Repos, Risk Aversion, and Haircuts*

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Abstract

Repurchase agreements ("repos") play a significant role in global credit market activity. Therefore, the individual decisions of repo market participants can weigh heavily on the broader economy. In order to analyze the decisions of these participants, our framework studies security repurchase agreements between risk-averse borrowers and lenders. In this manner, repo agreements are part of an efficient risk-sharing arrangement. In turn, properties of the agreements depend on environmental conditions such as the degree of risk aversion, default risk, and the market value of collateral. Previous work based upon risk neutrality cannot capture these features. Haircuts, in particular, arise naturally in the model in order to promote risk-sharing. From this perspective, our work is in stark contrast to previous contributions which determine the extent of over-collateralization in response to asymmetric information between market participants.

Keywords: Repurchase Agreements, Haircuts, Risk Sharing

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1 Introduction

Due to their significant role in credit market activity, there has been a lot of attention devoted towards understanding repo markets in recent years. Notably, Bernanke (2010) states: "Leading up to the crisis, the shadow banking system, as well as some of the largest global banks, had become dependent on various forms of short-term wholesale funding...In the years immediately before the crisis, some of these forms of funding grew especially rapidly; for example, repo liabilities of U.S. broker dealers increased by 2-1/2 times in the four years before the crisis." In addition, Hördahl and King (2008, p. 37) estimate "gross amounts outstanding [of repos] at year-end 2007 of roughly $10 trillion in each of the US and Euro markets, and another $1 trillion in the UK repo market."

Following the bankruptcy of Lehman Brothers, repo markets seized up significantly affecting credit market activity. As Copeland, Martin, and Walker (2010) note, it is critically important for policymakers to understand the underlying factors which contribute to failures in repo markets. In particular, Adrian et al. (2013) stress that the relationships between (a) funding amount, (b) repo rates, (c) haircuts, and (d) counterparty risk need to be studied in order to convincingly address the structure of repo markets and the potential for instability in the financial system.

Repo lending consists of two basic parts. In the first leg of a repo, cash-lenders provide loans to borrowers in exchange for collateral. The second leg completes the transaction with borrowers repurchasing the collateral from the cash-lender at a higher price.[1] For example, a money market fund (cash-lender) purchases collateral from a broker-dealer (borrower) with an agreement by the dealer to repurchase the collateral at a higher price on a later date. The higher price reflects interest on the loan or the "repo rate."

Along with repo rates, the level of "haircuts" is another important component of security repurchase agreements. Haircuts are the extent to which repo transactions are *overcollateralized.* Imagine a case where a borrower sells $100 worth of collateral to a lender for a loan of $90. This transaction is said to be overcollateralized by 10% or, alternatively, there is a 10% haircut on the collateral sold by the borrower. If the borrower repurchases the collateral for $95, then the repo rate would be roughly 5.5%.

Though the structure of a security repurchase agreement may appear to be relatively straightforward, policymakers and regulators have little guidance for predicting how repo markets are likely to respond to different economic conditions. For example, Dang, Gorton and Holmström (2013b, p. 3) explain: "The existence of repo haircuts is a puzzle, as standard finance theory would suggest that risk simply be priced and the market price reflects risk and risk aversion of the market." That is, Dang et al. (2013b) intimate that standard finance theory only accounts for uncertainty ex-ante rather than ex-post when

[1] The lender in essence is purchasing the collateral. If the borrower defaults in the agreement, the lender can immediately sell the collateral. By doing so, the lender is able to avoid lengthy Chapter 11 bankruptcy proceedings. For more discussion of bankruptcy policy rules and repo markets, see Antinolfi et al. (2012).

default occurs. Consequently, standard finance theory cannot be used to study repo markets.

Adrian et al. (2013, p. 13) state that "in response to a rise in perceived risk... cash lenders might ask for higher interest rates, higher-quality collateral, increased haircuts, shorter maturities, or all of the above." However, empirical analysis studying changes in market risk is limited due to gaps in the data. In particular, Bernanke (2012) emphasizes: "Unfortunately, data on the shadow banking sector, by its nature, can be more difficult to obtain. Thus, we have to be more creative to monitor risk in this important area."

The objective of this paper is to study the structure of repurchase agreements in the absence of commitment. In particular, we demonstrate that the level of haircuts endogenously depends on default risk and the value of collateral. In contrast to previous research based on risk neutral agents, we study settings in which all participants in the repo market are risk averse. To be specific, lenders in our framework must be compensated for the risk that they bear in the process of lending funds to borrowers. As a result, activity in repo markets critically depends on the degree of risk aversion among participants in the financial system.

