phi+\beta)}{(1+\beta)}r_t^1 w_t\right)\right] \tag{15}$$

$$\text{s.t. } r_t^0 = \frac{m_t}{\pi w_t}\left(\frac{P_t}{P_{t+1}}\right) \tag{16}$$

$$r_t^1 = \frac{i_t\rho + (w_t - m_t - i_t)r}{(1-\pi)w_t} \tag{17}$$

$$w_t = m_t + i_t + i_t^h \tag{18}$$

In order for the bank to invest in both sectors of the economy, a no-arbitrage condition between capital in both sectors must be satisfied:

$$\rho_t = r \tag{19}$$

Money demand is given by:

$$m_t = \frac{\pi w_t}{(1-\pi)\beta + 1} \tag{20}$$

In comparison to standard Diamond and Dybvig type models with log-preferences, the probability of not being subject to liquidity risk $(1-\pi)$ factors into the banks' demand for money. Each bank's problem involves providing income to different groups of individuals after the realization of liquidity shocks. For example, patient people value income which they use in their choice of housing. Their anticipated housing wealth in old-age finances their old-age consumption. Consequently, the rate of time preference towards old-age utility is a component of money demand by each bank.

2.2 General Equilibrium

We now define the equilibrium for our baseline economy. One of the goals of the model is to determine the amount of housing supply and the relative price of housing. The amount of residential capital in each period depends on previous investment in the residential sector of the economy:

$$i_t^h = k_{t+1}^h \tag{21}$$

A similar relationship occurs in the non-residential sector:

$$i_t = k_{t+1} \tag{22}$$

2.2.1 Residential Investment

Using the bank's balance sheet in conjunction with the (19), (3), (21), and (22), we obtain the derived investment demand for residential capital by a bank. The derived investment demand depends on prices and productivity in the residential sector because each intermediary factors anticipated housing demand among patient individuals in choosing the portfolio of assets to acquire on behalf of its depositors:

$$K_{t+1}^h = A(1-\alpha)\left(1-\pi\right)\left(\frac{A\alpha}{P_{h,t+1}B}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1+\beta}{1+(1-\pi)\beta}\right) - \left(\frac{A\alpha}{P_{h,t+1}B}\right)^{\frac{1}{1-\alpha}} \tag{23}$$

Notably, residential investment is lower at higher prices of housing. At higher prices, individuals who have savings to acquire homes will have lower housing demand. The intermediary factors anticipated demand conditions in choosing the level of residential investment.

2.2.2 Housing Demand and Consumption

Based upon the portfolio choice of the bank, the rate of return to patient people can be expressed in terms of the price of housing. After substitution into (9), the demand for housing by a middle-aged patient person is:

$$h_{t+1}^* = \left(\frac{BA(1-\alpha)(\phi+\beta)(1-\pi)}{1+\beta(1-\pi)}\right)\left(\frac{A\alpha}{P_{h,t+1}B}\right)^{\frac{\alpha}{1-\alpha}} \tag{24}$$

First, housing demand will depend on net income available after money balances. Higher financial returns would then improve the demand for housing.

We turn the effects of productivity in the housing sector, B. While it would be natural to assume that productivity is a supply-side factor and only affects housing demand through the price of housing ($P_{h,t+1}$), housing productivity affects housing demand in our framework through factor markets and financial returns. Notably, the higher the return to residential capital in the housing sector (higher values of $P_{h,t+1}B$), the less capital will be allocated to non-residential sector. As a result of lower amounts of investment in the non-residential sector, wages will be lower since labor and non-residential capital are complements in the production of non-residential goods. However, there is a competing factor due to the higher financial returns on savings for patient agents which promotes the demand for housing in (24). Higher levels of total factor productivity (productivity in the non-residential sector) have two effects on an individual's housing demand. First, productivity in the non-residential sector raises wages because it attracts more investment. Second, it leads to higher financial returns.

In addition, there will be more demand for homes (above their role in wealth accumulation) if individuals derive a higher level of utility from homeownership. Higher housing prices are associated with less demand for housing. Armed with financial returns and housing demand, we may now determine the levels of consumption among all individuals as a function of housing prices:

$$c^0_{t+1} = \left(\frac{A(1-\alpha)}{1+\beta(1-\pi)}\right)\left(\frac{A\alpha}{P_{h,t+1}B}\right)^{\frac{\alpha}{1-\alpha}} \tag{25}$$

$$c^1_{t+1} = \left(\frac{A\alpha}{P_{h,t+1}B}\right)^{\frac{\alpha}{1-\alpha}}\left[\frac{A\alpha(1-\alpha)(1+\beta)}{(1-\alpha)(1-\pi)[1+\beta-\delta(\phi+\beta)]} - \left(\frac{BA(1-\alpha)(\phi+\beta)(1-\pi)}{1+\beta(1-\pi)}\right)P_{h,t+1}\right] \tag{26}$$

$$c_{t+2} = (1-\delta)P_{h,t+1}\left(\frac{BA(1-\alpha)(\phi+\beta)(1-\pi)}{1+\beta(1-\pi)}\right)\left(\frac{A\alpha}{P_{h,t+1}B}\right)^{\frac{\alpha}{1-\alpha}} \tag{27}$$

2.2.3 Equilibrium in the Housing Market

The total demand for housing comes from aggregating the housing demand across all of the patient middle-aged population:

$$D^h_t = (1-\pi)h_t \tag{28}$$

We next seek to determine the equilibrium relative price of housing. In each period, new housing supply depends upon residential investment: $H = BK^h$. On the other side of the housing market, demand for new housing depends on the amount that depreciates over time: δD^h.

