Phillips Curves, Monetary Policy, and a Labor Market Transmission Mechanism

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Abstract

This paper develops a general equilibrium monetary model with performance incentives to study the inflation-unemployment relationship. A long-run downward-sloping Phillips curve can exist with perfectly anticipated inflation because workers' incentives to exert effort depend on financial market returns. Consequently, higher inflation rates can reduce wages and stimulate employment. An upward-sloping or vertical Phillips Curve can arise instead, depending on agents' risk aversion and the possibility of capital formation. Welfare might be higher away from the Friedman rule and with a central bank putting some weight on employment.

Keywords: Phillips curve; Efficiency wages; Involuntary unemployment; Labor and financial market frictions; Central Bank mandate

JEL Codes: E24, E31, E52, E58, J21, J64, M5

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1. Introduction

The recent financial crisis and global recession have generated tremendous pressure on central banks to adopt policies to promote labor market activity. How should a central bank conduct policy to stimulate labor markets? Typically, central banks resort to policies that stimulate inflation and lower the unemployment rate by moving the economy along its Phillips Curve.

Why does the Phillips Curve tradeoff exist? How does the conduct of monetary policy in general equilibrium affect unemployment? Without a rigorous understanding of this relationship, it is difficult for policymakers to implement optimal monetary policy. There are numerous arguments that seek to explain the "short-run" tradeoffs. Virtually all of them rely on some kind of price rigidity. However, as a policymaker confronts the post-2008 crisis economic climate—one of *prolonged* slack in the labor market—explanations based upon the inability of prices to adjust simply do not apply. Consequently, micro-founded explanations that are robust to a longer time frame are needed.

This article fills the gap in the existing literature by presenting a general-equilibrium monetary model in which the impact of monetary policy is transmitted to real variables through the wage-contracting/employment process in the labor market. However, in contrast to the existing literature, monetary policy affects real activity without relying on wage rigidities. Thus, the mechanism that we put forward is a candidate that could be used even in the post-crisis economic climate.

The model is used to address three questions: First, how does monetary policy affect labor-market contracts and thus employment? Second, what is the optimal monetary policy in the face of involuntary unemployment? Third, what weight should a central bank with a dual mandate put on each mandate?

We study these questions in a setting where individuals face two sources of idiosyncratic risk. One source of risk stems from liquidity risk in which individuals are uncertain about their need to withdraw their savings. The other source of risk is income risk due to the presence of equilibrium unemployment. Private information about an agent's effort on the job results in firms setting efficiency wages and hiring only a subset of available workers, leaving the rest involuntarily unemployed.

As modeled in Schreft and Smith (1997, 1998), spatial separation, limited

communication, and restrictions on asset portability generate a transactions role for money so that monetary policy can be analyzed. In the spirit of Diamond and Dybvig (1983), relocation shocks represent a type of liquidity shock and create a special role for banks to provide insurance against this form of idiosyncratic risk. Agents can partially self-insure against income risk by exerting effort on the job, and asset returns affect agents' desire to do so. For example, if workers earn high returns from their savings, they have less incentive to insure themselves against the risk of unemployment by exerting labor effort. Thus, the core transmission mechanism for monetary policy to the labor market centers on a "wealth" effect. Inflation, acting as a tax on wealth, affects workers' incentives because they have less real income in the event of job loss. In this manner, this work ties into a large literature studying the impact of wealth effects for macroeconomic activity.[1]

The Phillips curve is the set of inflation rate-unemployment rate pairs that are steady states. The slope of the Phillips curve and the optimal monetary policy (modeled as a choice of an inflation target) depend on fundamentals of labor and financial markets, on agents' risk aversion, and on whether capital formation is possible.

In Section 2, we present a version of the model that abstracts from capital formation. The model has three general features. First, varying any of the parameters that characterize the labor-market or financial-market frictions or affect their impact on unemployment, holding the others fixed, generates a family of long-run Phillips curves.[2] At a higher steady-state inflation rate, agents know that the return on their assets will be lower; hence, they have greater incentive to

[1] In their initial contribution, Mankiw and Zeldes (1991) point out that stock market wealth can have an important impact on consumption behavior. In light of the strong gains in housing prices before the crisis, much research examined the implications of housing wealth for consumption. Hryshko, Luengo-Prado, and Sorensen (2010) look at the consumption effects of housing wealth after labor market displacement. Homeowners experience high levels of consumption in periods of rising house prices even after job displacement. Consequently, in periods of rising housing wealth, homeowners would have lower incentives to avoid unemployment. Our model shows that monetary policy can have an important impact on labor market activity through the wealth effect. Cunningham and Reed (2012) study the impact of housing wealth on labor market activity in the presence of equilibrium unemployment.

[2] There are different possible interpretations of the time period to which the long-run Phillips curves in the model pertain. One interpretation relies on the fact that in a steady state, the economy's growth rate equals its potential growth rate. This translates to a period of about three years, as reflected by the Federal Open Market Committee's November 2007 decision to release its members' projections over a three-year horizon (Bernanke 2007). A second interpretation takes a period in the model to be about 20 years long, and the Phillips curves pertain to periods of similar length. A third interpretation is that the model captures the part of the human life cycle that occurs shortly before and after retirement. Under this interpretation, a period in the model is five to 10 years, and the Phillips curves apply to that period. Whatever the interpretation chosen, the Phillips curves in the economy of Section 2, which can be thought of as having a fixed capital stock, should pertain to a shorter period than the curves in the Section 3 economy, with capital formation. In addition, monetary policy should have effects in the short run as well as the long run in this model.

put forth full effort on the job. Firms, in turn, can offer lower wages and still induce full effort, so the equilibrium unemployment rate is lower. Second, points along each Phillips curve can be ranked based on welfare, which is taken to be the expected utility of a representative agent. Third, the optimal monetary policy, defined as the inflation target that maximizes welfare, can be identified.

The model of Section 2 yields striking results. If agents are relatively risk averse (meaning more risk averse than with logarithmic utility), the long-run Phillips curves are always downward sloping. The inflation target that maximizes welfare can vary dramatically across Phillips curves within a given family, being the target that achieves either full employment or the Friedman rule (the rate that drives the gross nominal interest rate to one), or some inflation rate in between. For a central bank with a dual mandate that requires balancing an inflation goal against an unemployment goal, the weight that should be given to each goal depends on the parameters for the frictions, the opportunity cost of employment, and the disutility of labor. Equal weight on each goal is not in general optimal. For example, if the disutility of labor is sufficiently high, the Friedman rule is always optimal.

If instead agents are less risk averse (meaning that they have log preferences), the Phillips curves are vertical at a natural rate of unemployment that depends only on real variables and the labor-market frictions but is independent of financial-market frictions. The Friedman rule is always optimal, in contrast to the case where agents are relatively risk averse. As a result, a central bank should put weight only on its inflation objective.

Comparing the optimal inflation target across Phillips curves in the Section 2 economy shows that, regardless of agents' risk aversion, agents are better off when the frictions are low or when agents have good alternatives to employment (that is, the opportunity cost of employment is high). A central bank, however, cannot control those factors.

Section 3 presents a variant of the model that allows for capital formation. If agents are relatively risk averse, the additional complexity of the model makes closed-form solutions unobtainable. Numerical analysis indicates, however, that the long-run Phillips curves are upward sloping.[3] The welfare-maximizing inflation target is the lowest inflation rate achievable and for which an equilibrium exists, which means that a central bank with a dual mandate should

[3] This is consistent with Friedman (1977), which finds evidence in cross-country data supporting an upward-sloping Phillips curve.

put all its emphasis on its inflation goal. This inflation rate might be the one that achieves the Friedman rule, but it might not be. Along some Phillips curves, there is no steady-state equilibrium under the Friedman rule. Intermediate values of the parameters governing the frictions and the opportunity cost of employment are optimal as well because if those parameters are too low, then no steady-state equilibrium exists, and if they are too high, then the steady-state inflation rate is associated with a higher steady-state unemployment rate than the ones achievable if those parameters were lower. Alternatively, if agents have log preferences (i.e., are not too risk averse), then as in the model without capital formation, the Phillips curve is vertical at the natural rate of unemployment, and the Friedman rule is always optimal. However, with capital formation, the natural rate of unemployment depends on financial-market frictions as well as labor-market frictions.

In addition to these results, several lessons stand out from Sections 2 and 3. The first is that monetary policy influences the performance incentives (e.g., the wage rate, incentive pay) that firms offer through policy's effect on worker's financial-market returns and thus their incentive to exert full effort on the job. This is explicit in the model and not surprising. The decline in equity prices after the September 11, 2001, terrorist attacks in the United States drove some retired workers back into the labor market and caused others to postpone retirement. That type of impact on worker incentives reduces the wages firms must offer to induce work effort.

A second and related lesson is that monetary policy can have real effects even in the long run without any nominal rigidities or unanticipated inflation. This is consistent with some research indicating that most U.S. firms adjust prices fairly frequently, suggesting few nominal rigidities, at least for prices.[4] In addition, theoretical research suggests that with perfect foresight, there would be no inflation-unemployment trade-off; the Phillips curve would be vertical.[5]

A third lesson from the model is that parameters characterizing fundamental features of labor and financial markets are critical to an economy's inflation-unemployment relationship, altering the position and slope of the long-run Phillips curves.[6] These same parameters, along with the disutility of labor effort, are critical to determining the welfare-maximizing inflation

[4] Bils and Klenow (2004) find that half of prices are changed every 5.5 months or less. Dennis (2006) finds that about 60 percent of firms change prices each quarter, although most also use a rule-of-thumb in setting prices.
[5] See Lucas (1972) and Kydland and Prescott (1977).
[6] Similarly, although in a model abstracting from unemployment, Davig (2007) finds that changing costs of price adjustment change the slope of the Phillips curve, with lower costs making the curve steeper.

target. The implication is that the impact of a central bank's choice of a particular inflation target on employment is dependent on such parameters. Because these parameters are difficult to estimate, a central bank is challenged in choosing its inflation target, regardless of the formality with which such a target is specified. In particular, the Friedman rule is not necessarily optimal.[7] In some cases, a low and positive inflation rate is optimal, and in other cases an even higher inflation rate is optimal.[8]

Fourth, the analysis when agents are not too risk averse (i.e., have log preferences) highlights features of vertical Phillips curves. First, the position of a Phillips curve that is vertical at the natural rate of unemployment depends on labor-market and financial-market fundamentals. Consequently, if parameters characterizing these frictions and fundamentals change, the vertical Phillips curve also shifts. This is consistent with empirical evidence on the volatility of the natural rate of unemployment.[9] Second, the possibility of capital formation enhances the impact of financial-market frictions on the natural rate of unemployment.

A final striking lesson from the model is the pervasiveness of the effect unemployment has on the economy. The model teaches that when the possibility of involuntary unemployment is added to a random-relocation model, aggregate income and aggregate deposits depend on the level of employment. The lower the unemployment rate, the more deposits banks have available to invest in the economy's assets. Therefore, the employment and unemployment rates, the wage rate, and the opportunity cost of employment, which jointly determine the volume of deposits, enter into banks' asset demand functions. In contrast, when capital is added to a simple random-relocation model, abstracting from any employment decisions, banks have an additional asset (capital) in which they can invest and thus an additional first-order condition, but their demand functions for other assets are no more complex.[10]

There is relatively little work that explicitly introduces micro-foundations for

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[7] Mishkin (2007) summarizes the reasons why models have found that a zero or negative inflation rate is not optimal.

[8] These findings are consistent with existing research. Billi (2007) finds the optimal inflation rate to be positive but robustly below 1 percent. Akerlof et al. (1996) and Andersen (2002), for example, find that a positive and low inflation rate, something around 2 percent or 3 percent, is optimal. Cavalcanti and Villamil (2003) finds that in an economy with structural imperfections, such as the existence of an underground economy, the optimal inflation rate is between 0 percent and 22 percent.

[9] King and Morley (2007) finds that in post-war U.S. data, that the natural rate of unemployment, measured as the time-varying steady state of a structural vector autoregression, is quite volatile, and its movements can be related to variables associated with labor-market search, labor productivity, real wages, and sectoral shifts in the labor market.

[10] Schreft and Smith (1997) model the latter. Agents are endowed with one unit of time that they supply inelastically to production.

unemployment and money demand. However, our work is most closely related to the overlapping generations model of Jullien and Picard (1998). As in our framework, moral hazard in the labor market leads to involuntary unemployment. Individuals only have two stages of life—the two periods in the model. Yet, they value consumption in each. Consequently, workers' incentives are closely tied to the desire for intertemporal substitution. Nevertheless, money is the only store of value in their economy. Moreover, individuals are only subject to the income risk from the labor market. In our setting, multiple assets serve as a store of value, and individuals are susceptible to liquidity risk. In particular, money is dominated in rate of return. The presence of liquidity risk plays a key role in motivating workers' incentives in our model. As a result, inflation is an important "worker discipline device" in our framework.

Moreover, in contrast to our analysis, the Phillips Curve is upward sloping in Jullien and Picard. However, Fair (2000), Akerlof et al. (2000), and King and Watson (1994) present evidence that the long-run Phillips Curve is downward sloping. King and Watson also show that the Phillips curve's elasticity adjusts to changing economic circumstances. Akerlof et al. finds that the tradeoff is strongest at lower inflation rates and disappears at sufficiently high inflation rates. DiNardo and Moore (1999) find a long-run tradeoff for nine OECD countries.

Following from Jullien and Picard, our work contributes to a growing literature incorporating frictions in the labor market in overlapping generations economies. For example, Bencivenga and Smith (1997) construct a model with adverse selection in the labor market to show that frictions are an important factor in unemployment during the process of urbanization. Betts and Bhattacharya (1998) demonstrate that adverse selection in the labor market magnifies distortions from credit rationing, inhibiting the process of economic development. Bhattacharya and Chakraborty (2005) expand on Betts and Bhattacharya by studying the implications of moral hazard in the labor market for unemployment and credit activity.