The absence of commitment among repo market participants is an important friction in our framework. As emphasized by Mills and Reed (2012), repo markets are limited by two-sided moral hazard. Typically, models of collateralized lending are only concerned about strategic default by the borrower (Lacker 2001; Manove, Padilla, Pagano 2001; Rampini 2005; Kehoe and Levine 2008). In a repurchase agreement, however, lenders can sell collateral in a secondary market. This option reflects the insurance role of collateral. In addition, absent a commitment mechanism, lenders can enter the secondary market at any time. Therefore, both borrowers and lenders must receive appropriate incentives in order to avoid strategic default.

Gorton and Metrick (2012) observe that increases in the risk of default lead to higher haircuts imposed on collateral, indicating that there is greater reliance on the insurance role of collateral in environments with more risk. Our model can be used to rationalize such behavior. Higher probabilities of default risk in our framework lead to less repo funding and higher haircuts – thus, an increase in required collateral per unit of funding.

One could also argue that market participants before the crisis became more risk averse at the same time that perceived risk increased. In contrast to previous work based upon risk neutrality, our framework is particularly well suited to study the impact of higher risk aversion. Interestingly, we find that higher degrees of risk aversion can also cause repo markets to collapse. As a result of the increase, there is an increased reliance on collateral along with higher haircuts imposed on those seeking funding.

Previous work such as Dang et al. (2013b) motivate the need for haircuts through the concept of "information sensitivity" which relies on an additional layer of uncertainty – institutions holding collateral may be uncertain about its

value.[2] Haircuts emerge to compensate lenders needing to sell the collateral to sophisticated secondary market buyers who have access to information regarding the collateral's true value. Cash-lenders, unable to access the information, set haircuts in the first leg of the repo to protect them from potential downside risk.

In comparison to their work, lenders in our model have complete information about the value of the collateral they possess. Haircuts emerge as part of an efficient risk-sharing arrangement between repo market participants. Consequently, there are distinct policy implications for the repo market from our model in comparison to Dang et al. (2013b)

The paper is organized as follows. Section 2 describes the model. Section 3 derives closed-form solutions for all variables. In section 4 we stress how default risk, risk aversion, and the market value of collateral affect the level of haircuts and overall funding in the repo market through numerical exercises. Section 5 concludes. The Appendix provides proofs of major results.

2 The Model

2.1 Endowments, Preferences, and Technologies

The economy lasts for two periods. There are two different goods available for consumption, α and β. There are also two different types of agents, A and B, each with a population mass of 1. The type A individual receives an endowment of good α while type B agents are endowed with β. The preferences of each agent are:

$$U^A(c_\alpha^A, c_\beta^A) = \frac{\left(c_\alpha^A\right)^{1-\rho}}{1-\rho} + \frac{\left(c_\beta^A\right)^{1-\rho}}{1-\rho}$$

$$U^B(c_\beta^B) = \frac{\left(c_\beta^B\right)^{1-\rho}}{1-\rho}$$

with coefficient of risk aversion ρ. That is, type A individuals derive utility from consumption of both goods while type B only wants to consume good β.

Type A individuals have access to a risky investment technology which generally produces $R > 1$ units of good β at $T = 2$ for every unit of good β invested at $T = 1$. However, with probability η, the risky investment technology does not yield any returns. Type A and B both have safe storage technologies for storing goods not allocated to $A's$ risky investment technology.

Differences in the endowments of each agent and differences in access to the investment technology lead to the potential for funding in $A's$ risky investment technology by a type B individual. However, there is not an enforcement mechanism in the economy. As a result, a type B will require a mutual transfer of

[2] For more on "information-sensitivity," see Dang, Gorton, and Holmström (2013a).

good α from A to be willing to lend. In this way, good α acts as a form of collateral. Since a type A individual will seek to borrow funds from a type B, hereafter we also refer to $A's$ as borrowers and $B's$ as lenders.

Moreover, because collateral is transferred at the same time that funding is provided to borrowers, the lending arrangement represents a repurchase agreement between a borrower and a lender. One might think of a lender in this manner as a cash-rich intermediary such as a money market mutual fund and a borrower as a broker-dealer. In turn, η reflects the default or counterparty risk of a dealer-bank. Notably, after the bankruptcy of Lehman Brothers, market participants perceived that default risk was significantly higher.