The equilibrium price of housing is achieved when total supply and total demand for new housing is the same:

$$P^*_{h,t} = \frac{\alpha\left[1+\beta(1-\pi)\right]}{B\left[1+\beta-\delta(\phi+\beta)\right](1-\pi)} \tag{29}$$

At higher levels of productivity in the residential sector, the equilibrium price of housing is lower due to the increase in housing supply. Steady-state housing prices also depend on conditions in the non-residential sector. Notably, the capital-share of production in the non-residential sector is associated with higher prices in the housing market. If production in the non-residential sector is more capital-intensive, the capital intensity pulls resources away from the housing market and causes prices to be higher. Prices are also higher if individuals place a higher valuation on utility in old-age which contributes to an increase in housing demand.

2.2.4 Steady-State Equilibrium Macroeconomic Activity

We choose to study macroeconomic activity in the steady-state. As we have shown, steady-state macroeconomic activity is highly dependent on conditions in the housing sector. Therefore, we are able to determine steady-state outcomes across all sectors in the economy with the solution for equilibrium housing prices (P^*_h) in (29). After imposing steady-state on the system of equilibrium conditions, we proceed by studying the steady-state levels of investment in each sector which are synonymous with the residential and non-residential capital stocks:

Proposition 1. *Let $\delta < \frac{(\phi+\beta)}{(1+\beta)}$. Under this condition, a (non-degenerate) steady-state equilibrium exists. In the steady-state, the equilibrium levels of residential and non-residential capital are:*

$$K^* = \left[\frac{A\left[1+\beta-\delta(\phi+\beta)\right](1-\alpha)(1-\pi)}{1+\beta(1-\pi)}\right]^{\frac{1}{1-\alpha}} \tag{30}$$

$$K^{h^*} = \left(\frac{A(1-\alpha)(1-\pi)\left(1+\beta\right)}{1+\beta\left(1-\pi\right)}\right)^{\frac{1}{1-\alpha}}\left(\frac{1+\beta-\delta(\phi+\beta)}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}} - K^* \tag{31}$$

As previously emphasized, it is important to disaggregate the components of the overall capital stock as the production function varies across sectors. Such differences contribute to substantially different levels of activity in the steady-state as witnessed by (30) and (31). Moreover, it is also clear that each sector competes for resources – the residential capital stock is the total amount of investment in each period net

of investment in the non-residential sector. In addition, fundamentals in each sector have different effects on investment in the steady-state. Notably, higher levels of productivity in the non-residential sector stimulate investment in both sectors but productivity in the residential sector does not affect investment in either sector.

We seek to study how financial returns and consumption are determined in the steady-state. For tractability, we look at a special case of capital intensity in the non-residential sector where $\alpha = 1/2$:

Lemma 1. *Suppose that $\alpha = 1/2$. In addition, let $\delta < \frac{(\phi+\beta)}{(1+\beta)}$. Under these conditions, the steady-state levels of residential and non-residential capital are:*

$$K^* = \left(\frac{A^2}{4}\right)\left(\frac{\left[1+\beta-\delta(\phi+\beta)\right](1-\pi)}{\left[1+\beta(1-\pi)\right]}\right)^2 \tag{32}$$

$$K^{h^*} = \left(\frac{\delta(\phi+\beta)}{\left[(1+\beta)-\delta(\phi+\beta)\right]}\right)K^* \tag{33}$$

In this case, the connections between both sectors of the economy are quite apparent – the residential capital stock is directly proportional to the non-residential stock of capital. Total factor productivity (A) clearly drives investment in both sectors higher. In addition, there is more investment in both sectors of the economy if liquidity risk is not as severe (π lower).

It is also clear that housing fundamentals have asymmetric effects across sectors. Durability of housing leads to less residential investment and more non-residential investment. By comparison, higher valuations for housing services drive residential investment up and lower non-residential investment.

We continue by studying steady-state returns in the banking sector and consumption across segments of the population.

Returns to Deposits:

$$r^{0^*} = \frac{1}{1+\beta(1-\pi)} \tag{34}$$

$$r^{1^*} = \frac{\alpha(1+\beta)}{(1-\alpha)(1-\pi)\left[1+\beta-\delta(\phi+\beta)\right]} \tag{35}$$

Returns paid to the impatient in the steady-state (r^{0^*}) are primarily independent of the return to capital in either sector of the economy. This property reflects the logarithmic form of preferences in which the substitution and income effects of higher returns to capital offset eachother.

In contrast, returns paid to patient agents (r^{1^*}) in the banking sector depend on conditions in both capital sectors. For example, returns are higher if the non-residential sector is more capital-intensive. Moreover, returns to patient individuals are higher if fundamentals in the housing sector favor higher housing prices (due to higher rates of depreciation of housing, a greater consumption value of housing, and a greater desire to invest in housing as a form of wealth accumulation). Higher housing prices raise returns to investment in the residential sector of the economy and support the ability of banks to pay higher rates of return to deposits.

Steady-State Consumption:

$$c^{0^*} = \left(\frac{(1-\pi)}{1+\beta(1-\pi)}\right)\frac{A^2\left[1+\beta-\delta(\phi+\beta)\right]}{4\left[1+\beta(1-\pi)\right]} \tag{36}$$

$$c^{1^*} = \frac{A^2\left[(1+\beta)-(\phi+\beta)(1-\pi)\right]}{4\left[1+\beta(1-\pi)\right]} \tag{37}$$

$$c^{2^*} = \frac{A^2(1-\pi)(1-\delta)\left(\phi+\beta\right)}{4\left[1+\beta(1-\pi)\right]} \tag{38}$$

As a result of the large amount of housing price appreciation before the housing bust, there has been much attention to studying the marginal propensity to consume out of housing wealth. Based upon aggregate data from U.S. states, Case, Quigley, and Shiller (2005) find a marginal propensity to consume equal to around 4 cents. Campbell and Cocco (2004) look at micro-level data for households in the UK and find that higher housing prices lead to increased consumption among homeowners, but not renters. In contrast, Carroll (2004) finds that the long-run marginal propensity to consume out of housing wealth to be around 9 cents per dollar.