The remainder of the paper is as follows. Section 2 presents our benchmark model, abstracting from capital accumulation. Section 3 expands to a neoclassical growth framework. Section 4 provides some concluding remarks.

2. A Model of the Phillips Curve

This section presents a two-asset overlapping generations model in which the presence of moral hazard and use of performance incentives in the labor market results in monetary policy having real effects and generates either a downward-sloping or vertical long-run Phillips curve, depending on agents' risk aversion, which affects the elasticity of the demand for money. As a benchmark, we begin by abstracting from capital formation.

2.1 The Environment

The economy exists at discrete dates $t = 1, 2, \ldots$ and consists of two separate geographic locations. Each location is populated by an infinite sequence of identical two-period-lived generations. Each new generation of young agents is of unit mass. Each location also has two sectors, known as the primary and secondary sectors, each with its own labor market, following Bulow and Summers (1986).

Agents born at $t \geq 1$ are each endowed with one unit of labor time when young, which they supply to the primary labor market unless they are unemployed. If they obtain employment in the primary labor market, they either exert full effort or shirk, exerting no effort. Their effort on the job, represented by $e_t \in \{0,1\}$, is private information as in Shapiro and Stiglitz (1984) and is the source of one friction in the economy. An agent's disutility from formal employment is given by $v > 0$, where v is assumed to be identical for all young agents and known to firms. Those young agents who are unemployed, having failed to obtain a job in the primary labor market, "work" in the secondary labor market, which can be thought of as home production.[11] Effort exerted in home production is costless.

Agents when old derive utility from consumption of the economy's single consumption good. The utility of a member of generation *t* is represented by the function

$$U(c_{t+1}, e_t) = \begin{Bmatrix} u(c_{t+1}) - ve_t & \text{if } e_t = 1 \\ u(c_{t+1}) & \text{if } e_t = 0 \end{Bmatrix}, \tag{1}$$

where c_{t+1} denotes the agent's consumption. In particular, utility from consumption is:

[11] Alternatively, the secondary sector can be thought of as the minimum-wage sector, and unemployment can be interpreted as the lack of employment in the primary sector.

$$u(c_{t+1}) = \begin{Bmatrix} \dfrac{c_{t+1}^{1-\theta}}{1-\theta} \text{ if } \theta > 1 \\ \ln c_{t+1} \text{ if } \theta = 1 \end{Bmatrix}. \quad (2)$$

Attention is restricted to values of $\theta \geq 1$, consistent with Schreft and Smith (1997, 1998) and empirical estimates of individuals' degree of risk aversion, which are typically around two.[12]

At the end of each period, a randomly selected fraction, $\pi \in (0,1)$, of young individuals learns that they will be forced to relocate to the other island at the beginning of the next period. The probability of relocation, π, is exogenous, publicly known, the same across both islands, and independent of a worker's employment status.

2.1.1 Employment and the Labor Market

In the primary labor market, firms have monitoring technologies that sometimes allow them to detect shirking behavior.[13] The conditional probability of a worker being caught shirking is *q*. Workers caught shirking are fired, while those not caught receive the real wage w_t from the firm. Unemployed workers supply their labor time to home production, for which they receive a "wage" of τ. Without loss of generality, τ is time invariant.[14] It is also assumed to be positive and finite, which is necessary for wages in the primary market to be positive and finite, as will be apparent below. All unemployment is involuntary in that agents are ex ante identical, but only a fraction of them is employed in the primary sector and earns the higher wage rate w. See Bulow and Summers (1986).

In standard efficiency-wage models, a worker's effort is determined by (i) the severity of the information friction (i.e., the inefficiency of monitoring) and (ii) the disutility of labor effort. Here, in contrast, the financial returns from working also play a role. For firms to maximize profits, the wage rate must be set to induce workers to exert full effort. This occurs when the

[12] For example, using Italian data, Chiappori and Paiella (2006) compute a median estimate equal to 1.7. From the consumer expenditures survey, Mazzocco (2006) obtains similar findings.

[13] In subsequent sections, where environments with capital are considered, primary sector firms operate constant-returns-to-scale technologies and earn zero profit. However, in this section, where labor is the only explicit factor of production, firms can earn positive profits. To keep the model tractable, firms in this section are taken to be a second type of agent, each with a linear utility function and consuming any profits earned. These agents will be ignored in the welfare analysis to follow; however, as will be shown, profits are higher when the expected utility of the two-period lived agents is higher, so monetary policies that maximize the latter's expected utility also would make these firms better off.

[14] If the model were extended to include more periods in the lifecycle, individuals could rely on their accumulated interest income in addition to income from home production.

utility from working and exerting full effort is at least as great as the expected utility from working and shirking, each of which depends on the rate of return on deposits, R_t^d :[15]

$$U\left(R_t^d w_t\right) - v \geq (1-q)U\left(R_t^d w_t\right) + qU\left(R_t^d \tau\right). \tag{3}$$

This expression reduces to

$$q\left[U\left(R_t^d w_t\right) - U\left(R_t^d \tau\right)\right] = v \tag{4}$$

because profit maximization requires that (3) be satisfied with equality. The connection between the rate of return on deposits and labor-market performance parameters is apparent, and can be summarized as follows:[16]

Lemma 1: ***The Nonshirking Wage Rate***

Given workers' preferences, as represented by (1), satisfaction of (4) yields the nonshirking wage:

$$w_t^{ns} = \left[\frac{\tau^{(\theta-1)}}{1-(\theta-1)\frac{v}{q}\left(R_t^d\right)^{(\theta-1)}\tau^{(\theta-1)}}\right]^{\frac{1}{\theta-1}}. \blacksquare \tag{5}$$

For a given rate of return on deposits and $\theta > 1$, the response of the nonshirking wage to labor-market conditions follows standard efficiency-wage models. If labor-market frictions, measured by the ratio v/q, are more severe, workers have less incentive to work and firms must pay a higher wage to prevent shirking. These frictions are higher when the disutility of labor effort (v) is higher and/or when the technology for monitoring workers is less efficient (q lower). The nonshirking wage is also higher when the alternative to work in the primary labor market, measured by τ, the secondary market wage, is higher.

Interestingly, in contrast to standard efficiency-wage models, financial-market conditions and the degree of risk aversion also affect the nonshirking wage. If agents are sufficiently risk

[15] Although the nonshirking wage provides conditions under which workers will not shirk, it does not guarantee that workers will choose to work instead of being unemployed (employment in the secondary sector). Workers participate in the primary labor market if $U\left(R_{t+1}^d w_t\right) - v \geq U\left(R_{t+1}^d \tau\right)$ is satisfied. This participation constraint always holds because $v(1-q)/q \geq 0$.

[16] Campbell (1993) presents empirical evidence supporting firms' payment of efficiency wages.

averse $(\theta > 1)$, a higher rate of return to deposits lowers the cost of shirking. Those who obtain a job in the primary sector can self-insure against the risk of job and income loss by exerting effort on the job. However, asset returns affect the desire to self-insure. For example, when returns to saving are higher, agents have less need to insure against risky labor income, so firms must pay workers more to elicit labor effort. Thus, the model has a novel channel or transmission mechanism through which monetary policy operates to have real effects, as discussed further below.

2.1.2 Production and the Goods Market

Production of the economy's single consumption good occurs in both sectors. In the secondary sector, output is simply τ. In the primary sector, the good is produced by firms that have access to a technology that converts effective labor, $(1-\psi)L_t$, into output, Y_t, of the consumption good according to the function

$$Y_t = F\left((1-\psi)L_t\right) = A\left((1-\psi)L_t\right)^{1-\eta}, \tag{6}$$

where ψ is the fraction of employees who exert no effort (shirk) and L is the total number of employees. A is a productivity factor, which can be interpreted as incorporating the contribution of an exogenous and nondepreciating fixed stock of capital.[17] Satisfaction of (5) implies that $\psi = 0$.

The population of firms is of mass one. Therefore, the demand for labor by a representative firm is the same as the market demand. The number of workers hired comes from the implicit demand function for labor:

$$w_t = A(1-\eta)(1-\psi)^{1-\eta}\left(L_t\right)^{1-\eta}. \tag{7}$$

In turn, the unemployment rate is given by

$$\mu_t \equiv 1 - L_t. \tag{8}$$

2.1.3 Assets and the Banking Sector

There are two primary assets in this simple economy: money (fiat currency) and a linear storage technology. The real rate of return to the storage technology is constant and given by R;

[17] See, for example, Gray (1976).

in nominal terms, it is represented by I_t. The aggregate nominal stock of money in each location at date *t* is given by M_t. The monetary authority selects a gross inflation target of σ and achieves that target by allowing the money supply to grow at the fixed rate of σ; hence, $M_t = \sigma M_{t-1}$. The price level at *t* is p_t, and $m_t = M_t / p_t$ is the real money stock. The initial money supply held by each old agent of the initial generation is $M_0 > 0$.

Two additional features of the environment give rise to trading frictions. First, only currency can be transported across locations. Second, agents cannot communicate across islands. These features prevent private credit markets from operating and, along with the risk of relocation, give rise to a transactions demand for money in that agents demand a liquid asset that they can take with them when they move, even if money is dominated in rate of return. Because the fraction of agents who relocate is known even though the identity of who must move is not, a role for banks arises to provide insurance against the risk of relocation, as in Diamond and Dybvig (1983). Banks hold the economy's assets directly, with young agents depositing all of their earnings, whether from work in the primary sector or home production.

In contrast to standard random-relocation models (e.g., Schreft and Smith 1997, 1998), deposits in this model vary across agents because some are employed in the primary sector while the others, unemployed, engage in home production. Employed agents deposit $w_t^{ns} L_t$, and unemployed agents deposit $\tau(1 - L_t)$. Agents' employment status when young can be shown to be immaterial for a bank's allocation of their deposits across assets. Banks acquire money balances of m_t^d and invest i_t^d in the storage technology. As a result, the bank's balance sheet condition is:

$$m_t^d + i_t^d \leq w_t^{ns} L_t + \tau(1 - L_t), \quad t \geq 1. \tag{9}$$

There is free entry into the banking sector, and banks are Nash competitors in the market for deposits, announcing rate of return schedules for deposits and taking the announced schedules of other banks as given. Banks thus announce deposit-return schedules (r_t^m, r_t^n), where r_t^m and r_t^n are the rates of return paid to relocated agents, "movers," and to "nonmovers," respectively, and

$$R_t^d \equiv \pi r_t^m + (1 - \pi) r_t^n$$

is the rate of return on deposits.

Profit-maximization requires that banks maximize the expected utility of a representative depositor. That is, $\left(r_t^m, r_t^n\right)$ is chosen to maximize

$$L_t\left[\frac{\pi}{1-\theta}\left(r_t^m w_t^{ns}\right)^{1-\theta}+\frac{1-\pi}{1-\theta}\left(r_t^n w_t^{ns}\right)^{1-\theta}\right]-vL_t+\left(1-L_t\right)\left[\frac{\pi}{1-\theta}\left(r_t^m \tau\right)^{1-\theta}+\frac{1-\pi}{1-\theta}\left(r_t^n \tau\right)^{1-\theta}\right], \quad (10)$$

given the preferences specified in (2) for $\theta > 1$ and the fact that a fraction L_t of agents is employed and a fraction $1-L_t$ is unemployed. The deposit-return schedule must satisfy two conditions in addition to the bank's balance sheet constraint. First, a bank's currency holdings must be sufficient to meet the demand for money by the fraction π of agents that must move:

$$\pi r_t^m \le \left(\frac{m_t^d}{w_t^{ns} L_t + \tau\left(1-L_t\right)}\right)\left(\frac{p_t}{p_{t+1}}\right), \quad (11)$$

where p_t / p_{t+1} is the rate of return on real balances between *t* and *t* +1. Second, because the focus in analyzing this economy is on equilibria in which money is dominated in rate of return, which requires $I_t > 1$ for all *t*, banks hold only enough currency to satisfy the withdrawal demand of movers, which implies that (11) is satisfied with equality and that investment in storage on behalf of nonmovers satisfies

$$\left(1-\pi\right) r_t^n \le \left(\frac{i_t^d}{w_t^{ns} L_t + \tau\left(1-L_t\right)}\right) R. \quad (12)$$

Since bank profit-maximization requires that (9) be satisfied at equality, constraints (11) and (12) can be rewritten as

$$r_t^m \le \frac{\gamma_t p_t}{\pi p_{t+1}}, \quad (13)$$

and

$$r_t^n \le \frac{\left(1-\gamma_t\right)}{1-\pi} R, \quad (14)$$

where $\gamma_t \equiv m_t^d \Big/ \left(w_t^{ns} L_t + \tau\left(1-L_t\right)\right)$ is the bank's reserve-deposit ratio and $1-\gamma_t$ is its storage investment-deposit ratio. Since holdings of currency and storage must be nonnegative, $\gamma_t \ge 0$.

It follows that the solution to the bank's optimization problem is

$$\gamma_t \equiv \gamma\left(I_t\right) = \frac{1}{1+\left(\frac{1-\pi}{\pi}\right)\left(I_t\right)^{\frac{1-\theta}{\theta}}}, \quad \forall I \geq 1. \tag{15}$$

When $I_t = 1$, which occurs under the Friedman rule, γ is indeterminate because there is no opportunity cost of holding currency, leaving banks indifferent between currency and storage investments. Hereafter, although $\gamma_t \geq \pi$ when $I_t = 1$, $\gamma_t = \pi$ is taken to be the demand for real balances in that case.