It is well known that cash-lenders like money market funds do not actively participate in the markets for various types of debt obligations which serve as collateral in security repurchase agreements. Consequently, the lender may face difficulties selling a collateral security in the event of default by a borrower.[3] In order to capture the discounted sale, lenders only possess a weak transformation technology for converting the collateral good into units of β. More specifically, the conversion rate is equal to $p \leq 1$. Note, however, the lender can choose to access the weak technology at any time. Such a lack of commitment, as articulated by Mills and Reed (2012), introduces frictions into repo arrangements. As a result, both borrowers and lenders must face appropriate incentives in order to avoid strategic default.

2.2 Timing of Actions

At the beginning of period 1, agents receive their endowments. Afterwards, agents A and B meet. A social planner determines the quantities of goods α and β to be exchanged between the agents. Assuming that the agents accept the proposal suggested by the planner, exchange takes place. Agent A promptly places the acquisition of β into his risky investment technology. All remaining endowments are placed into $A's$ and $B's$ respective storage technologies. After endowments are allocated to the various technologies, period 1 ends.

At the beginning of period 2, returns from the investment technology occur. If $A's$ technology does not yield any output, A defaults on the lender. Following a productive return, both agents meet and the planner suggests the amount of good β to be transferred by the A in order to re-acquire the amount of α that was put forward as collateral.

It is important to note neither agent is required to accept any trade proposal. Therefore, the suggestions offered by the planner can be rejected at any stage.[4] If the trade is acceptable by both agents, then the repurchase occurs and both

[3] These difficulties may include transactions costs such as search costs and bargaining costs. Moreover, at times during the crisis, fire-sales of various types of assets took place which also caused their values to decline significantly.

[4] In comparison to the bilateral repos studied in our model, an additional third party known in a tri-party repo acts as a clearing bank by providing intermediation services for the cash-lender and borrower. In particular, the clearing bank (JPMorgan Chase or the Bank of New York Mellon in the United States) stewards the transaction by providing collateral management and repurchase settlement services.

agents consume. If the trade is unacceptable or agent A was forced to default, then agent B utilizes the transformation technology and converts all of the collateral into units of good β. After consumption, period 2 concludes.

3 Optimal Choices

The model is solved using a backwards-induction approach. We begin by considering the options available to agent B in the event that A did not receive any productive returns or simply defaulted on B earlier in period 2. It is also possible that agent B did not accept repayment of the loan from A. Upon converting the collateral good (α) to units of β at rate p, the lender's consumption would equal $y - d_\beta + p \cdot d_\alpha$. Alternatively, $B's$ consumption of β without converting the collateral would equal $y - d_\beta$. Thus, it is trivial to determine that an agent B in this scenario would convert the collateral good into units of β since lenders only derive utility from consumption of β.

We proceed to studying the decision of a borrower to repay the loan from a lender during the second leg of the repurchase agreement. A social planner will propose a level of repayment that maximizes aggregate utility across both parties:

$$\max_r \left[\frac{x^{1-\rho}}{1-\rho} + \frac{(Rd_\beta - r \cdot d_\beta)^{1-\rho}}{1-\rho} \right] + \frac{(y - d_\beta + r \cdot d_\beta)^{1-\rho}}{1-\rho} \tag{1}$$

Solving the optimization problem yields:

$$r^* (d_\beta) = \frac{(R+1)}{2} - \frac{y}{2d_\beta}. \tag{2}$$

From (2), the Planner's suggestion for the repo rate reflects the desire to maximize aggregate welfare through the distribution of income between both the borrower and lender. A higher interest rate on the loan provides the lender with more consumption of good β but lowers the amount that the borrower would obtain. However, by participating in the second leg of the repo, the borrower is able to consume all of his endowment of good α.

In the absence of commitment, either agent may walk away from the social planner's proposal. For the borrower, incentive compatibility requires:

$$\frac{x^{1-\rho}}{1-\rho} + \frac{(Rd_\beta - r^*(d_\beta) \cdot d_\beta)^{1-\rho}}{1-\rho} \geq \frac{(x - d_\alpha)^{1-\rho}}{1-\rho} + \frac{(Rd_\beta)^{1-\rho}}{1-\rho} \tag{3}$$

By comparison, the lender's utility from accepting repayment of the loan is:

$$\frac{(y - d_\beta + r^*(d_\beta)d_\beta)^{1-\rho}}{1-\rho} \geq \frac{(y - d_\beta + p \cdot d_\alpha)^{1-\rho}}{1-\rho}$$