Time-series approaches used to estimate the marginal propensity to consume are constructed in the following way. First, estimate a process for changes in housing wealth. Second, look at a series of changes in consumption. The marginal propensity to consume looks at the slope of the latter process over the former.

Interestingly, our framework can be used to draw insights into this phenomenon. In particular, we derive the MPC out of housing wealth when changes in housing wealth are driven by higher levels of productivity. The impact of productivity in the non-residential sector on housing prices is:

$$\frac{\partial(P_h^* * h^*)}{\partial A} = \frac{A\left(\phi+\beta\right)(1-\pi)}{2\left[1+\beta(1-\pi)\right]} \tag{39}$$

Notably, productivity has a larger impact on housing wealth if individuals derive higher levels of utility from homeownership and they value owning as a form of wealth accumulation because they have a higher rate of time preference.

We next turn to studying how productivity affects the consumption of patient individuals who are also homeowners:

$$\frac{\partial c^{1^*}}{\partial A} = \frac{A\left[(1+\beta)-(\phi+\beta)(1-\pi)\right]}{2\left[1+\beta(1-\pi)\right]} \tag{40}$$

$$\frac{\partial c^{2^*}}{\partial A} = \frac{A(1-\pi)(1-\delta)\left(\phi+\beta\right)}{2\left[1+\beta(1-\pi)\right]} \tag{41}$$

While the effect of productivity on a middle-aged patient person is decreasing in the value of homeownership, the effect is increasing for the old who finance their consumption out of housing wealth.

Aggregating across both periods is equivalent to looking at the aggregate consumption across both groups of homeowners in the steady-state. As a result, the MPC out of housing wealth is:

$$MPC^* = \frac{1+\beta-\delta\left(\phi+\beta\right)\left(1-\pi\right)}{\left(\phi+\beta\right)\left(1-\pi\right)} \tag{42}$$

Interestingly, the MPC is exclusively dependent on fundamentals in the housing sector. Of course, the durability of housing is a significant factor. If housing is more durable, housing is a more productive form of wealth accumulation and individuals can spend more as housing wealth increases. It is decreasing in the utility from homeownership – as productivity drives up housing wealth, it is also associated with greater housing expenditures which detracts from the desire of individuals to spend on consumption. The rate of time preference is also a key component of the MPC.

In this environment money is super-neutral. However, available evidence indicates that inflation does have important effects on housing market activity. We turn to the relationship between inflation and housing market activity in the following section.

3 Non-Superneutral Effects of Monetary Policy in the Housing Market

In the preceding section, all revenues from seigniorage were consumed by the government. Since the revenues from the inflation tax were not redistributed back to the economy, monetary policy was superneutral and did not have any effect on real economic activity. By stripping out real effects from monetary policy, the benchmark framework elucidates how the fundamentals of the housing market affect overall macroeconomic activity.

However, the superneutrality of money is clearly at odds with empirical evidence demonstrating that inflation has a significant impact on economic activity through the housing sector. For example, both Summers (1981) and Piazessi and Schneider (2012) find that housing investment becomes more attractive relative to corporate capital in inflationary episodes. Moreover, Ahmed and Rogers (2000) find evidence of a long-run Tobin effect for the United States.

Thus, the available evidence points to two important observations to address. First, it is important that a model of investment activity produces a positive relationship between inflation and investment. Second, monetary policy produces asymmetric effects on investment – residential investment should show a stronger response to inflation than non-residential investment.

In the following two sections, we seek to address the non-superneutral effects of monetary policy in the housing market and the consequences for macroeconomic performance. Rather than promoting government consumption, in this section, all seigniorage revenues are redistributed to the economy in the form of lump-sum transfers to the young.[5] In the next section, seigniorage revenues promote inflation-financed public credit obligations as in Schreft and Smith (1997, 1998).

Due to the revenues from money creation, the government's budget constraint is:

$$\tau_t = \frac{\sigma}{1+\sigma} m_t \tag{43}$$

Seigniorage revenues provide the government with income which it redistributes to young individuals in the lump-sum amount, τ_t. Consequently, young individuals have two sources of income. Income from the labor market equal to w_t and income from transfers, τ_t.

Due to the increase in income, a representative patient person's housing demand is:

$$h_{t+1} = \left(\frac{(\phi+\beta)}{(1+\beta)}\right)\left(\frac{r_t^1\left(w_t+\tau_t\right)}{P_h}\right) \tag{44}$$

Consumption across time-periods directly follows the analysis in the benchmark model:

$$c_{t+1}^1 = \frac{(1-\phi)}{(1+\beta)} r_t^1 (w_t+\tau_t) \tag{45}$$

$$c_{t+2} = \left(\frac{(1-\delta)(\phi+\beta)}{(1+\beta)}\right) r_t^1 (w_t+\tau_t) \tag{46}$$

[5] The redistribution of seigniorage to the young is pretty standard in monetary models with liquidity risk. See, for example, Bhattacharya and Singh (2010).

Obviously, housing demand and consumption are all affected by the transfers from the government.

The portfolio choices of the bank are virtually the same as the benchmark model. Again, a no-arbitrage condition implies that the returns to capital in either sector of the economy are the same:

$$P_{h,t}B = A\alpha K_t^{\alpha-1} \tag{47}$$

Money balances and the bank balance sheet are affected by the size of the transfers:

$$m_t = \frac{\pi(w_t + \tau_t)}{(1-\pi)\beta + 1} \tag{48}$$

$$(w_t + \tau_t) = m_t + i_t + i_t^h \tag{49}$$

3.1 Steady-State General Equilibrium

As previously stated, workers are paid their marginal product of labor in equilibrium. Combining (47), (49), (43), (48), and (3) we attain a steady-state relationship between residential capital and housing prices that determines the level of residential investment:

$$K^h = A(1-\alpha)\left(\frac{A\alpha}{P_h B}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{(1-\pi)(1+\beta)}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi}\right) - \left(\frac{A\alpha}{P_h B}\right)^{\frac{1}{1-\alpha}} \tag{50}$$

For a given price of housing, higher inflation raises seigniorage. With the additional level of deposit income received by the bank, residential investment is also higher.