Given $\gamma\left(I_t\right)$, the rate of return on deposits is

$$R_t^d = \left(1-\gamma\left(I_t\right)\right)R + \gamma\left(I_t\right)\frac{p_t}{p_{t+1}}. \tag{16}$$

With $\theta > 1$, $\partial\gamma\left(I\right)/\partial I > 0$, indicating that the income effect from a change in the nominal interest rate on the demand for real balances dominates the substitution effect. When $\theta = 1$, money demand is perfectly inelastic. Both cases will be considered below.

2.1.4 The Timing of Economic Activity

The features of the economy described above are consistent with the following timing of events each period. At the beginning of each date, a new generation of workers is born. The central bank injects currency to implement its chosen monetary policy, and the return to storage investments made the previous period is realized. Old agents who did not relocate withdraw their deposits. Next, firms choose the number of workers to hire and a wage rate to attract the desired number of workers and ensure full effort from each. Young agents either receive job offers at the going wage or are involuntarily unemployed and involved in home production. Production occurs in the primary sector and within the homes of the unemployed, and then the goods market operates. Old agents who relocated trade their currency holdings for goods, and all old agents then consume. Young workers receive their earnings in the form of goods, all of which they deposit. All young agents, regardless of their employment status, learn whether they must relocate. Those who must move withdraw their deposits in the form of currency before moving.

2.2 Equilibrium with $\theta > 1$: A Phillips Curve Tradeoff

In analyzing equilibria of this economy, attention is restricted to steady states. With an inflation-targeting central bank conducting monetary policy according to the fixed-money-growth-rate rule specified in section 2.4, in a steady state the real rate of return on currency, p_t / p_{t+1}, equals $1/\sigma$, and $I = R\sigma$. It follows that monetary policy affects the steady-state rate of return on deposits, as shown in (16), in two ways. First, a higher inflation rate lowers money's rate of return, $1/\sigma$, which directly lowers the return to deposits given the bank's demand for money. Second, higher inflation drives up the nominal interest rate. With $\theta > 1$, agents are relatively risk averse (more so than with logarithmic preferences), so the income effect of a change in the nominal interest rate dominates the substitution effect. Consequently, higher inflation causes banks to partially insure movers against inflation's effects by holding more real balances. Because money is dominated in rate of return, this results in a lower rate of return on deposits and thus a lower rate of return to nonmovers.

2.2.1 Steady States

To obtain a closed-form solution for the steady state, $\theta = 2$ is assumed. With this assumption, the steady-state expected rate of return on deposits is

$$R^{d*} = \left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right), \tag{17}$$

and the nonshirking wage is

$$w^{ns} = \left(\frac{\tau}{1 - \frac{v}{q} R^d \tau} \right). \tag{18}$$

Substituting the expected return to deposits into (18) leads to the following lemma:

Lemma 2: ***The Steady-state Nonshirking Wage Rate***

If the disutility of effort (v) is sufficiently low or the probability of detecting shirking (q) is sufficiently high, then the nonshirking wage rate is finite and given by

$$w^{ns*} = \left[\frac{\tau}{1 - \tau \frac{v}{q} \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right)} \right]. \tag{19}$$

Proof. For the wage rate to be positive and finite, the denominator of (19) must be positive. Therefore, it must be the case that

$$\frac{q}{v} > \tau \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right). \blacksquare$$

In this economy, the nonshirking wage is decreasing in the economy's inflation rate. As the inflation rate rises, the expected return to deposits falls, which reduces the return to labor effort. To offset this loss, agents must be willing to put forth full labor effort. The resulting impact on employment is provided in the following proposition:

Proposition 1: ***Existence of a Long-run Phillips Curve Trade-off***

If the disutility of labor effort (v) is sufficiently low or the probability of detecting shirking (q) is sufficiently high, so that the nonshirking wage (w^{ns}) is positive and finite, then a steady state with equilibrium unemployment exists and is unique. Steady-state employment is given by

$$L^* = \left(\frac{(1-\eta) A}{\tau} \right)^{\frac{1}{\eta}} \left[1 - \tau \frac{v}{q} \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right) \right]^{\frac{1}{\eta}}. \tag{20}$$

From Proposition 1, it follows that $\partial\mu/\partial\sigma \equiv \partial\left(1-L_t\right)/\partial\sigma < 0$: the Phillips curve is always downward sloping.■

Because at higher inflation rates the rate of return to deposits (saving) is lower, agents have more incentive to work at such inflation rates and equilibrium employment is higher. This is the case even though firms pay lower wages, giving workers less incentive to perform. With less employment, output is lower. This is the mechanism that generates a steady-state inflation-unemployment tradeoff and real effects of monetary policy.

Two employment levels will be of particular interest in analyzing the welfare-maximizing monetary policy. Full employment (that is, $L^* = 1$) requires, from (20), that

$$\frac{v}{q} = \left[\frac{A\left(1-\eta\right)-\tau}{A\left(1-\eta\right)\tau}\right]\left[\frac{\sqrt[2]{\sigma R}+\dfrac{1-\pi}{\pi}}{\left(\dfrac{1-\pi}{\pi}\right)R+\sqrt[2]{\dfrac{R}{\sigma}}}\right]. \tag{21}$$

At higher values of *v/q* and τ, the inflation target that achieves full employment is also higher. For $\pi < 0.5$, as π increases, the full-employment inflation rate is higher. For $\pi > 0.5$, the opposite is the case.

When inflation is set to achieve the Friedman rule (that is, $\sigma = 1/R$),

$$L^* = \left(\frac{A\left(1-\eta\right)}{\tau}\right)^{\frac{1}{\eta}}\left(1-\tau\frac{v}{q}R\right)^{\frac{1}{\eta}}. \tag{22}$$

At the Friedman rule, agents are fully insured against liquidity risk; hence, the financial-market friction does not affect agents' incentives on the job or unemployment. However, higher *v/q* and τ are associated with higher unemployment.

2.2.2 Optimal Monetary Policy

Closed form solutions cannot be obtained for the optimal inflation target because welfare is sensitive to the parameters reflecting the labor and financial-market frictions and corner solutions abound. As in standard random-relocation models, the gross nominal interest rate must exceed one in any equilibrium because the demand for real balances is indeterminate otherwise. In addition, with the possibility of unemployment, as here, and a population of young agents of

mass 1, employment must be between zero and one in equilibrium. Consequently, the optimal monetary policy is identified numerically instead.

For the numerical analysis, $\theta = 2$, $\eta = 0.3$, and $R = 1.02$ are used, consistent with U.S. data or empirical estimates. As a baseline, $\pi = 0.5$, $\tau = 0.5$, and $v/q = 1.0$. The Phillips curves, consisting of inflation rate-unemployment rate combinations that are steady states, depend on the ratio *v/q*, not on *v* or *q* independently. Welfare, however, measured as the expected utility of a representative agent, must be calculated to identify the optimal monetary policy along and across the Phillips curves.[18] It does depend on *v* through the disutility of labor effort, as indicated in equation (1). The welfare analysis is discussed for two representative values of *v*: 0.001 and 0.5. *A*, generally calculated as a residual (Abel and Bernanke, 1998), is treated as a residual here too and set to 1.25 so there exists an equilibrium with a reasonable unemployment rate at an inflation rate (target) of 2 percent (that is, $\sigma = 1.02$). Because agents are either employed or unemployed in the model—no one is out of the labor force—the unemployment rate in the model is the equivalent of the measured share of the civilian population that is unemployed plus the share not in the labor force. Thus, a reasonable equilibrium unemployment rate is 36 percent, given a labor force participation rate of about 64 percent in the data.[19]

For these parameterizations, families of Phillips curves are graphed as functions of the labor-market friction parameters (τ and *v/q*) and the financial-market friction parameter (π), given the disutility of labor (*v*). For each Phillips curve, welfare is calculated for each point along the curve, and the welfare-maximizing combination is identified. Figures 1 through 6 show the steady states and optimal monetary policies for $v = 0.001$, while Figures 7 through 12 do so for $v = 0.5$. As is discussed below, an equilibrium with full employment, at the Friedman rule, or somewhere in between, can be optimal, implying a corresponding optimal monetary policy.[20]

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[18] This concept of the optimal monetary policy is consistent with the Federal Reserve's (Mishkin 2007b).

[19] The assumed values for σ, π, *R*, and θ with $v = 0.001$ are also an equilibrium for a 5 percent unemployment rate, with $A = 1.405$. The article's qualitative conclusions do not change if this value of *A* is used in the numerical analysis.

[20] As discussed in footnote 13, firm profits can be positive in this version of the model. In measuring welfare as the expected utility of a representative agent, meaning a two-period lived agent, the expected utility of the one-period-lived firms is ignored. However, higher inflation always results in higher profits and higher utility for the firms. As a result, any weight put on the firms' expected utility in determining the optimal monetary policy would reinforce the optimality of the full-employment equilibrium when that equilibrium maximizes the expected utility of a representative agent. In contrast, when an equilibrium with a relatively low inflation rate maximizes the expected utility of a representative agent, such as when the Friedman rule is optimal, the welfare of firms is relatively low,

Figure 1 displays part of the family of Phillips curves obtained by varying the opportunity cost of employment (the secondary wage rate, τ), given a low disutility of labor. For the parameters studied, steady states do not exist for $\tau < 0.47$ because the opportunity cost at that point is so low that $L > 1$ and $\mu < 0$ is required for a steady state to exist, which is not feasible. For $\tau \in [0.47, 0.502]$, the optimal inflation rate achieves full employment. At $\tau = 0.5$, for example, an inflation rate of 39 percent maximizes the expected utility of a representative agent. For $\tau \in [0.503, 0.507]$, the optimal inflation rate lies between the full-employment rate and the Friedman Rule. For $\tau \in [0.508, 0.98]$, the Friedman rule is optimal, with the highest unemployment rate achievable. When τ is in that range, unemployment is a relatively desirable state for many agents. For $\tau > 0.98$, the alternative to working in the primary sector is so lucrative that unemployment is too desirable—the unemployment rate that solves the model is negative, which is not feasible—so no steady state exists.

With the optimal inflation rate so dependent on the opportunity cost of employment, a natural question to ask is how expected utility compares across Phillips curves. Figure 2 graphs expected utility under the optimal monetary policy for each Phillips curve in Figure 1 (for each τ). Expected utility is highly nonlinear in τ, depending on the fraction of agents working versus not working and the marginal utility from employment versus unemployment, and each component moves in opposite directions. Hence, expected utility under the optimal monetary policy is convex, with $\tau = 0.98$ maximizing welfare across values of τ, as shown in Figure 2.

What values of the opportunity cost of employment are most likely to be observed? τ can be thought of as the value of home production (for example, the cost that a worker not employed in the primary market saves by doing housekeeping rather than paying a housekeeper and perhaps of caring for children rather than paying a nanny), the income earned at a minimum-wage job, or unemployment compensation. This suggests that lower values of τ are more likely to be observed than higher ones, which in turn suggests that the full-employment monetary policy is more likely to be the optimal monetary policy than the Friedman rule.

Figure 3 shows the family of Phillips curves that arises as labor-market frictions associated with the disutility of labor and the effectiveness of firm monitoring of worker effort,

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offsetting to some extent the optimality of that equilibrium. Therefore, considering the welfare of the firms makes the Friedman rule less likely to be an optimum than the analysis here indicates.

represented by the ratio v/q, vary. For $v/q < 0.85$, no steady state exists with a nonnegative unemployment rate. For $v/q \in [0.85,1.05]$, full- or near-full employment is optimal. For $v/q \in [1.06,1.145]$, the optimal steady state has less than full employment and inflation higher than the Friedman rule. As v/q increases beyond 1.145, the Friedman Rule is always optimal.

Not surprisingly, welfare is higher when the labor-market frictions represented by v/q are lower, as Figure 4 shows. Agents prefer full employment and the low inflation rate available when $v/q = 0.85$, which also allows the nominal interest rate to be fairly low, limiting the welfare cost of having to relocate, over all other steady states, even the one where the Friedman rule is followed and financial frictions are neutralized.

It is tempting to conclude that better firm monitoring, which lowers v/q given v, is desirable. However, in reality, v rises when q rises: more intense and accurate firm monitoring lowers job satisfaction, which equates to greater disutility of labor. See, for example, Ottensmeyer and Heroux (1991) and Wood (1998).

Figures 5 and 6 display the last set of numerical results for this economy with $v = 0.001$. Figure 5 shows how the Phillips curves vary with the financial-market friction, π. For $\pi \geq 0.5$, the full-employment steady state is optimal. The higher π is in that range, the lower the inflation rate associated with that equilibrium. For $\pi < 0.5$, the unemployment rate rises at the welfare-maximizing steady state. Initially, the inflation rate also rises, but eventually it declines with reductions in π. As π approaches zero, the optimal monetary policy approaches the Friedman rule.[21]

Figure 6 displays expected utility under the optimal monetary policy as π varies. Welfare is maximized at $\pi = 1.0$, meaning that all agents must relocate at the end of their youth. Welfare is minimized as $\pi \to 0.0$, where no agents relocate. Interestingly, the steady state achieved when $\pi = 1.0$ is much like that as $\pi \approx 0.0$ in that there is virtually no uncertainty about relocation, or equivalently, there are no liquidity shocks. Banks' portfolios are very different across the two steady states, however. When $\pi = 1.0$, banks hold all their assets in the form of currency, whereas at $\pi \approx 0.0$, hardly anyone relocates, so banks invest almost all of their assets in storage. In the latter case, the inflation rate has almost no effect on the rate of return on deposits. The welfare gain from achieving the Friedman rule when $\pi \approx 0.0$ is insufficient to

[21] At $\pi = 0$, the wage rate is undefined.

offset the welfare cost of the high unemployment rate that must obtain for the Friedman rule to be the steady-state inflation rate.