Obviously, this constraint is satisfied if the return on investment exceeds the value of liquidating the collateral:

$$r^*(d_\beta)d_\beta \geq pd_\alpha$$

Or, using the solution for r^*, the lender will find the repayment to their benefit if the level of lending meets the following condition:

$$d_\beta \geq \frac{y + 2pd_\alpha}{(R+1)} \tag{4}$$

In order to determine the subgame perfect equilibrium for the model, we proceed by working back to studying actions in period 1. In the initial leg of the repo, a social planner suggests an amount of funding (d_β) to be provided by the type B (the lender) to the type A (the borrower). In order to help resolve the lack of commitment friction, the planner also suggests an amount of collateral (d_α) to be provided by the borrower to the lender. Consequently, the planner solves the following problem:

$$\max_{d_\alpha, d_\beta} (1-\eta)\left[\frac{\left(\frac{y+(R-1)d_\beta}{2}\right)^{1-\rho}}{1-\rho}\right] + \eta\left[\frac{(y - d_\beta + pd_\alpha)^{1-\rho}}{1-\rho}\right] + \tag{5}$$
$$(1-\eta)\left[\frac{x^{1-\rho}}{1-\rho} + \frac{\left(\frac{y+(R-1)d_\beta}{2}\right)^{1-\rho}}{1-\rho}\right] + \eta\left[\frac{(x - d_\alpha)^{1-\rho}}{1-\rho}\right].$$

The planner's suggestion for d_β is:

$$d_\beta = \frac{y + pd_\alpha - \frac{y}{2}\left(\frac{\eta}{(1-\eta)(R-1)}\right)^{\frac{1}{\rho}}}{\left[1 + \frac{(R-1)}{2}\left(\frac{\eta}{(1-\eta)(R-1)}\right)^{\frac{1}{\rho}}\right]} \tag{6}$$

Proposition 1. *On the basis of (6), d_β is increasing in d_α and p, and decreasing in η.*

Naturally, the planner suggests that the lender provide more funding to the borrower if a larger amount of collateral is offered. On the same note, more funding is also offered if the liquidation value of the borrower's collateral is higher (p higher). An increased liquidation value means the lender obtains a greater amount of insurance against default. Given the higher degree of protection, the planner suggests that the borrower receive more funding from the lender. On the other hand, increases in default risk (η higher) lead to less credit funding between the borrower and lender.

Proposition 2. *Let η be such that $\underline{\eta} \leq \eta \leq \overline{\eta}$. Under this condition, d_β is increasing in y and R.*

In contrast to the results in Proposition 1, the optimal response of funding with respect to the lender's endowment and the return to the risky technology depends on the probability of default. This takes place because funding decisions depend on the marginal utilities of both the borrower and lender across good and bad states. An intermediate level of default risk creates an environment with a relatively high marginal rate of substitution (MRS) from the bad state towards the good state.[5] Subsequently, the high MRS leads to more investment when the lender's endowment increases or the risky technology becomes more profitable.

The planner's suggestion for d_α is:

$$d_\alpha = \frac{x - p^{\frac{\rho-1}{\rho}} (y - d_\beta)}{\left(1 + p^{\frac{\rho-1}{\rho}}\right)} \tag{7}$$

Proposition 3. *By equation (7), d_α is increasing in d_β and x. It is decreasing in y and p. The collateral decision is unaffected by η.*

Proposition 3 summarizes the amount of collateral to be offered in the repo. If the borrower seeks more funding, the amount of collateral to be posted should increase. Similarly, the borrower should offer more collateral if he has a larger endowment of the collateral good. By comparison, if the lender has a relatively high endowment of the good β, then the planner proposes that the borrower offers less collateral.

[5] The MRS is relateively high compared to the marginal gains in expected income.

If the collateral good has a higher market value, then its insurance value is also greater. Since the role of collateral is to insure the lender against the possibility of default, the amount of collateral to be retained does not need to be as high if the collateral provides a higher level of insurance. As borrowers do not post as much collateral if p is higher, this effect detracts from the ability of collateral to promote credit activity as observed in Proposition 1. In other words, *the value of higher quality collateral in promoting repo activity is diluted by the reluctance of borrowers to offer it.*[6]

In the event that the borrower's technology does not yield any returns, both the borrower and lender are affected by the default. Consequently, the amount of collateral to be offered is independent of the default probability.

This does not imply that default risk does not affect repo market activity. As observed in Proposition 1, rather than increasing their reliance on collateral if default risk is higher, lenders shed their exposure to default risk by decreasing funding to borrowers. In turn, Proposition 2 shows that as the amount of funding is lower, less collateral will be put forward by borrowers. Thus, collateral responds to default risk *indirectly* through the level of credit provided.