As in the benchmark model, the portfolio choice of the bank provides information about the rate of return to patient agents so that an individual's housing demand can be expressed as a function of the price of housing:

$$h = \left(\frac{A(1-\alpha)B(\phi+\beta)}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi}\right)\left(\frac{A\alpha}{P_h B}\right)^{\frac{\alpha}{1-\alpha}} \tag{51}$$

In turn, consumption across the different segments of the population is:

$$c^0 = \left(\frac{A(1-\alpha)}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi}\right)\left(\frac{A\alpha}{P_h B}\right)^{\frac{\alpha}{1-\alpha}} \tag{52}$$

$$c^1 = \left(\frac{A(1-\alpha)P_h B(1-\phi)}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi}\right)\left(\frac{A\alpha}{P_h B}\right)^{\frac{\alpha}{1-\alpha}} \tag{53}$$

$$c^2 = \left(\frac{(1-\delta)(\phi+\beta)A(1-\alpha)P_h B}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi}\right)\left(\frac{A\alpha}{P_h B}\right)^{\frac{\alpha}{1-\alpha}} \tag{54}$$

For a given price of housing, a higher money growth rate stimulates consumption across all segments of the population.

3.2 Equilibrium in the Housing Market

The amount of residential investment affects the total supply of new housing while total demand is equal to δD^h. In steady-state equilibrium, prices clear the housing market:

Lemma 2. *Let* $\delta < \frac{(\phi+\beta)}{(1+\beta)}$. Under this condition, the steady-state equilibrium price of housing is:

$$P_h^* = \left(\frac{\alpha}{B\,(1-\alpha)}\right)\left(\frac{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi}{[(1+\beta) - (\phi+\beta)\delta]\,(1-\pi)}\right) \tag{55}$$

A higher money growth rate lowers the price of housing. Further, this impact is stronger if liquidity risk in the economy is higher.

Presumably, the lower price of housing in response to higher inflation reflects an increase in housing supply. In order to sort this out, we turn to the following:

Proposition 2. *Let* $\delta < \frac{(\phi+\beta)}{(1+\beta)}$. *Under this condition, the steady-state stocks of capital across sectors are:*

$$K^* = \left(\frac{A\alpha \left[(1+\beta) - (\phi+\beta)\delta \right] (1-\pi)}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi} \right)^{\frac{1}{1-\alpha}} \tag{56}$$

$$\begin{aligned} K^{h^*} &= \frac{(\phi+\beta)}{(1+\beta)} \left(\frac{\delta}{BP_h^*} \right) \left[A\alpha \left(K^* \right)^{\alpha} - BP_h^* K^* \right] + \\ & \left(\frac{(\phi+\beta)\delta A(1-\alpha)}{(1+\beta)} \right) \left(\frac{(1-\pi)(1+\beta)}{(1-\pi)\beta + 1} \right) \left(\frac{(1-\pi)(1+\beta)}{(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi} \right) (K^*)^{\alpha} \end{aligned} \tag{57}$$

Proposition 2 demonstrates how monetary intervention in the economy dramatically changes how housing market activity depends on macroeconomic conditions. In the absence of transfers from seigniorage, (31) is simply the residual amount of capital after non-residential investment. However, (57) is clearly non-linear in the non-residential stock and housing prices. In this manner, our framework demonstrates that policy intervention by monetary authorities is likely to make the interplay between the housing market and macroeconomic activity much less transparent.

Nevertheless, analytical solutions for all variables are obtainable in the special case in which the non-residential sector is equally capital and labor-intensive:

Lemma 3. *Let $\alpha = 1/2$. Assuming Proposition 1 holds, a (non-degenerate) steady-state equilibrium exists. In the steady-state, the equilibrium stocks of residential and non-residential capital are:*

$$K^* = \left(\frac{A^2}{4} \right) \left(\left(\frac{(1-\pi)\left((1+\beta) - (\phi+\beta)\delta \right)}{1 + (1-\pi)\beta - \frac{\sigma}{1+\sigma}\pi} \right) \right)^2 \tag{58}$$

$$K^{h^*} = \left(\frac{\delta(\phi+\beta)}{(1+\beta) - \delta(\phi+\beta)} \right) K^* \tag{59}$$

As previously mentioned, Ahmed and Rogers (2000) find evidence of a long-run Tobin effect for the United States. It is clear from (58) and (59) that higher rates of inflation stimulate investment activity in both sectors of the economy. However, where does monetary policy have the biggest impact? We turn to this issue in the following corollary to Lemma 3:

3.2.1 The Effects of Monetary Policy

The principal objective of this section is to focus on the effects of monetary policy. Based upon Lemma 3, it is easy to see that the effects of monetary policy in the housing sector are proportional to the effects of policy in the non-residential sector:

$$\frac{\partial K^{h^*}}{\partial \sigma} = \left(\frac{\delta(\phi + \beta)}{(1+\beta) - \delta(\phi + \beta)} \right) \frac{\partial K^*}{\partial \sigma} \tag{60}$$

Corollary 1. *Assume that the conditions in Lemma 3 hold. Further, let* $\delta > \frac{(1+\beta)}{2(\phi+\beta)}$. *If these conditions hold,* $\frac{\partial K^{h^*}}{\partial \sigma} > \frac{\partial K^*}{\partial \sigma} > 0$.