Figures 7 through 12 show the steady states and optimal monetary policies when the disutility of labor effort is relatively high (that is, at $v = 0.5$). The Phillips curves shown in Figures 1, 3, and 5 are identical to those in Figures 7, 9, and 11, respectively: v independent of q does not affect the inflation-unemployment relationship. Only expected utility differs at the higher value of v, and differs sufficiently that the Friedman rule is the optimal policy along each Phillips curve in Figures 7, 9, and 11, and a central bank should put full weight on its inflation objective. Intuitively, as the inflation rate rises, the unemployment rate falls, so employment is higher. With a higher v, the disutility from labor effort of the representative agent is lower, even though the utility from consumption is not altered. As a result, welfare is lower at higher inflation rates for higher v than for lower v, other things equal.

Figures 8, 10, and 12 show expected utility under the optimal monetary policy across the Phillips curves in Figures 7, 9, and 11, respectively. The values of τ and v/q that maximize expected utility are the same regardless of the value of v, as seen by comparing Figure 8 to Figure 2 and Figure 10 to Figure 4. That is not the case for π, however. Expected utility is highest when $\pi = 1.0$ when the disutility of labor effort is low ($v = 0.001$, Figure 6) but is independent of π when the disutility of labor effort is high ($v = 0.5$, Figure 12). When work is sufficiently unpleasant that the welfare-maximizing steady state is always at the Friedman rule, as is the case in Figure 11, the gross nominal interest rate is 1.0, which neutralizes the impact of the financial-market friction, π, whatever its value. Hence, welfare is the same at the Friedman rule regardless of the value of π.

It is not clear what values are reasonable for the disutility from labor effort. The true value could vary by job (e.g., v might be lower for empowered workers in pleasant work environments), based on national culture (e.g., differences between American workers and their European counterparts), or over time for a variety of reasons (e.g., a worker's age, trends in employer-provided benefits). There seems little way for a central bank to know the true value of v, yet the value matters if a central bank is to adopt the optimal inflation target. A central bank that knows τ, v/q, and π still might not know v and therefore would not know how to conduct monetary policy to maximize welfare.

2.2.3 Discussion

The numerical analysis of the optimal policy is striking because of the extent to which labor-market and financial-market frictions affect the optimal inflation target. The optimal monetary policy can be the inflation target that achieves full employment or the Friedman rule or some target in between. The greater the disutility of labor, the more likely it is that the Friedman rule is optimal.

One implication of these results is that an inflation target chosen without good estimates of the magnitude of the frictions, the opportunity cost of employment, or the disutility from labor could achieve a steady state far from the one that maximizes welfare, contrary to a central bank's intentions. In addition, the weight that a central bank with a dual mandate puts on inflation versus unemployment depends on the labor-market and financial-market frictions present. When the optimal policy is to achieve full employment, the central bank should emphasize its employment goal, but when the Friedman rule is the optimal monetary policy, all weight should be put on price stability. When optimal monetary policy involves an inflation target somewhere between the full-employment rate and the Friedman rule, the central bank must balance its dual goals.

2.3 Equilibrium with $\theta = 1$: A Vertical Phillips Curve

When $\theta = 1$, $\gamma = \pi$ from (15): money demand is perfectly inelastic, so the income and substitution effects of a change in the nominal interest rate offset each other. In addition, the nonshirking wage rate only depends on the severity of labor-market frictions:

$$w^{ns*} = \tau e^{\frac{v}{q}}. \tag{23}$$

This implies that agents' incentives in the labor market do not depend on financial returns. The same is true for the Phillips curve, which is vertical at a "natural" rate of unemployment that depends only on real factors:

$$\mu^* \equiv 1 - L^* = 1 - \left(\frac{\left(A(1-\eta) \right)}{\tau e^{\frac{v}{q}}} \right)^{\frac{1}{\eta}}. \tag{24}$$

With a vertical Phillips curve, the Friedman rule is always optimal, and a central bank should focus only on its price-stability goal, as the following proposition shows.

Proposition 2: ***Optimality of the Friedman Rule When*** $\theta = 1$**.**

When $\theta = 1$*, welfare is maximized when* $\sigma = 1/R$.

Proof. With $\theta = 1$, $\gamma^* = \pi$, $r^{m*} = 1/\sigma$, and $r^{n*} = R$. Expected utility of the representative agent is

$$\left[L^* \ln\left(w^*\right) + \left(1 - L^*\right)\ln\left(\tau\right)\right]\left[\pi \ln\left(r^{m*}\right) + \left(1-\pi\right)\ln\left(r^{n*}\right)\right] - vL^*. \tag{25}$$

The first bracketed term is independent of the money growth rate. The second bracketed term is $\pi \ln\left(1/\sigma\right) + \left(1-\pi\right)\ln\left(R\right)$, which is maximized when the central bank sets $\sigma = 1/R$, which drives I to one.■

3. A Model of the Phillips Curve with Capital Accumulation

This section extends the analysis of Section 2 to a neoclassical growth framework in which physical capital replaces storage as the economy's real asset. As in standard monetary models such as Stockman (1981), the economy exhibits a reverse Tobin effect. However, anticipated inflation also has an impact on investment through the labor-market effects of unemployment.

3.1 The Environment

The environment is the same as in Section 2, except that in the primary sector firms have access to a constant-returns technology that combines capital, K_t, with effective labor, $\left(1-\psi\right)L_t$, to produce output:

$$Y_t = F\left(K_t, \left(1-\psi\right)L_t\right).$$

For simplicity, capital completely depreciates at the end of the period. If $k_t \equiv K_t \big/ \left(\left(1-\psi\right)L_t\right)$ denotes the capital-*primary-sector* labor ratio (hereafter, the "capital-labor ratio"), then

$$f\left(k_t\right) \equiv A\left(k_t\right)^{\eta}, \quad A > 0, \ \eta \in \left(0,1\right). \tag{26}$$

In addition to choosing wages and employment to maximize profits, as described previously, firms also choose the amount of capital to rent. The rental decision implies that the

gross real return to capital satisfies

$$R_t = f'(k_t) = \eta A k_t^{\eta-1}, \tag{27}$$

and the demand for labor is given by

$$w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t) = (1-\eta) A k_t^{\eta} = (1-\eta) A \left(\frac{K_t}{L_t}\right)^{\eta}. \tag{28}$$

In all other respects, the economy is as in Section 2.

3.2 Equilibrium with $\theta > 1$: An Upward-sloping Phillips Curve

In a steady-state equilibrium,

$$R = \eta A k^{\eta-1}, \tag{29}$$

which provides the demand for capital:

$$k = \left(\frac{\eta A}{R}\right)^{\frac{1}{1-\eta}}. \tag{30}$$

As a result, the wage rate as a function of the rental rate on capital is

$$w(R) = (1-\eta) A \left(\frac{\eta A}{R}\right)^{\frac{\eta}{1-\eta}}. \tag{31}$$

Higher rental rates result in less capital being used per worker and thus a lower marginal product of labor in the primary sector and a lower wage rate.

From the analysis in Section 2, the nonshirking wage is

$$w^{ns}(R,\sigma) = \left(\frac{\tau}{1 - \tau \frac{v}{q} \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right)} \right). \tag{32}$$

In a steady-state equilibrium for this economy, R^* is the unique value of R that equates (31) and (32), and from (30), pins down the steady-state capital-labor ratio, k^*.

As in Section 2, the bank's balance-sheet constraint must hold with equality. In turn, this implies that steady-state investment satisfies

$$i^* = \left(1-\gamma^*\right)\left(w^*L^* + \tau\left(1-L^*\right)\right) \tag{33}$$

and that the demand for real balances satisfies

$$m^* = \gamma^*\left(w^*L^* + \tau\left(1-L^*\right)\right).$$

Total investment, i^*, is equal to the capital stock, K^*, in the steady state:

$$K^* = L^*k^*\left(R^*\right) = \left(1-\gamma^*\right)\left(w^*L^* + \tau\left(1-L^*\right)\right). \tag{34}$$

Given the economy's steady-state capital-labor ratio, $k^*\left(R^*\right)$, steady-state employment is:

$$L^*\left(R^*,\sigma\right) = \frac{\tau\left(1-\gamma\left(R^*,\sigma\right)\right)}{k^*\left(R^*\right) - \left(1-\gamma\left(R^*,\sigma\right)\right)\left(w^*\left(R^*,\sigma\right)-\tau\right)}. \tag{35}$$

From the foregoing, the effect of capital accumulation on the economy can be analyzed. As in Section 2, monetary policy affects performance incentives. At higher inflation rates, workers bear a higher cost of shirking, so wages are lower and employment is higher. However, at higher inflation rates, the nominal interest rate is higher, and with $\theta > 1$, banks hold more currency and invest less in capital formation. This is a reverse-Tobin effect. Since capital and labor are complements, less capital accumulation is accompanied by less employment and a higher unemployment rate. The net impact on employment depends on which effect dominates. Although closed-form solutions cannot be obtained, it is expected that the second effect dominates when $\theta > 1$, resulting in Phillips curves that are upward sloping.

Figures 13 through 18 present the results from numerical analysis of this economy, assuming $\theta = 2$ and $\eta = 0.335$, the latter approximately the same as its value in the analysis of Section 2. As a baseline, $\pi = 0.5$, $\tau = 0.5$, and $v/q = 1.0$ are again used. Although steady states only depend on the ratio of v to q, and expected utility also depends on v independent of q, the analysis is only presented for $v = 0.001$, for reasons presented below. R is now endogenous. The analysis is extremely sensitive to changes in A, which is set at 0.725.

Figures 13, 15, and 17 show the Phillips curve families as τ, v/q, and π, respectively, are varied. All Phillips curves are upward sloping, with the slope increasing at an increasing rate as the unemployment rate rises. The curves shift to the right as the opportunity cost of work and the friction parameters increase.

The analysis of the optimal monetary policy is the same qualitatively whether the impact

of τ, v/q, or π is studied because of the model's corner solutions and the slope of the Phillips curves. In each case, for sufficiently small values of each parameter, given the baseline values of the others, the Friedman rule cannot be implemented in a steady state because it would require the unemployment rate to be negative. Welfare maximization instead requires a central bank to set its inflation target to the lowest inflation rate for which a steady state exists (Figures 13, 15, and 17). This achieves the lowest equilibrium unemployment rate. Consequently, the qualitative results regarding the optimal monetary policy do not depend qualitatively on whether the disutility of labor (v) is low or high, so only the results for $v = 0.001$ are presented.

The critical values for the parameters are 0.2 for τ, 1.0 for v/q, and 0.5 for π (Figures 14, 16, and 18, respectively). The Friedman rule is optimal for values at and above the critical values (Figures 13, 15, and 17, respectively), at which point $I = 1.0$ and the impact of the financial-market friction is neutralized.[22] However, unemployment continues to increase in τ, v/q, and π for values above the critical levels. Intuitively, any steady state in which the Friedman rule is implemented equates the intertemporal marginal rates of substitution across movers and nonmovers, who are ex ante identical. However, the Friedman rule does not eliminate the existence of the financial-market friction; a fraction π of the population still must relocate after they finish working, and agents' awareness of this, but not of which individuals must relocate, affects their incentives to work. Agents know that banks will hold more currency and invest less in capital formation the higher is π. Because labor and capital are complements, less capital formation implies that fewer agents will be employed in equilibrium and that less output will be available to consume when old. Thus, even though agents' intertemporal marginal rates of substitution are equated in all equilibria in which the Friedman rule is achieved, unemployment is higher and welfare lower when either τ, v/q, or π is higher, given baseline values of the others. Only at the critical levels of τ, v/q, and π are financial-market frictions neutralized and income risk minimized.

For all values of τ, v/q, and π studied, a central bank with a dual mandate should focus only on achieving its inflation goal. "Price stability" should be defined as the lowest inflation rate consistent with the existence of a steady-state equilibrium. That might not be consistent with the Friedman rule, but it achieves the lowest equilibrium unemployment rate.

[22] In the analysis of Section 2, with a fixed capital stock, the Friedman rule requires setting $\sigma = 1/R$ to achieve $I = 1.0$. With capital formation, a central bank must set $\sigma > 1/R$ to achieve $I = 1.0$ if $I = 1.0$ is a steady state equilibrium.

3.3 Equilibrium with $\theta = 1$: A Vertical Phillips Curve

As in the economy without capital, when $\theta = 1$, the demand for real balances is perfectly inelastic and equal to π, independent of the rates of return on any assets. Consequently, the nonshirking wage rate only depends on the severity of labor-market frictions:

$$w^{ns} = \tau e^{\frac{v}{q}}, \tag{36}$$

and the wage condition is

$$w(R) = (1-\eta) A \left(\frac{\eta A}{R} \right)^{\frac{\eta}{1-\eta}}. \tag{37}$$

Because $w^{ns} = w(R)$ in a steady state, the steady-state real return to capital is

$$R^* = \frac{\eta}{\left(\tau e^{\frac{v}{q}} \right)^{\frac{1-\eta}{\eta}} (1-\eta)^{\frac{1-\eta}{\eta}} A^{\frac{1-2\eta}{\eta}}}, \tag{38}$$

and the equilibrium capital-labor ratio is

$$k^* = \left(\frac{\tau e^{\frac{v}{q}}}{(1-\eta) A} \right)^{\frac{1}{\eta}}. \tag{39}$$

As in the economy without capital formation, the Phillips curve is vertical at the natural rate of unemployment:

$$\mu^* \equiv 1 - L^* = \frac{\frac{1}{\tau} \left(\frac{\tau e^{\frac{v}{q}}}{(1-\eta) A} \right)^{\frac{1}{\eta}} - (1-\pi) e^{\frac{v}{q}} + (1-2\pi)}{\frac{1}{\tau} \left(\frac{\tau e^{\frac{v}{q}}}{(1-\eta) A} \right)^{\frac{1}{\eta}} - (1-\pi) \left(e^{\frac{v}{q}} - 1 \right)}. \tag{40}$$

This natural rate of unemployment is higher if labor-market frictions are more severe (q lower, v and τ higher). However, in contrast to the natural rate for the Section 2 economy, depicted in (24), this natural rate also depends on the intensity of the financial-market friction. The higher the probability of relocation, the higher the natural rate of unemployment. With a greater

fraction of depositors having to move, banks must hold more real balances and invest less in capital formation, which implies less employment in equilibrium.