Based upon (6) and (7), we turn to the overall solutions for funding and collateral. We begin with a simple benchmark where activity can be studied analytically:

Lemma 1. (Repo Solutions). *Suppose that $\rho = 1$ and $\eta = \frac{1}{2}$. Further, let $x > x_0$. Under these conditions, the (non-degenerate) solutions to the Planner's proposals for repo activity are:*

$$d_\beta^* = \frac{px + y\left[2 - p\left\{\frac{2R-1}{2(R-1)}\right\}\right]}{3-p} \tag{8}$$

$$d_\alpha^* = \frac{3x + y\left[\frac{R-2}{(R-1)}\right]}{2\,(6-p)} \tag{9}$$

$$r^*\left(d_\beta^*\right) = \frac{(R+1)}{2} - \frac{y}{2d_\beta^*} > 1 \tag{10}$$

[6] This is similar to the problem in monetary economics in regard to commodity money. Individuals would be less inclined to offer objects as a means of payment if it has greater intrinsic value. In this manner, the ability of commodity money to promote trade may be limited.

Proposed funding $\left(d_\beta^*\right)$ *is increasing in* p, R, *and* y. *Proposed collateral* (d_α^*) *is increasing in* p *and* y, and *decreasing in* R. *The solution for the repo rate is increasing in* p *and decreasing in* y.

By Proposition 3, for a given collateral amount, repo funding is increasing in the market value of collateral. However, the borrower is less willing to offer collateral if it has a higher market value. Yet, Lemma 1 illustrates that the overall impact on repo volumes by the collateral's liquidation value is positive.

The conclusion that more collateral should be offered if it has a higher liquidation value may appear to be counter-intuitive – in practice, lenders impose smaller haircuts on securities of higher liquidation value. We offer the following explanation. By Lemma 1, the amount of funding is increasing in the value of the collateral. Also, the total amount of collateral to be posted is increasing in the size of the loan. The observation for collateral in the Lemma is reached due to the ability of collateral to promote repo financing (as d_β is increasing in p from Proposition 3). That is, the relationship between posted collateral and its liquidation value emerges simply because higher value collateral promotes repo market activity. As borrowers obtain higher amounts of funding, lenders in turn will ask for more collateral. In addition, as we discuss following Corollary 1 below, logarithmic preferences also play a role.

The variable R captures the possible investment gains from short-term funding. For a given amount of collateral, higher R is associated with more funding. The social planner proposes that more funding resources should be allocated to the investment project as R increases because the expected return is higher. Interestingly though, there is no direct effect of R on collateral. The intuition is similar to the effects of default risk on collateral as discussed following Proposition 3. Investment gains only affect collateral through the amount of funding obtained. So, there is a limited impact of R on the allocation. Again, the insurance role of collateral in a repo is highlighted – collateral allows the lender in a repo to hedge risk.

By Proposition 2, the amount of funding to be obtained is increasing in the lender's endowment. On the other hand, Proposition 3 shows that the amount of collateral to be posted is decreasing in the size of the endowment. In this manner, the overall impact of the lender's resources on the funding solution in Lemma 1 is not as strong as implied by Proposition 2.

The lender's endowment (y) can easily be thought of as the pool of resources available to a money market fund (MMF). Larger pools of resources translate into larger exchanges of funding and collateral. So, as more resources become available for funding in the repo market, total credit increases.

Lemma 1 also provides some understanding into the determinants of the repo rate. The repo rate, shown in (10), is increasing in p. The primary mechanism for the liquidation value of collateral to affect the repo rate occurs through the amount of lending. Simply put, if the lender provides the borrower with more funding, then the planner proposes that the borrower compensate the lender with a higher interest rate.

In terms of the lender's resources, a higher value of y would be associated with a lower repo rate for a given amount of funding. For a given loan size, the planner proposes that the lender share some of his income with the borrower in the form of a lower interest rate. However, y also affects the repo rate through the size of funding. Lemma 1 shows that the latter effect of y dominates to the extent that a higher repo rate is subsequently offered.

Following Gorton and Metrick (2012) and Dang et al. (2013b), the solution for the haircut imposed, given (8) and (9), is equal to:

Corollary 1. (Haircuts) *Suppose that* $\rho = 1$ and $\eta = \frac{1}{2}$. *Further, let* $\underline{R} < R < \overline{R}$ *and* $p < \overline{p}$. *Under these conditions, the level of haircuts is:*

$$H^*(d^*_\alpha, d^*_\beta) = \frac{d^*_\alpha - d^*_\beta}{d^*_\alpha} = \frac{(3x + 2p\,(y - x))\,(R - 1) + y\,(4 - 3R)}{3x\,(R - 1) + y\,(R - 2)} \tag{11}$$

The haircut is increasing in p *and decreasing in* y.