According to the corollary, monetary policy will have asymmetric effects on the residential and physical capital stocks. In particular, as observed in the empirical evidence, inflation has a stronger effect on the residential capital stock than the physical capital stock.

Thus, monetary policy plays a bigger role in investment in the housing sector than the non-residential sector. It is often argued that the reason for such behavior is due to the interest-rate sensitivity of the housing sector. Alternatively, mortgage-interest deductibility through taxes if often cited. However, our framework demonstrates that the durability of the housing stock is a significant factor. Simply put, housing is a durable asset. As a result, inflation promotes the asset with the highest present discounted stream of income. Arnott (1980) also stresses that the durability of housing should be an important consideration when evaluating the effects of public policy.

While Corollary 1 demonstrates that the effects of monetary policy are stronger in the housing sector than the non-residential sector, the following focuses on conditions in the non-residential sector:

Corollary 2. *Assume that the conditions in Lemma 3 hold. The physical capital locus, (58), behaves such that:*

$$\frac{\partial K}{\partial \sigma} > 0, \frac{\partial^2 K}{\partial \sigma \partial A} > 0, \frac{\partial^2 K}{\partial \sigma \partial \phi} < 0$$

While Corollary 1 demonstrates that inflation promotes capital accumulation, Corollary 2 demonstrates that the effects of policy are stronger if productivity in the economy is higher. However, the effects of inflation are weaker if preferences for housing are higher. Yet, our principal motivation is to study the impact of policy on housing market activity in general equilibrium:

Corollary 3. *Assume that the conditions in Lemma 3 hold. The equation denoting the residential capital stock, (59), behaves as follows:*

$$\frac{\partial K^h}{\partial \sigma} > 0, \frac{\partial^2 K^h}{\partial \sigma \partial A} > 0, \frac{\partial^2 K^h}{\partial \sigma \partial \pi} > 0, \frac{\partial^2 K^h}{\partial \sigma \partial \phi} > 0 \text{ if } \phi < \frac{(1+\beta)(1-\pi) - 2\delta\beta}{2\delta}$$

Inflation raises the residential capital stock by way of increasing the income of young workers. The revenues from the inflation tax are also higher if wages are higher which explains the complementarity between monetary policy and productivity. Thus, the model demonstrates that monetary policy would be less effective in promoting housing market activity in periods of low productivity than high productivity. For example, attempts to promote economic activity during the productivity slowdown of the 1970s were largely unsuccessful. However, residential investment was consistently higher during the high levels of productivity encountered in the "New Economy" from 1993 - 2003. The latter period has also been characterized as a period in which homeownership rates climbed. For example, in 1970, the U.S. Census reports that the homeownership rate was 62.9%. It climbed to 64.2% in 1990 and 66.2% in 2000. The apparent increase in preferences for homeownership also contributed to the strong impact of policy on housing market activity.

We next turn to the impact on policy on consumption.

Steady-State Consumption:

$$c^{0^*} = \frac{A^2 \left[(1+\beta) - (\phi+\beta)\delta\right](1-\pi)}{4\left[(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi\right]^2} \tag{61}$$

$$c^{1^*} = \frac{A^2(1-\phi)}{4\left[(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi\right]} \tag{62}$$

$$c^{2^*} = \frac{A^2(1-\delta)(\phi+\beta)}{4\left[(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi\right]} \tag{63}$$

As in the previous section, we are particularly interested in finding the determinants of the MPC from housing wealth. In contrast to the benchmark model, monetary policy has real effects on the housing market:

$$\frac{\partial(P_h h)}{\partial\sigma} = \frac{-\pi A^2(\phi+\beta)}{4\left(1+\sigma\right)^2\left[(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi\right]^2} \tag{64}$$

That is, although monetary policy leads to an increase in residential investment, the increase in supply lowers the total value of the housing stock. We next turn to the effects of policy on consumption across the consumption path:

$$\frac{\partial c^{1^*}}{\partial\sigma} = \frac{-\pi A^2(1-\phi)}{4\left(1+\sigma\right)^2\left[(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi\right]^2} \tag{65}$$

$$\frac{\partial c^{2^*}}{\partial\sigma} = \frac{-\pi A^2(1-\delta)(\phi+\beta)}{4\left(1+\sigma\right)^2\left[(1-\pi)\beta + 1 - \frac{\sigma}{1+\sigma}\pi\right]^2} \tag{66}$$

By inducing individuals to allocate more income towards housing, consumption declines. However, the marginal propensity to consume out of housing wealth remains the same as the benchmark model:

$$MPC = \frac{1 + \beta - \delta(\phi + \beta)}{(\phi + \beta)} \tag{67}$$

Moreover, the MPC is the same regardless of whether housing wealth is driven by real factors such as productivity or nominal factors such as monetary policy. This is an important argument that policymakers need to consider when trying to understand how consumption patterns respond to changes in housing market activity over time.

4 The Economy with Government Bonds

In recent years, the line between fiscal policy and monetary policy has become blurred as many have argued that the high amount of money growth that has occurred across countries simply represents a redistribution to fiscal authorities. In order to consider this possibility, we extend the model to account for inflation-financed government bonds as in Schreft and Smith (1997, 1998). In this manner, there is another asset for banks to add to their portfolios. As in the previous setting with transfers from seigniorage, monetary policy in this setting is not super-neutral.

With the introduction of government debt, banks now allocate deposits between money, physical capital, residential capital, and bonds. The real per capita bond supply is $b_t \equiv \frac{B_t}{P_t}$, and the real return on bonds is $R_t = I_t \left(\frac{P_t}{P_{t+1}}\right)$, where I_t is the nominal interest rate. We assume there are no government transfers or taxes. Therefore, the government budget constraint requires:

$$R_{t-1} b_{t-1} = \frac{(M_t - M_{t-1})}{P_t} + b_t \tag{68}$$

Government revenue must be sufficient to cover interest payments on previously issued bonds. Revenues come from newly issued bonds along with seigniorage.