The following proposition may now be stated:

Proposition 3. ***Optimality of the Friedman Rule When*** $\theta = 1$**.**

When $\theta = 1$ *and capital formation is possible, welfare is maximized when* $\sigma = 1/R$.

Proof. The proof is analogous to the proof of Proposition 2, but $r^{n*} = R^*$ from (38).■

It follows that a central bank with a dual mandate should put all its weight on its inflation objective.

4. Concluding Remarks

This article has shown that an upward-sloping, downward-sloping, or vertical long-run Phillips curve can arise in an overlapping-generations monetary model with random relocation and involuntary unemployment. The slope of the Phillips curve and optimal monetary policy depend on fundamentals of labor and financial markets, on agents' degree of risk aversion, and on whether capital formation is possible. They are independent of any wage or price stickiness and unanticipated inflation.

Although the model studied in this article is rich in many respects, it is abstract in others. For example, because only steady states are derived, the article is silent on the short-run dynamics of the inflation-unemployment relationship. Future research should also examine the behavior of dynamical equilibria in the model.

The model also abstracts from asset accumulation and from adverse selection. Agents who have worked previously and already accumulated some assets presumably would be more likely to shirk if they went back to work, affecting the wage rate firms would have to offer to induce such agents to take employment in the primary sector. Agents also could differ in their ability to perform on the job, and their abilities might be private information. Firms might offer higher wages to attract at least some better workers into the labor market. Monetary policy,

through its effects on asset returns, would reinforce or mitigate the impacts of asset accumulation, but analyzing this also must be left for future research.

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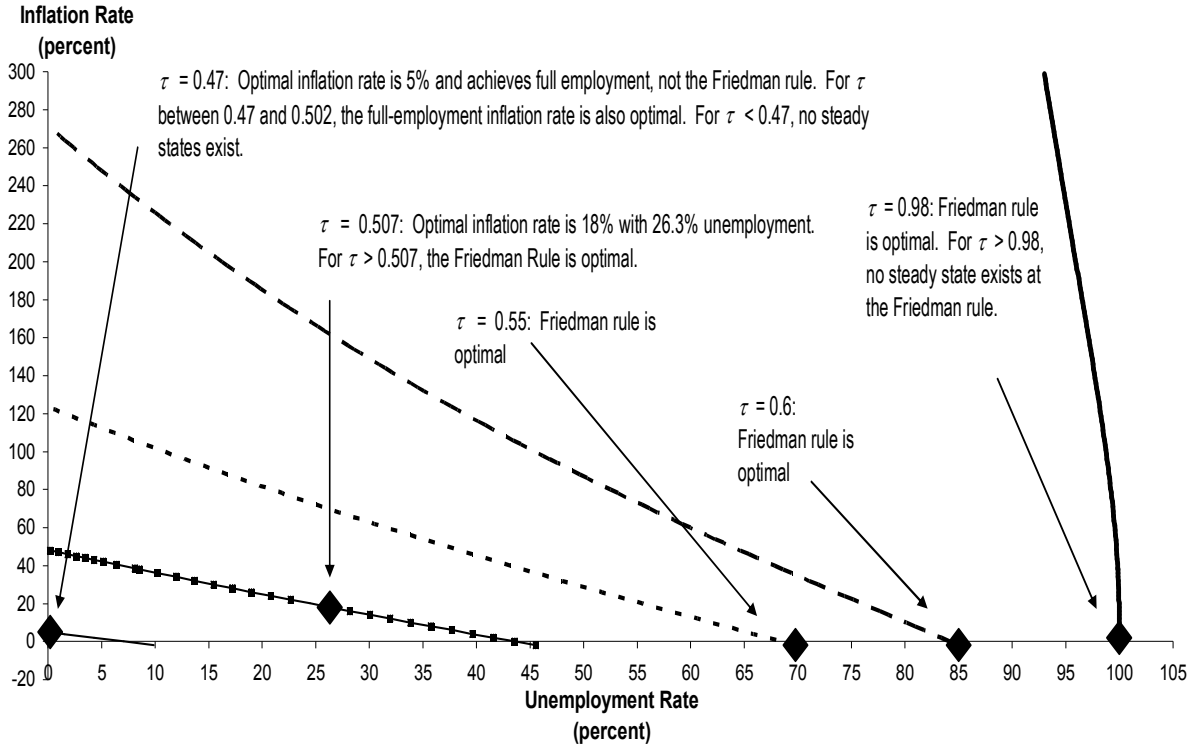
Figure 1

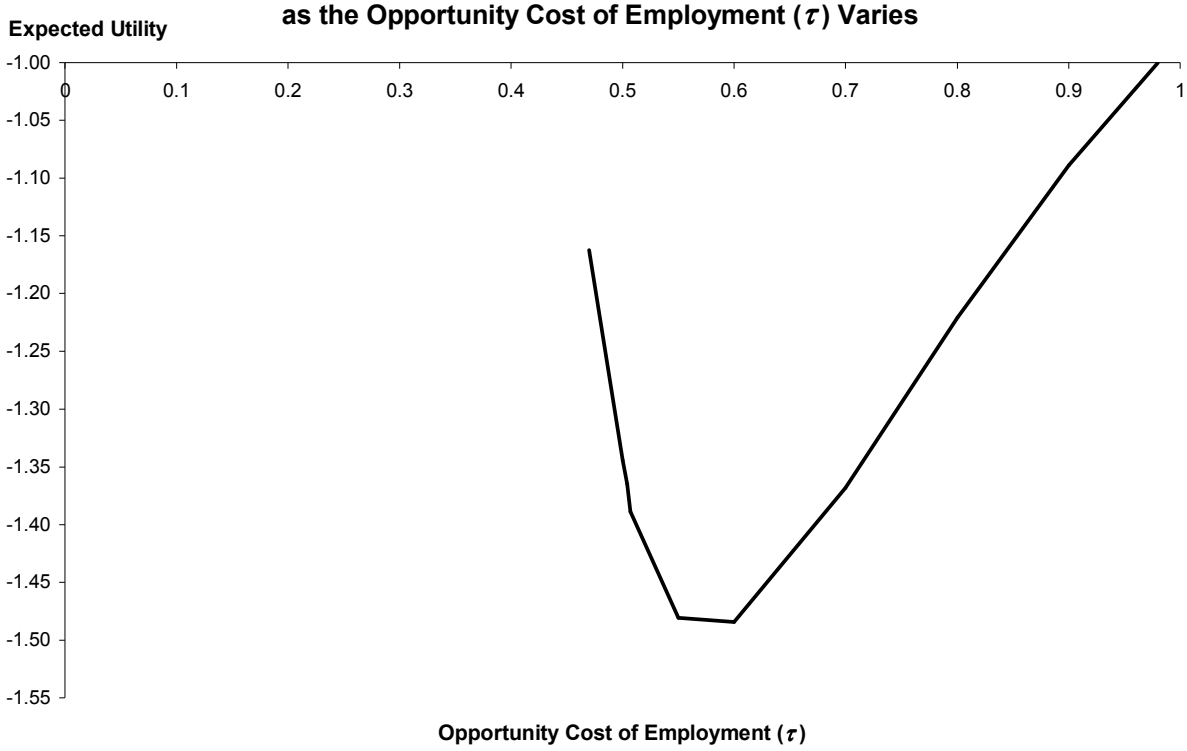
Phillips Curves for Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001. Optimal inflation rate denoted by ♦.

Figure 2

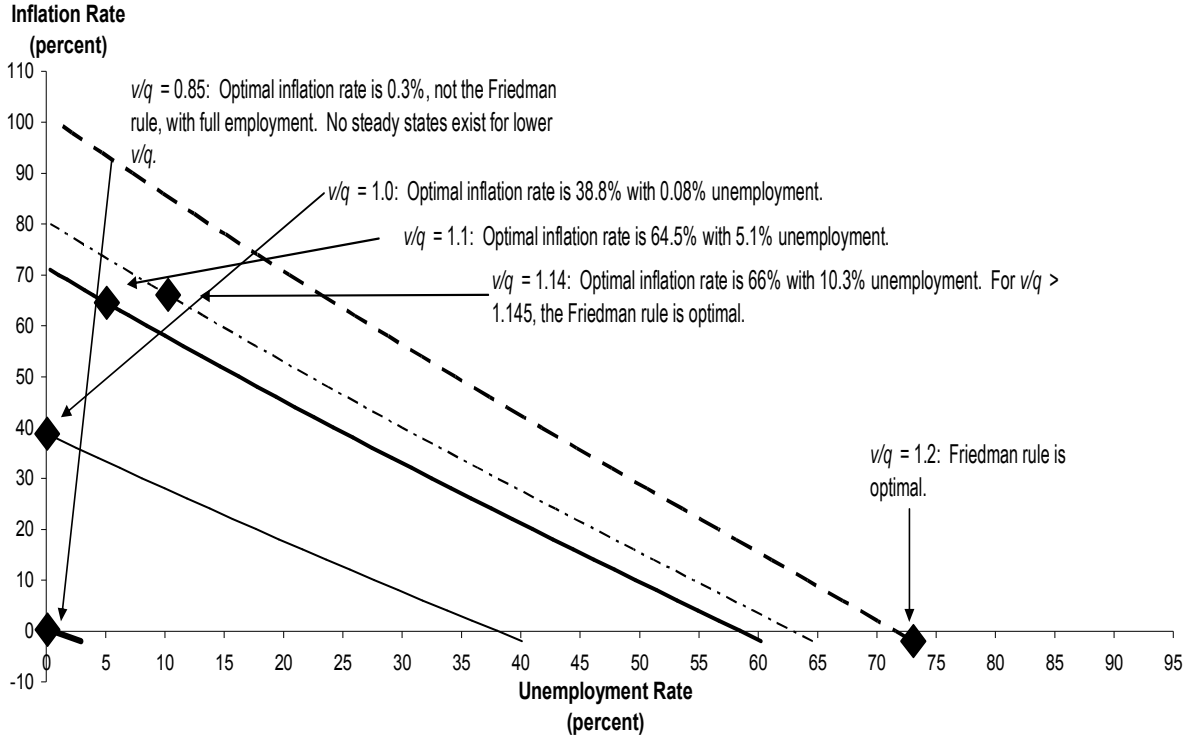
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001

Figure 3

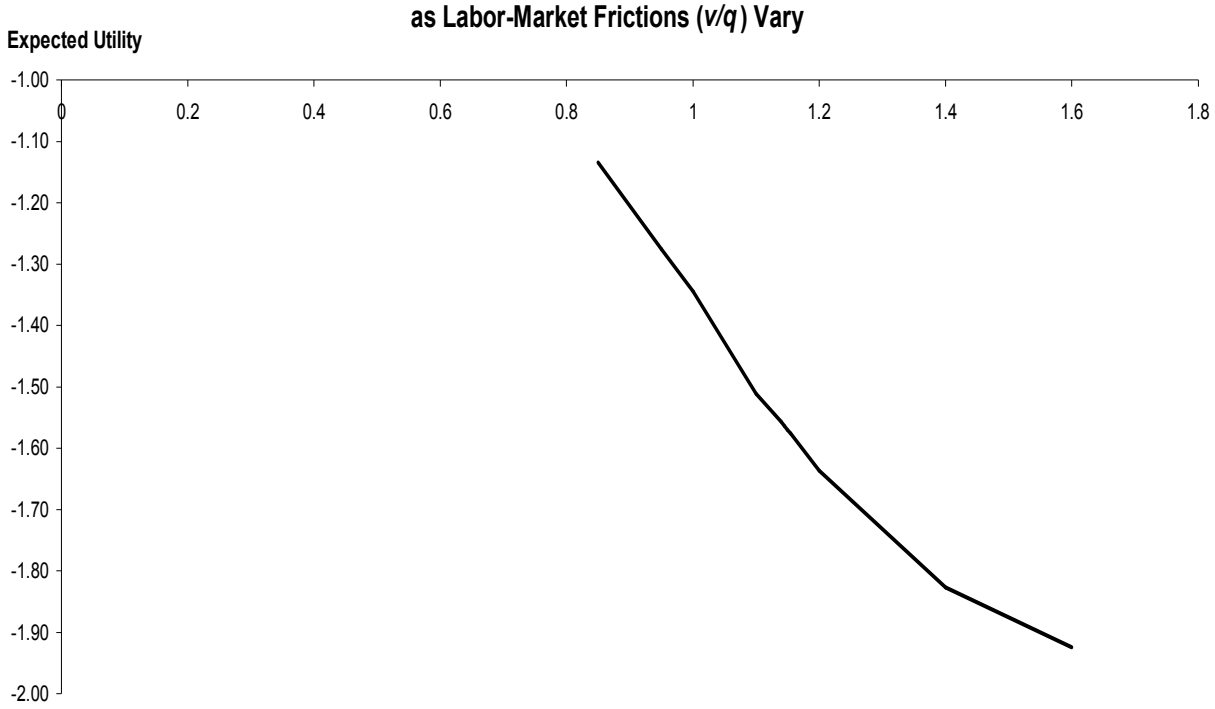
Phillips Curves for Economy with Fixed Capital Stock as Labor-Market Frictions (v/q) Vary



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $v = 0.001$; $\tau = 0.5$. Optimal inflation rate denoted by ♦.