In practice, haircuts in repo arrangements are purely determined on the basis of the collateral's value in secondary markets. The lower the value of the collateral to the lender, the greater the quantity of collateral required in the repurchase agreement to obtain funding. However, as we discuss below, the Corollary implies that the severity of the haircut should also depend on the amount of the loan. The differential below highlights both aspects:

$$\frac{d(d^*_\alpha)}{dp} - \frac{d(d^*_\beta)}{dp}$$

While Propositions 1 and 3 isolate the Planner's proposal for the borrower and lender individually, Lemma 1 accounts for the overall impact of the liquidation value on the repo solutions proposed by the Planner. The term $\frac{d(d^*_\alpha)}{dp}$ is the analogue to Proposition 3 in the aggregate. It shows how the liquidation value of the collateral affects the amount of collateral to be received by the lender. By comparison, $\frac{d(d^*_\beta)}{dp}$ reflects the behavior implied by Proposition 1. Solving the differential above determines the overall impact of the liquidation value on haircuts in the Corollary. As shown in the Corollary, haircuts are increasing in the liquidation value.

As alluded by Dang et al. (2013b) in the introduction, one might perceive that the impact of higher demand for credit funding would be completely absorbed through the interest rate. However, from our optimal risk-sharing

perspective, this conclusion would be much too narrow for funding obtained through a repo. Instead, the increase in the amount of risk borne by the lender (by foregoing income that is subject to default) should also be compensated by an increased reliance on collateral.

Moreover, the findings in the Corollary point to the importance of risk aversion for understanding repo markets. It can be easily seen from (7) in Proposition 3 that *only the size of the repo* (d_β) affects the amount of collateral if the coefficient of relative risk aversion is equal to 1. While logarithmic preferences provide a useful benchmark for studying repo activity when participants in the repo market are risk averse, the predictions regarding haircuts do not match up with available evidence. Consequently, we turn to studying behavior under higher risk aversion in the following section. In the numerical analysis below, we show that the predictions of the model match observations on haircuts if market participants are more risk averse than our benchmark. In this manner, our work highlights the importance of accounting for risk aversion in models of repurchase agreements.

We turn to the effects of the lender's resources (y) on the level of haircuts. By the Corollary, haircuts are lower if the supply of loanable funds in the repo market is higher. Haircuts are present as part of an efficient risk sharing arrangement. However, as lenders have more income, the marginal value of collateral per unit of funding diminishes.

Finally, both agents will agree to the planner's proposal as long as the following conditions are satisfied:

Agent A:

$$(1-\eta)\left[\frac{x^{1-\rho}}{1-\rho}+\frac{\left(\frac{y+(R-1)d_\beta^*}{2}\right)^{1-\rho}}{1-\rho}\right]+\eta\left[\frac{(x-d_\alpha^*)^{1-\rho}}{1-\rho}\right]\geq\frac{x^{1-\rho}}{1-\rho}$$

Agent B:

$$(1-\eta)\left[\frac{\left(\frac{y+(R-1)d_\beta^*}{2}\right)^{1-\rho}}{1-\rho}\right]+\eta\left[\frac{(y-d_\beta+pd_\alpha^*)^{1-\rho}}{1-\rho}\right]\geq\frac{y^{1-\rho}}{1-\rho}$$

4 Numerical Analysis

The analysis in the previous section studies repo agreements under the assumption of logarithmic preferences. The resulting degree of tractability allows us to

study repo agreements analytically. In this section, we expand on our previous work to incorporate some important aspects of repo market activity in recent years. For example, as shown quite forcefully during the financial crisis, how changes in investors' risk aversion can have a substantial impact on the repo market.

To do so, we choose a benchmark parameter set so that the predictions of the model match repo market variables observed in the data. Gorton and Metrick (2012) present evidence that haircuts (averaged across asset classes) were around 8% in November 2007, immediately preceding the beginning of the recession in the United States. Observations for repo rates are available from the Securities Industry and Financial Markets Association (SIFMA). Their data shows overnight repo rates on Treasury General Collateral averaged below 5% in 2007.