Given the bank now has four asset choices, the bank's balance sheet is:

$$w_t = m_t + i_t + i_t^h + b_t \tag{69}$$

The return paid to impatient agents comes from the bank's money balances:

$$\pi r_t^0 w_t = m_t \left(\frac{P_t}{P_{t+1}} \right) \tag{70}$$

In comparison, the return to patient agents is paid from the bank's return on physical capital investment, residential investment, and bond holdings.

$$(1-\pi) r_t^1 w_t = i_t \rho + i_t^h r + R_t b_t \tag{71}$$

The bank's objective is to maximize the expected utility of its depositors. In order for banks to invest in both types of capital and government bonds, the following two no-arbitrage conditions must hold:

$$P_{h,t} B = A \alpha K_t^{\alpha - 1} \tag{72}$$

$$\frac{I_t}{\sigma} = P_{h,t} B \tag{73}$$

As in the benchmark model, money demand is:

$$m_t = \frac{\pi w_t}{(1-\pi)(1-\beta) + \pi} \tag{74}$$

4.1 Steady-State General Equilibrium

By virtue of the no-arbitrage conditions which apply, the total demand for housing is the return on income after money balances:

$$D^h = \frac{(\phi + \beta)}{(1 + \beta)} \frac{1}{P_h} (w - m) r \tag{75}$$

In the steady-state, bond demand is:

$$b = \left(\frac{\sigma - 1}{I - \sigma} \right) m \tag{76}$$

Similar to the previous model, in equilibrium the supply of housing must equal the total demand for housing:

$$BK^h = \delta D^h \tag{77}$$

A steady-state equilibrium reduces to two conditions on the residential capital stock and the nominal return to bonds which must hold. The first condition derives from the government's budget constraint (GBC):

Proposition 3. (Government Budget Constraint) *Suppose that* $\frac{\alpha}{(1-\alpha)(1-\pi)} > \frac{I}{\sigma}$. *Also, assume that* $I < \sigma$. *Under these conditions, the residential capital stock is positive and described by:*

$$K^h = A(1 - \alpha) \left(\frac{A\alpha\sigma}{I} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(1 - \pi)(1 - \beta) - \frac{(\sigma-1)}{(I-\sigma)}\pi}{(1 - \pi)(1 - \beta) + \pi} \right) - \left(\frac{A\alpha\sigma}{I} \right)^{\frac{1}{1-\alpha}} \tag{78}$$

In addition, $\frac{\partial K^h}{\partial I}\,|_{78}$ *and* $\frac{\partial K^h}{\partial \sigma}\,|_{78} > 0$.

In equilibrium we are interested in analyzing the economy when $I > 1$. This insures that the return on money is dominated by the return from other assets. However, as $I < \sigma$, this implies that the revenues earned from money creation exceed the interest paid on government bonds. Hence, $b < 0$. That is, the residential capital stock only exists as long as the government is a net lender to the financial system.

Typically, in models such as Schreft and Smith (1997, 1998), the government budget constraint has two potential positions. If $I > \sigma$, the government is a net borrower since the interest to pay for its debt exceeds seigniorage revenues. The other position is similar to the result in Proposition 3. In the presence of government bonds, residential investment competes with bonds and the non-residential sector for resources. As suggested by the Proposition, if the government were a net borrower, it would require the housing stock to be negative. In contrast, a higher money growth rate provides the fiscal authority with more resources to support housing market activity.

The second condition that must hold is that the housing market must be in equilibrium:

Proposition 4. **(**Housing Market Equilibrium) *The relationship between* I *and* K^h *in which (77) is satisfied is described by:*

$$K^h = \left(\frac{(\phi+\beta)\delta}{(1+\beta)}\right)\left[A(1-\alpha)\left(\frac{A\alpha\sigma}{I}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{(1-\pi)(1-\beta)}{(1-\pi)(1-\beta)+\pi}\right)\right] \tag{79}$$

In addition, $\frac{\partial K^h}{\partial I}\,|_{79} < 0$ *and* $\frac{\partial K^h}{\partial \sigma}\,|_{79} > 0$.

Steady-state locus, (79), represents equilibrium in the housing market. More specifically, (79) demonstrates how the nominal interest rate must relate to residential capital to ensure the demand for housing is equivalent to the supply of housing. As demonstrated by the Proposition, there is a negative relationship between the

nominal interest rate and the residential capital stock. At higher nominal interest rates, the return to government bonds is higher. As a result, banks allocate more resources to bonds which results in lower residential investment. However, higher money growth reduces the real return to bonds and induces a substitution towards residential capital.

Based upon the above analysis, we arrive to the following Proposition:

Proposition 5. *Suppose that* $\left(\frac{(\phi+\beta)}{(1+\beta)}\right) A > (1-\pi)+\frac{\pi}{1-\beta}$ as $I \to \infty$ *and* $\left(\frac{(\phi+\beta)}{(1+\beta)}\right) \delta(1-\beta)+\frac{\alpha\sigma[(1-\pi)(1-\beta)+\pi]}{(1-\alpha)} < (1-\pi)(1-\beta)+\pi$ *as* $I \to 1$. *If these conditions hold, a unique steady-state exists in which* $I < \sigma$.

Figure 1 depicts the Government Budget Constraint and Housing Market Equilibrium loci in which an equilibrium exists.