Figure 4

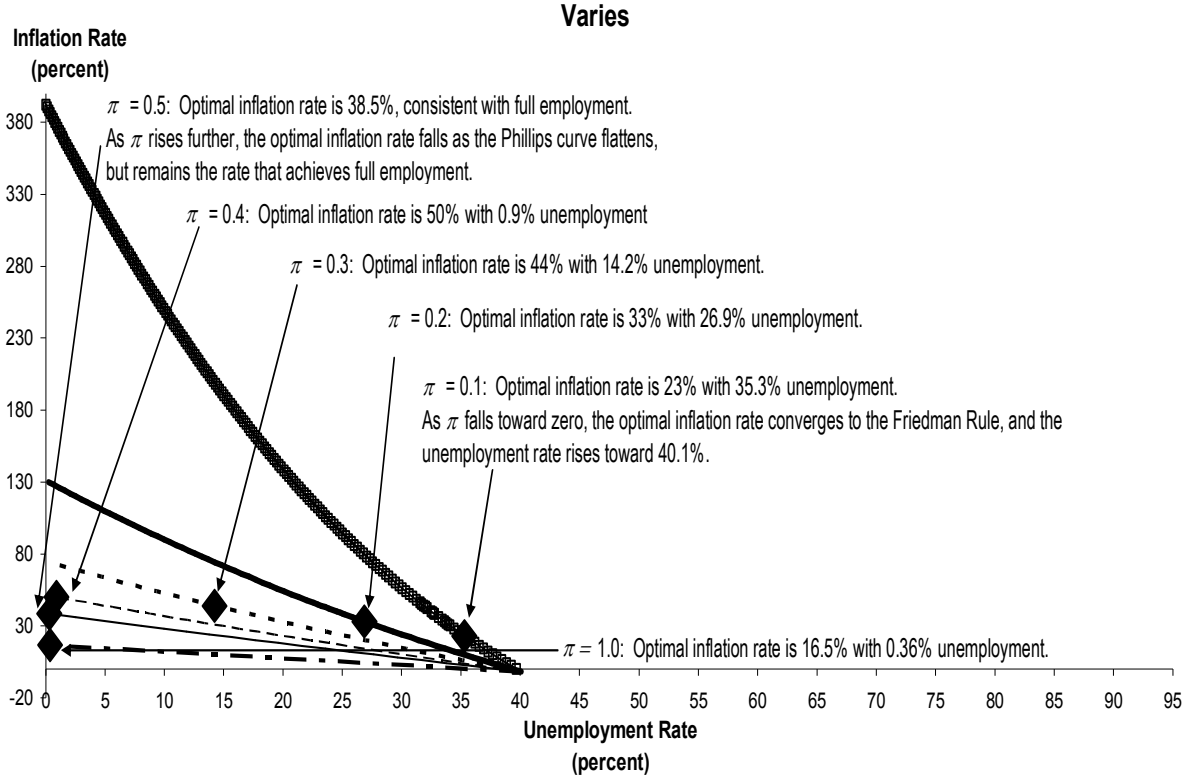
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Labor-Market Frictions (v/q) Vary



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $\tau = 0.5$; $v = 0.001$

Figure 5

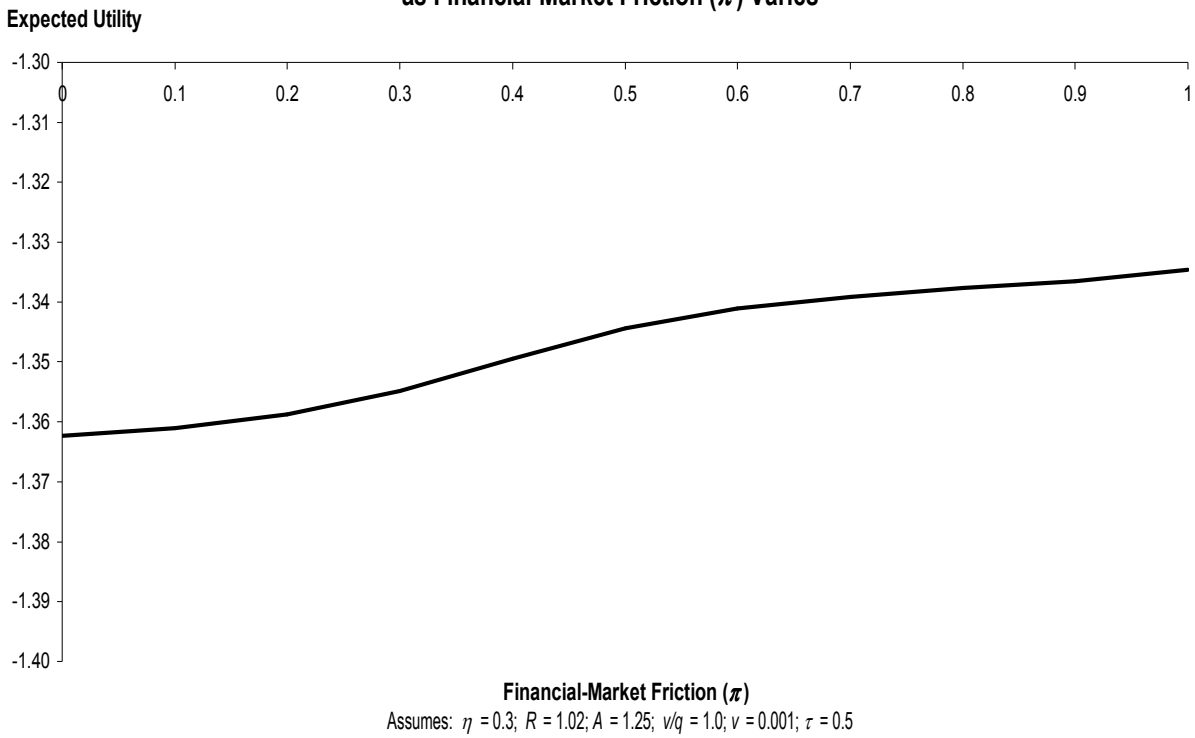
Phillips Curves for Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 6

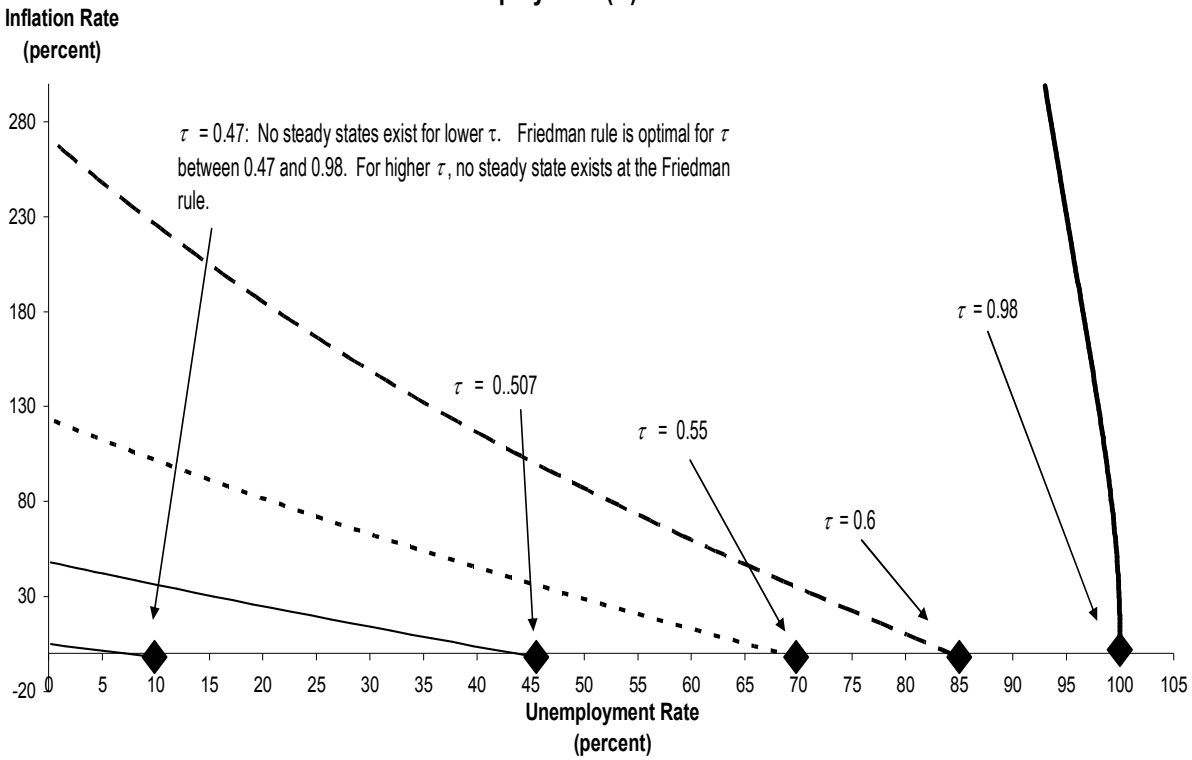
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5

Figure 7

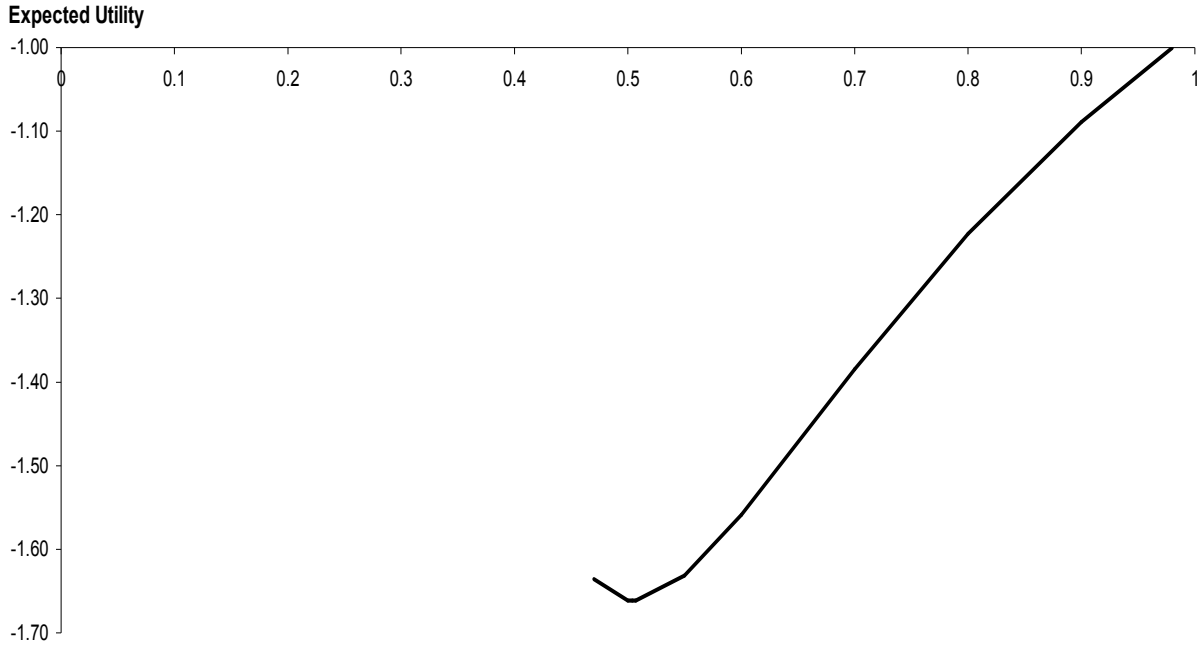
Phillips Curves for Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $v/q = 1.0$; $v = 0.5$. Optimal inflation rate denoted by ♦.

Figure 8

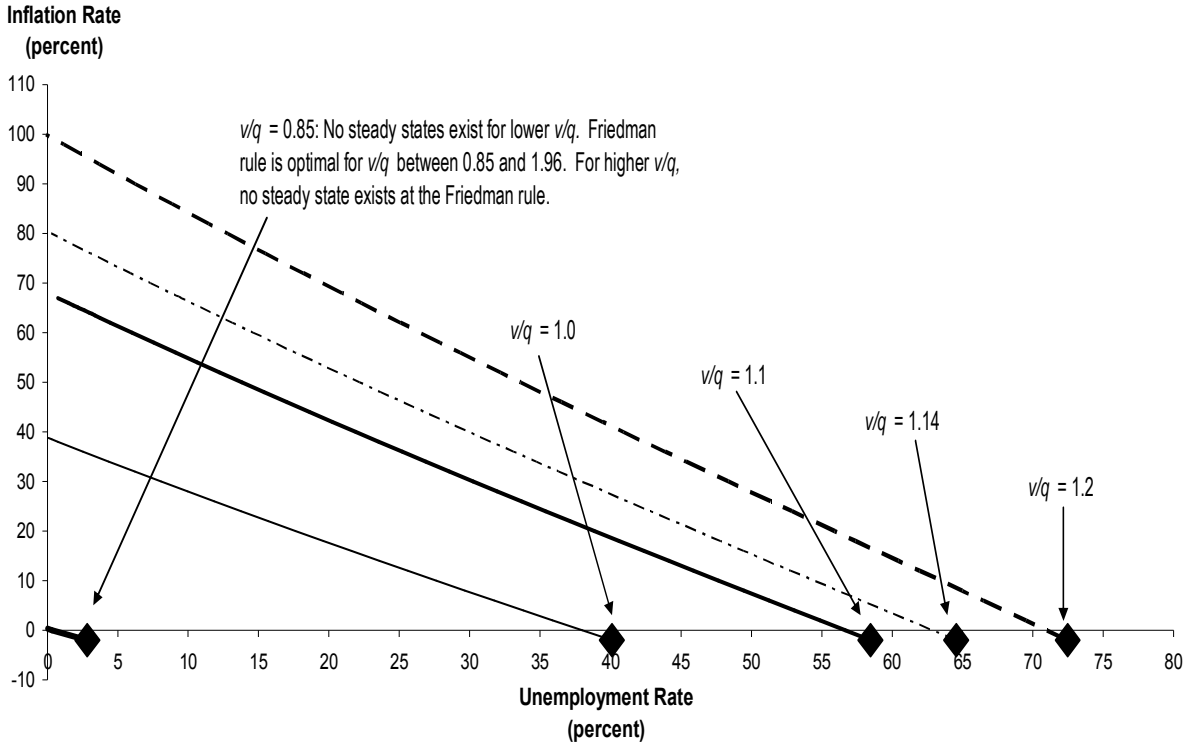
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5