We study variations on the following benchmark set of parameters: $x = 1$, $y = 1$, $R = 2.7$, $p = 0.25, \eta_0 = 0.02$, and $\rho_0 = 7$.[7] Under this set of parameters, we obtain:

Loan	Collateral	Haircut	Repo Rate
0.6225	0.6782	8.226%	4.673%

In comparison to the prediction of zero repo rates of Dang et al. (2013b), the level of haircuts and repo rates in our numerical analysis are consistent with the data from Gorton and Metrick (2012) and SIFMA. As emphasized by Mills and Reed (2012) and Ewerhart and Tapking (2008), the desire for risk-sharing is a strong motivating factor in the repo arrangements between the borrower and lender. For example, it will be apparent below that the lender will accept a lower repo rate in return for a higher haircut.[8]

It is widely accepted that risk aversion in repo markets increased in the years preceding the financial crisis. Notably, Gorton and Metrick (2010, p.5) catalogue the rise of haircuts from 2005 - 2008: "ABX data show that the deterioration of the subprime market began in early 2007."[9] Moreover, haircuts ranged from nearly 0 in early 2007 to 5% in the fall of 2007. In the following year, haircuts rose to 25%. Finally, after the bankruptcy of Lehman Brothers, they increased to as high as 45%. In order to capture these features of repo markets, we consider different measures of risk-aversion for repo market activity:

[7] Mehra and Prescott (1985) cite that values of the coefficient of relative risk aversion up to 10 are plausible. Lansing and LeRoy (2014) also consider a range of values up to 10.

[8] The trade off between the marginal rates of insurance and return is a common risk-sharing result – see also Ewerhart and Tapking (2008) for a related discussion on repo markets and counterparty risk.

[9] The ABX index, or asset-backed securties index, measures the overall value of morgages in the subprime mortgage market.

ρ	Loan	Collateral	Haircut	Repo Rate
7	0.6225	0.6782	8.236%	4.673%
7.1	0.6191	0.6781	8.710%	4.235%
7.2	0.6158	0.6780	9.183%	3.377%
7.3	0.6126	0.6779	9.644%	2.957%
7.4	0.6094	0.6779	10.094%	2.542%

As *both* agents become more risk averse, haircuts on collateral increase. Moreover, risk aversion dramatically affects repo market activity by causing the amount of lending to shrink. To this extent, increases in risk aversion can ultimately cause repo markets to collapse. The decrease in collateral naturally follows the decline in repo activity. In fact, loan offers decrease at a faster pace than the borrower's posting of collateral – leading to a larger haircut. Consequently, the predictions of our model line-up well with the change in investor sentiment prior to the financial crisis.

In this manner, our work makes a clear contribution to the literature on repo markets. Standard models are confined to studying repo activity under risk-neutrality and therefore are not suited for capturing changes in the degree of risk aversion among market participants. Rather, they study investor sentiment through changes in the risk of collateral or the riskiness of the borrower's investment opportunities. Yet, as we discuss below, it is clearly important to study default risk when investors are risk averse.

The model also allows us to consider the effects of an increase in the default rate among borrowers in repo markets. Consequently, the model provides insights about repo market behavior following the two pivotal moments during the financial crisis – the collapse of Bear Stearns and the failure of Lehman Brothers – which clearly raised concerns about default. Naturally, the results are qualitatively similar to the effects of higher risk aversion:

η	Loan	Collateral	Haircut	Repo Rate
0.020	0.6225	0.6782	8.236%	4.673%
0.021	0.6197	0.6776	8.550%	4.310%
0.022	0.6170	0.6770	8.861%	3.960%
0.023	0.6144	0.6764	9.160%	3.622%
0.024	0.6119	0.6758	9.724%	3.294%

That is, at higher default rates, there is less lending activity along with an increase in haircuts.

However, the twin distortions from higher default rates along with higher risk aversion have a dramatic impact on the repo market. At a slightly higher degree of risk aversion ($\rho_1 = 7.5 > 7.0 = \rho_0$), the market is much more fragile as default rates rise:

η	Loan	Collateral	Haircut	Repo Rate
0.020	0.6064	0.6778	10.534%	1.025%
0.021	0.6037	0.6772	10.844%	1.022%
0.022	0.6012	0.6766	11.141%	1.018%

Notably, the market completely breaks down at lower thresholds of default as lenders no longer participate. The expected return from lending in the repo market is not incentive compatible when there is an increase in default rates at higher degrees of risk aversion. Rather, lenders would receive higher utility from simply consuming their endowments than lending to borrowers. As risk aversion in the market was easily higher after the default of Lehman Brothers than the collapse of Bear Stearns, the model shows us that repo markets would break down more quickly at higher degrees of risk aversion.

Available data on repo activity is in-line with the predictions of our analysis. The decline in repo activity was much larger after the bankruptcy of Lehman Brothers than the collapse of Bear Stearns when risk aversion in the market was not as high. For example, available survey evidence on the amount of total repo activity declined marginally after the Bear Stearns event. To be specific, data from the International Capital Markets Association (ICMA) sample of firms in Europe was around 3.24 trillion euros in December 2007. Later, in June 2008, the number stood at 3.17 trillion euros. Following Lehman, the decline was substantial. In December 2008, repo activity fell to 2.31 trillion euros – more than a 25% drop.