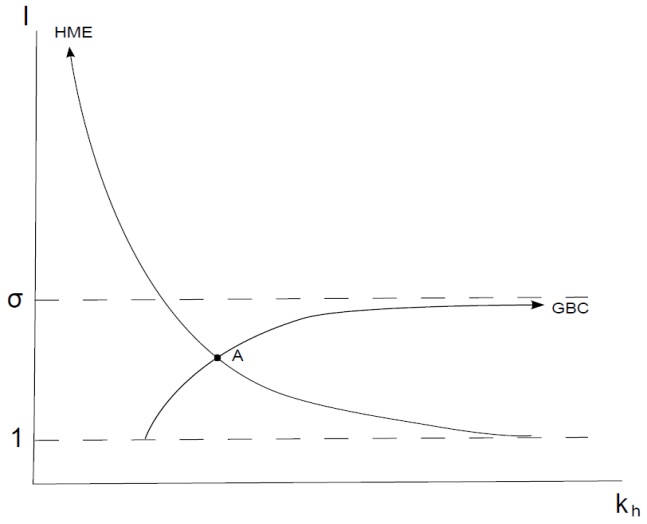

Figure 1: Steady-State Equilibrium

In the following, we discuss the effects of monetary policy in the presence of government bonds.

4.1.1 The Effects of Monetary Policy

We now focus on the effect of monetary policy on the residential capital stock. As opposed to Schreft and Smith (1997, 1998), only one steady-state exists so the effects of monetary policy on capital accumulation can be pinned down.

To begin, as previously mentioned in Proposition 3, a higher money growth rate shifts the locus (78) to the right. We refer to this shift as the *redistribution effect from monetary policy* as it is associated with the bank's allocation from nominal to illiquid assets and thereby induces redistribution from impatient agents to patient agents when residential investment increases.

From Proposition 4, a higher rate of inflation also shifts the locus associated with (79) to the right. This reflects the *substitution effect from monetary policy* as inflation reduces the real return to government bonds. Consequently, banks allocate more funds towards residential investment.

The total increase in residential investment due to both mechanisms is shown in Figure 2 below:

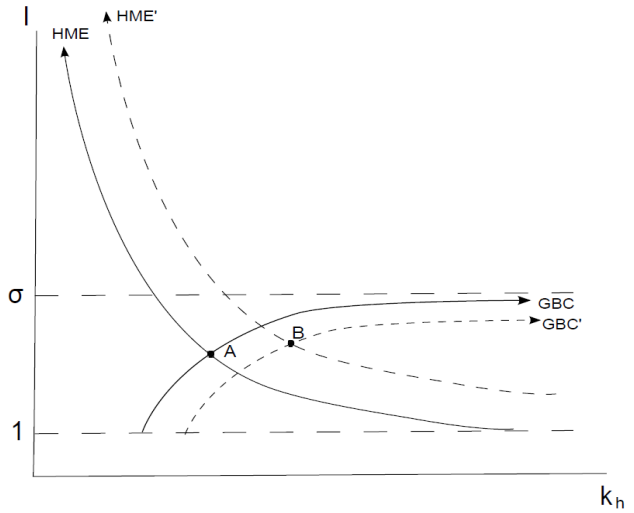

Figure 2: Increase in the Rate of Money Growth

5 Optimal Monetary Policy

The previous two sections provide mechanisms in which inflation promotes housing market activity. In fact, each model shows that the impact of inflation is *permanent* – that is, higher inflation rates promote investment and housing market activity from *any* inflation rate. However, the U.S. inflationary experience in the 70s shows

that there are limits to the potential benefits from higher inflation. For example, Bernanke (2004) contends that high inflation contributed to output volatility and high unemployment during the 1970s. Romer and Romer (2002) make the same arguments.

We incorporate the relationship between volatility and risk observed in the data by assuming that the incidence of liquidity risk is a function of monetary policy, $\pi(\sigma)$. In particular, we posit that the marginal increase in risk is higher at high inflation rates. Therefore, we consider the case where $\pi(\sigma) = \pi(1 + d\sigma^2)$. Similar ideas have been proposed in the literature. For example, Ghossoub and Reed (2010) study a model in which the incidence of liquidity risk is a function of the capital stock. They argue that such a mechanism captures the increased exposure to risk in poor countries relative to the stability of advanced economies.

Our objective is to show that introducing the relationship between policy and volatility allows us to formalize the trade-offs inherent in designing policy to stimulate housing market activity. In particular, we demonstrate that these trade-offs are important for thinking about how optimal monetary policy depends on conditions in the housing market. To show the impact of policy, we return to a setting in which seigniorage revenues are transferred to the young. We define welfare as the expected lifetime utility of an individual young agent:

$$U(c^0_{t+1}, c^1_{t+1}, h_{t+1}, c_{t+2}) = \pi \ln(c^0_{t+1}) + (1-\pi)\left[\phi \ln(h_{t+1}) + (1-\phi)\ln(c^1_{t+1}) + \beta \ln(c_{t+2})\right] \tag{80}$$

We characterize "optimal" monetary policy by the choice of σ that would maximize welfare.

In terms of our numerical analysis, we start by identifying the optimal rate of inflation given our choice of parameters. We consider the following parameter set: $A = 7.4$, $B = 4.8$, $\beta = .8$, $\delta = .33, \alpha = .5$, $d = 9.7$, $\pi = .35$, and $\phi = .3$. Also, in order to maintain consistency with the majority of the previous analytical results, we assume that the non-residential sector is equally capital and labor-intensive. The aggregate value of the housing stock is $\overline{h}$ which is the price of a single home $(P_h * h)$ adjusted for the total number of homeowners. The primary point of our parameter set is that the welfare-maximizing rate of inflation is $\sigma^* = 0.03$. This is demonstrated in Table 1 below and is consistent with the average inflation rates during the Volcker-Greenspan era of monetary policy, the "Great Moderation."