Figure 9

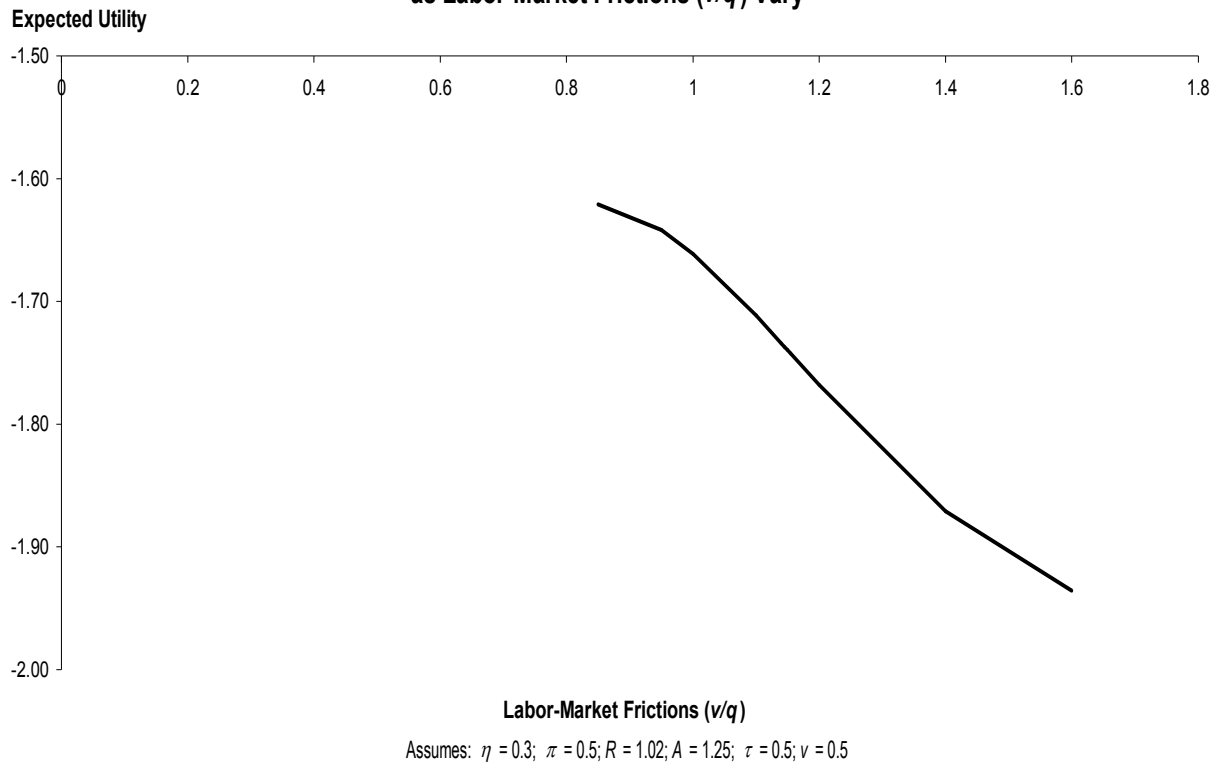
Phillips Curves for Economy with Fixed Capital Stock as Labor-Market Frictions (v/q) Vary



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $\tau = 0.5$; $v = 0.5$. Optimal inflation rate denoted by ♦.

Figure 10

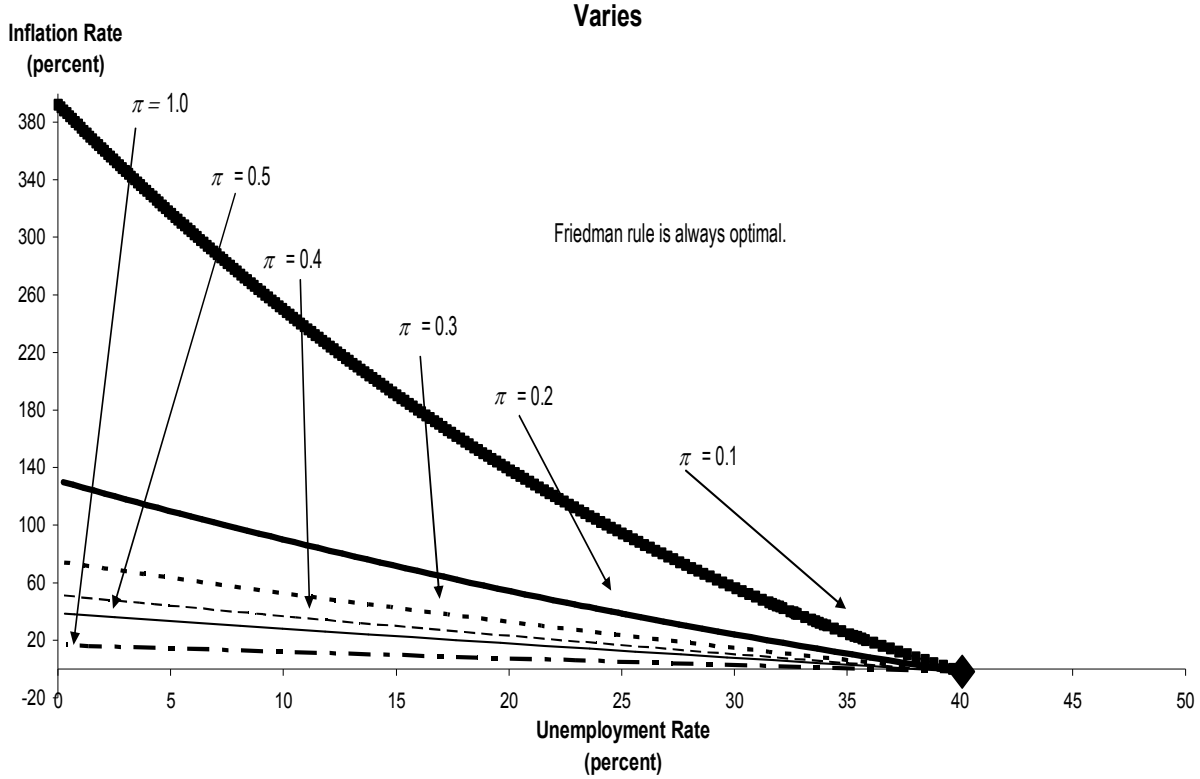
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Labor-Market Frictions (v/q) Vary



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $\tau = 0.5$; $v = 0.5$

Figure 11

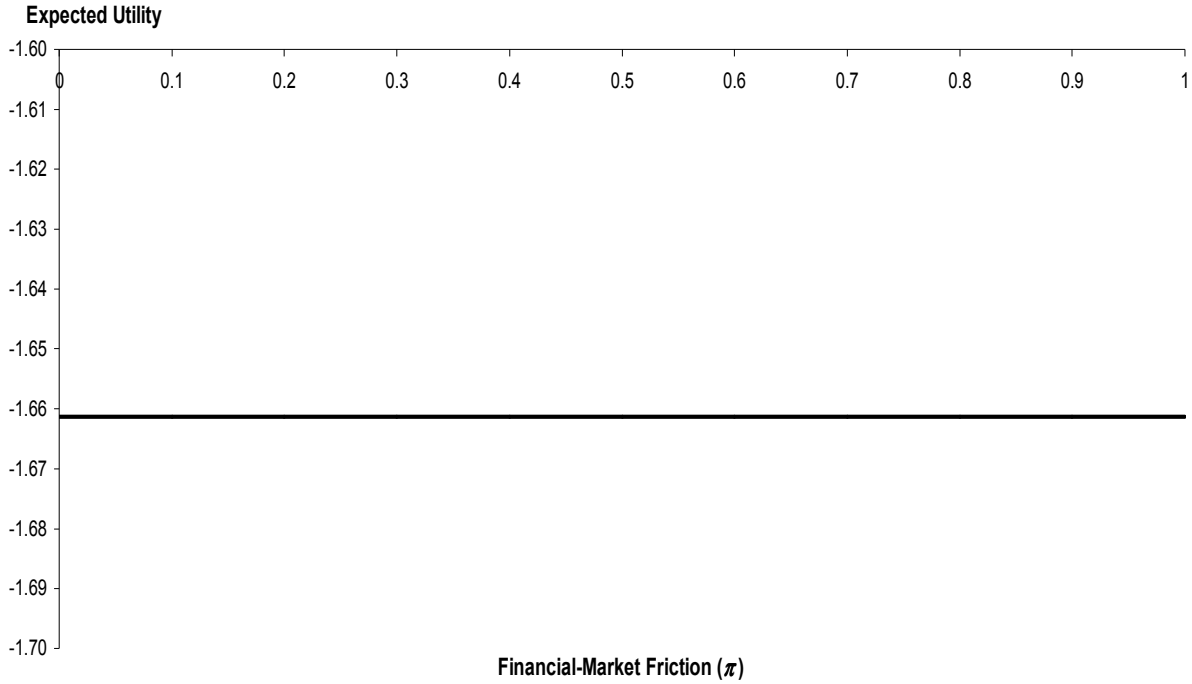
Phillips Curves for Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 12

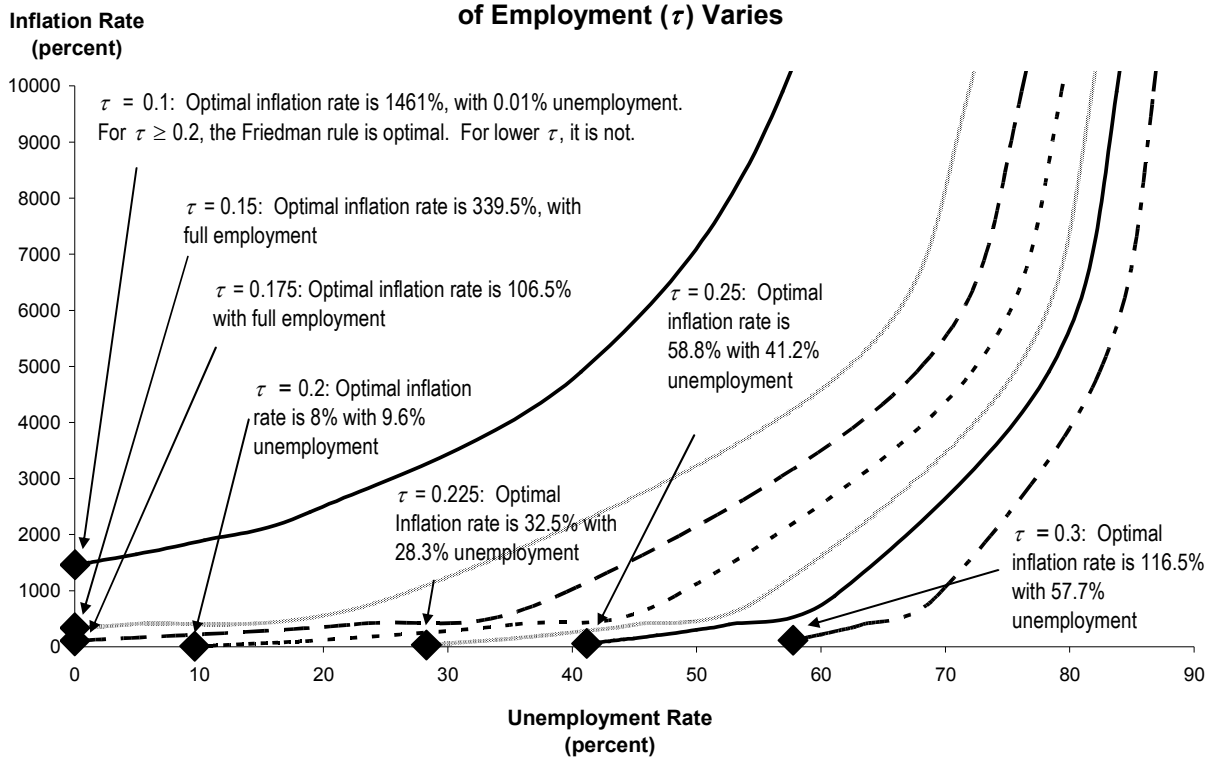
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5; τ = 0.5.

Figure 13

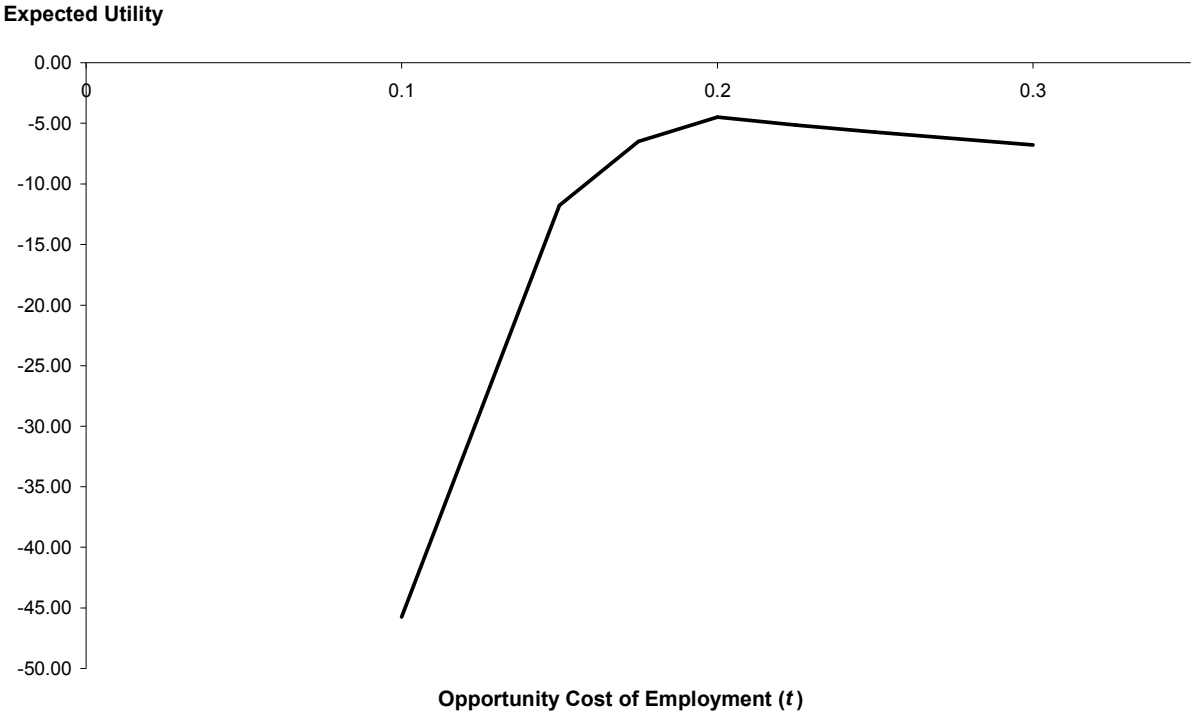
Phillips Curve for Economy with Capital Formation as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001. Optimal inflation rate denoted by ♦.

Figure 14

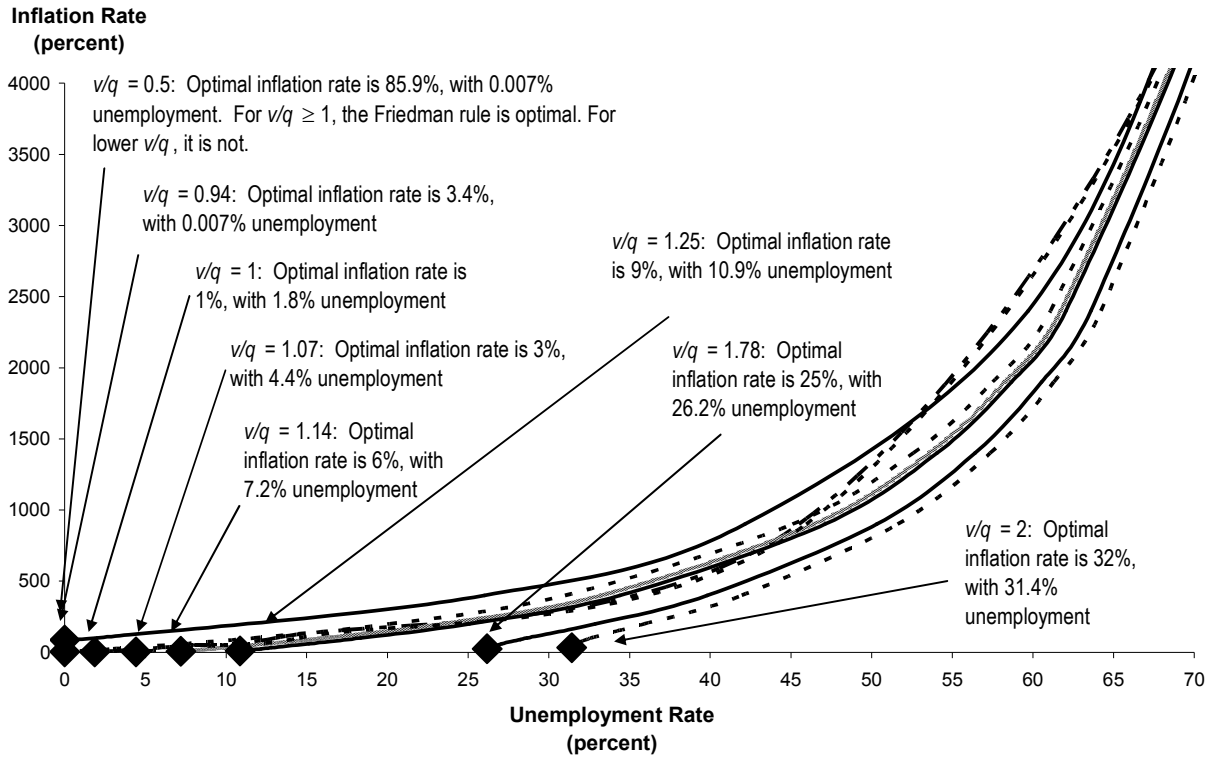
Expected Utility under the Optimal Monetary Policy in an Economy with Capital Formation as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; ν = 0.001

Figure 15

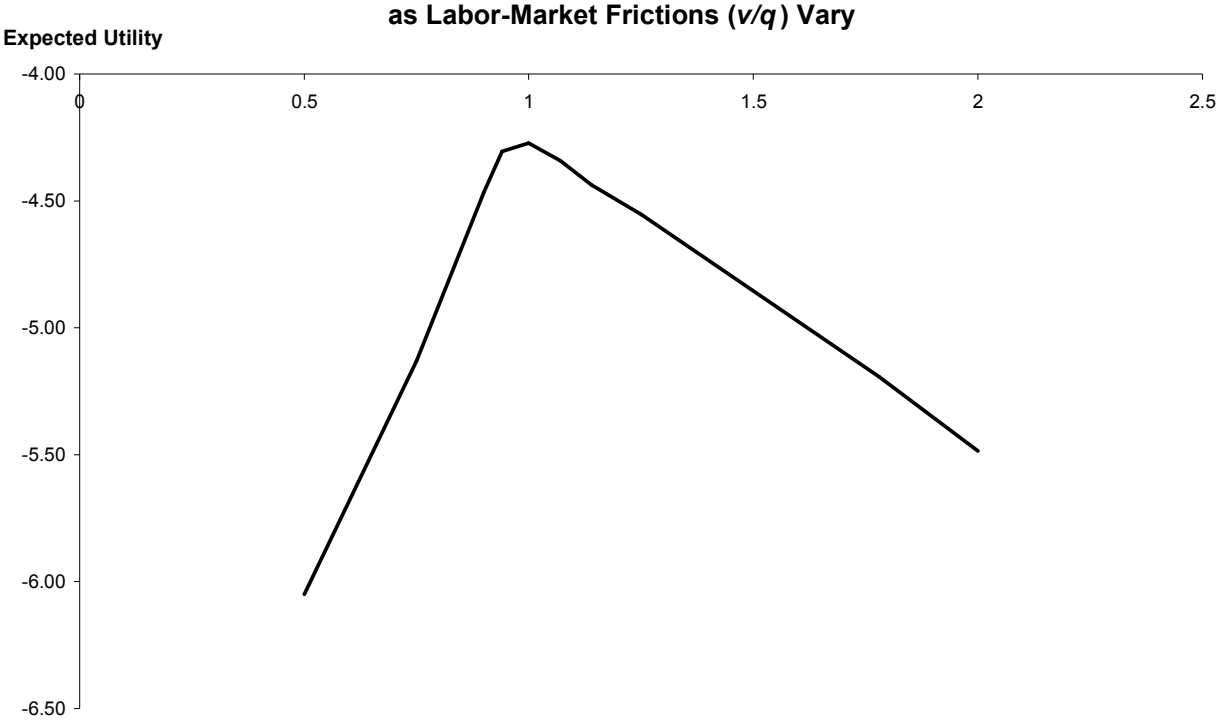
Phillips Curve for Economy with Capital Formation as Labor-Market Frictions (v/q) Vary



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 16

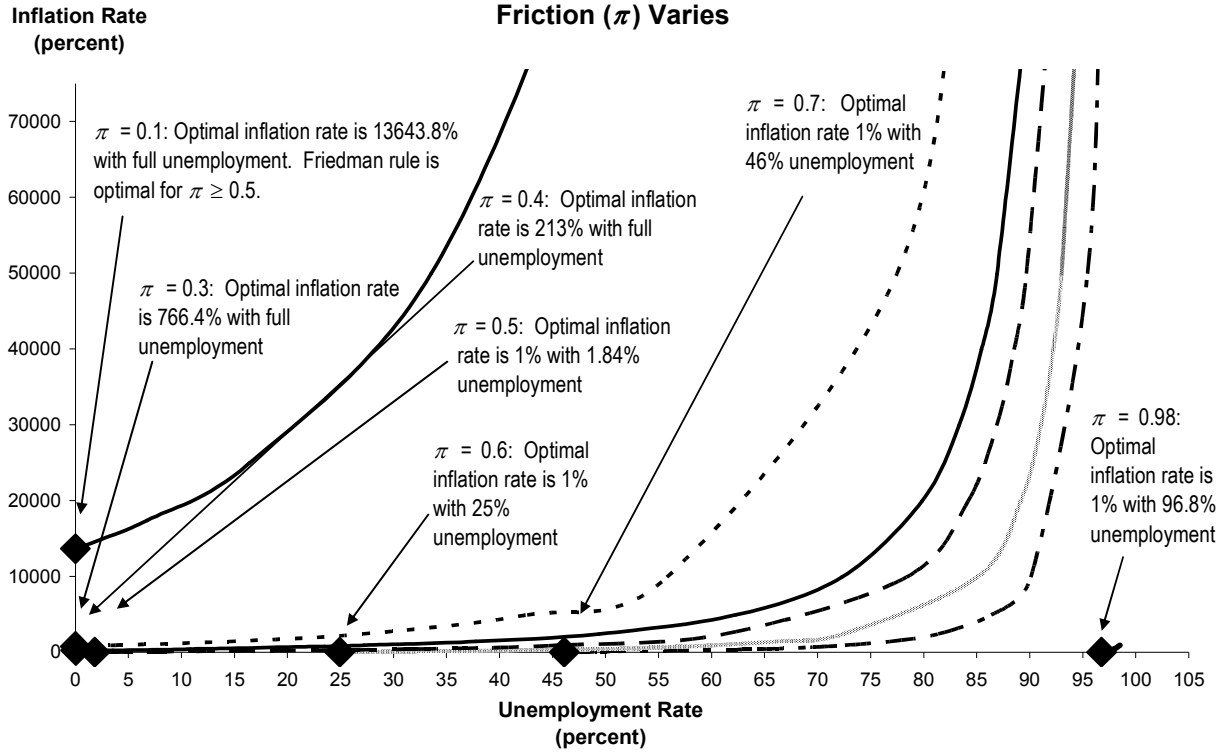
Expected Utility under the Optimal Monetary Policy in an Economy with Capital Formation as Labor-Market Frictions (*v/q*) Vary



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v = 0.001; τ = 0.5.

Figure 17

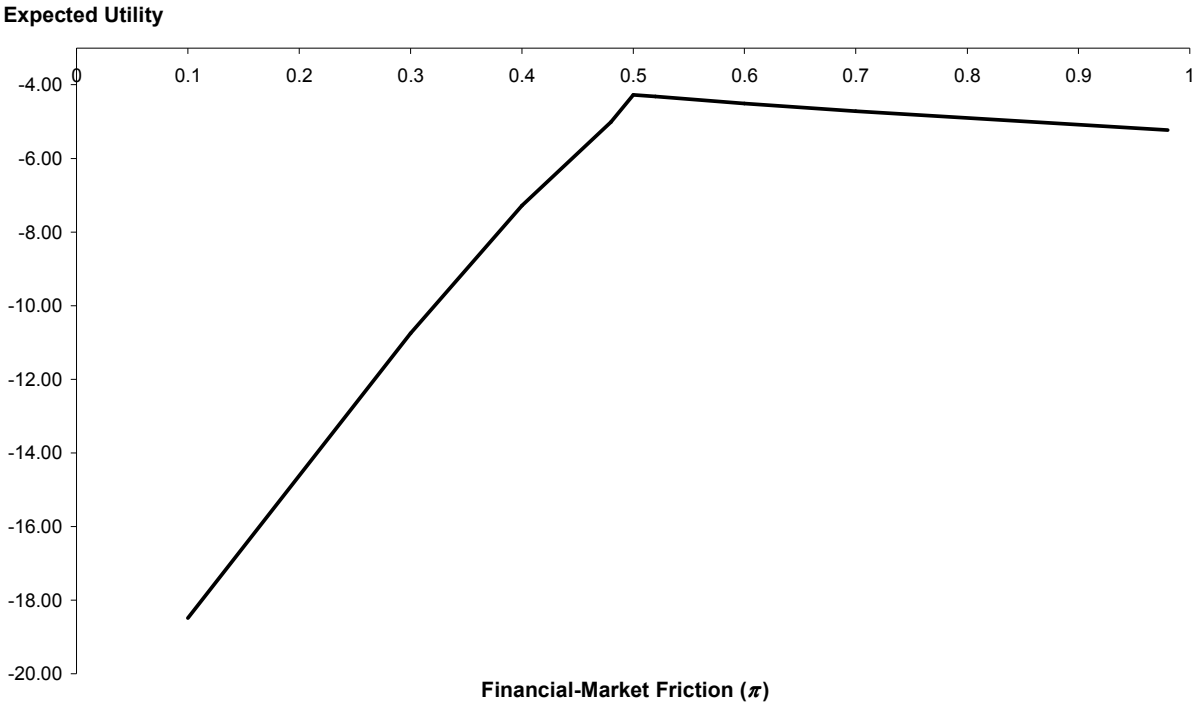
Phillips Curve for Economy with Capital Formation as the Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 18

Expected Utility under the Optimal Monetary Policy in an Economy with Capital Formation as the Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5.