Table 1

Optimal monetary policy

σ	0.015	0.02	0.025	0.03	0.035	0.04	0.045
K^h	1.74612	1.74883	1.75119	1.75319	1.75484	1.75614	1.75708
K	3.87525	3.87613	3.87472	3.87104	3.86507	3.85684	3.84635
m	1.68726	1.69275	1.69871	1.70517	1.7121	1.71953	1.72744
w	7.28369	7.28452	7.2832	7.27973	7.27412	7.26637	7.25648
P_h	.391571	.391527	.391598	.391784	.392086	.392504	.39304
h	25.3981	25.4376	25.4719	25.501	25.525	25.5438	25.5576
τ	.0249349	.0331911	.041432	.049665	.0578972	.0661356	.0743872
$\overline{h}$	6.45678	6.46015	6.46241	6.46355	6.46358	6.46249	6.46028
$welfare$	3.00459	3.00553	3.00607	3.0062	3.00593	3.00526	3.00419

In comparison to the model with transfers, higher rates of inflation promote residential investment but not investment in the non-residential sector. However, higher inflation rates are associated with higher housing prices which is consistent with available evidence on the transmission of monetary policy to conditions in the housing sector. It is interesting to see that σ^* is lower than the rate of money growth that maximizes total housing wealth in the economy, $\sigma = .035$. This takes place because optimal policy takes into account that inflation reduces the value of money and the extent of risk-sharing in the economy. In other words, optimal monetary policy is less aggressive than the rate of money growth that maximizes the total value of the housing stock.

We turn to examining the role that housing market fundamentals play in determining optimal monetary policy. We begin with a setting in which productivity in the housing sector is lower ($B = 2.4$). This may be due to increased land regulations over time. The results are listed in Table 2:

Table 2

Optimal monetary policy ($B = 2.4$)

σ	0.024	0.027	0.03	0.033	0.036	0.039	0.042
K^h	1.75075	1.75204	1.75319	1.75423	1.75513	1.75591	1.75656
K	3.87519	3.87352	3.87104	3.86733	3.86361	3.85867	3.85292
m	1.69748	1.70124	1.70517	1.70927	1.71355	1.718	1.72263
w	7.283149	7.28207	7.27973	7.27662	7.27274	7.26809	7.26267
P_h	.783149	.783317	.783568	.783903	.784321	.784823	.785409
h	12.7327	12.7421	12.7505	12.758	12.7646	12.7702	12.775
τ	.0397847	.0447258	.049665	.054604	.0595442	.0644871	.0694343
$\overline{h}$	6.46205	6.463	6.46355	6.4637	6.46345	6.46279	6.46172
$welfare$	2.87123	2.87152	2.87167	2.87169	2.87158	2.87133	2.87096

In response to the lower productivity, supply in the housing market will be lower. Consequently, access to housing would be lower. In order to promote housing market activity, the optimal rate of money growth is raised to $\sigma^* = 0.033$. As a result, residential investment will increase and promote housing supply in the face of low productivity. In this manner, optimal monetary intervention should be more aggressive in economies with adverse housing supply conditions.

We conclude by looking at a setting in which there is an increase in housing demand. This is due to the higher consumption-value of home ownership:

Table 3
Optimal monetary policy ($\phi = .5$)

σ	0.01	0.015	0.02	0.025	0.03	0.035	0.04
K^h	1.89136	1.89454	1.89726	1.89953	1.9013	1.9027	1.90362
K	3.26419	3.26631	3.26627	3.26409	3.25976	3.25329	3.24468
m	1.54456	1.54903	1.55389	1.55912	1.56475	1.57077	1.57717
w	6.68482	6.68698	6.68695	6.68471	6.68028	6.67364	6.66481
P_h	.426651	.426513	.426515	.426658	.426941	.427365	.427932
h	27.5106	27.5569	27.5965	27.6295	27.6559	27.6757	27.689
τ	.0152927	.0228921	.0304683	.0380274	.0455753	.053178	.0606604
$\overline{h}$	7.62535	7.63071	7.63472	7.6374	7.63875	7.63877	7.63748
$welfare$	3.15788	3.15907	3.15972	3.15985	3.15944	3.15852	3.15706

If individuals value homeownership more, the optimal monetary policy rule seeks to reduce volatility in the economy. This promotes the ability of individuals to save so that they face less exposure to liquidity risk. As a result, the optimal rate of inflation falls to $\sigma^* = 0.025$. That is to say, monetary policy should be more conservative if fundamentals in the economy favor housing demand.

6 Conclusion

In recent years, the impact of monetary policy on housing markets and the macroeconomy has received a large amount of attention. This paper provides a dynamic general equilibrium model with a microfoundation for money to study the transmission of monetary policy to the housing sector and macroeconomic activity. While previous empirical work has shown that inflation stimulates housing sector activity, our research develops a rich framework to show that there are important asymmetries resulting from monetary stimulus. As in a large volume of work on housing, we show that the durability of housing as an asset plays a huge role. Moreover, we study how optimal policy intervention depends on conditions in the housing sector. Interestingly, we find that optimal money growth is higher if productivity in the residential

sector is low. In this manner, we argue that policy should be designed according to supply conditions in the housing sector. By comparison, inflation should be lower when conditions favor housing demand.

There are a number of issues we intend to address in future work. For example, the model could be extended as in Arnott et al. (1999) to study the impact of monetary policy on investment in both housing quantity and quality. Investment in multi-family structures could also be incorporated. In addition, both Arnott and Braid (1997) and Harding et al. (2007) look at housing maintenance and depreciation over time. One could also use our framework to show how inflation plays a role in housing sector activity through endogenous maintenance expenditures. In addition, the model could be extended to a four period OLG framework in order to study the effects of policy on home equity lending. Finally, Mankiw and Weil (1989) point out that population demographics have important consequences for housing prices. Consequently, we intend to study how optimal monetary intervention depends on demographics in future work.

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