Data on haircuts during the crisis period is also highly suggestive of a breakdown in repo lending. In June 2007, Fitch Ratings data show that haircuts on AA and A- rated securities posted as collateral were equal to 25%. By comparison, two years later, haircuts increased to 100%, implying that lenders refused to lend against such securities.[10]

It is also well known that there were "fire-sales" of assets originating from the shadow banking system during the crisis. Therefore, as a final exercise, we turn to the effect of changes in the liquidation value of collateral on repo activity:

p	Loan	Collateral	Haircut	Repo Rate
0.25	0.6225	0.6782	8.236%	4.673%
0.24	0.6191	0.6860	9.760%	4.233%
0.23	0.6156	0.6940	11.296%	3.780%
0.22	0.6121	0.7022	12.840%	3.311%
0.21	0.6085	0.7107	14.387%	2.826%

Notably, our results are in stark comparison to Dang et al. (2013b). They argue that haircuts should only increase as *uncertainty* about the value of the

[10] Haircut data from Fitch as reported by the ICMA (Comotto, 2012).

collateralized asset increases. Therefore, because the lender does not initially know the true value of the collateral they, in turn, require a haircut. That is, in their model with risk-neutral agents, haircuts are used to pass-through the potential costs of information acquisition from a buyer in a secondary market to the borrower in a repo.

By comparison, in our framework, collateral and haircuts are part of an efficient risk-sharing arrangement between a risk-averse borrower and a risk-averse lender. In the event of default, lenders would like to have a certain amount of income. Haircuts are a way of over-collateralizing a secured credit arrangement in order to achieve optimal risk-sharing.

Our results are particularly interesting in the context of changes in counterparty (default) risk. At a slightly higher counterparty risk ($\eta_1 = .025 > 0.020 = \eta_0$), the market is much more fragile as the liquidation value of collateral falls:

p	Loan	Collateral	Haircut	Repo Rate
0.25	0.6096	0.6752	9.724%	2.976%
0.24	0.6063	0.6831	11.250%	2.528%
0.23	0.6029	0.6912	12.779%	2.064%

5 Conclusion

This paper studies security repurchase agreements in the presence of risk aversion. In so doing, we show that repo activity between borrowers and lenders reflects optimal risk sharing. Out of this risk sharing behavior, haircuts arise naturally in order to provide lenders with insurance against default risk. Interestingly, we find that the twin distortions from higher risk aversion along with counterparty risk or lower market values of collateral render repo markets highly susceptible to a collapse of activity. Therefore, policymakers must pay close attention to conditions in financial markets which can affect the repo market.

There are a number of interesting avenues for future research. First, the model could be extended to consider repos of different maturities when lenders are subject to liquidity risk as in Diamond and Dybvig (1983). Second, multiple types of collateral could be incorporated with varying degrees of liquidity in secondary markets. Consequently, as shown by Boulware, Ma, and Reed (2014), the level of activity, the type of collateral posted, and the maturity of repo arrangements would respond to changes in financial conditions.

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Appendix

1. Proof of Proposition 1.

Recall that:

$$d_\beta = \frac{y + pd_\alpha - \frac{y}{2}\left(\frac{\eta}{(1-\eta)(R-1)}\right)^{\frac{1}{\rho}}}{\left[1 + \frac{(R-1)}{2}\left(\frac{\eta}{(1-\eta)(R-1)}\right)^{\frac{1}{\rho}}\right]}$$

$$\frac{\partial d_\beta\left(d_\alpha\right)}{\partial \eta} = -\frac{\left(\frac{\eta}{(1-\eta)}\right)^{\frac{1-\rho}{\rho}}}{2\rho\left(1-\eta\right)^2}\left[\frac{y\left(R-1\right)^{-\frac{1}{\rho}} + \left(y + pd_\alpha\right)\left(R-1\right)^{\frac{\rho-1}{\rho}}}{\left[1 + \frac{1}{2}\left(\frac{\eta}{(1-\eta)}\right)^{\frac{1}{\rho}}\left(R-1\right)^{\frac{\rho-1}{\rho}}\right]^2}\right]$$

which is unambiguously negative.

2. Proof of Proposition 2.

Derivation of $\frac{\partial d_\beta}{\partial y}$

$$\frac{\partial d_\beta\left(d_\alpha\right)}{\partial y} = \frac{1 - \frac{1}{2}\left(\frac{\eta}{(1-\eta)(R-1)}\right)^{\frac{1}{\rho}}}{\left[1 + \frac{(R-1)}{2}\left(\frac{\eta}{(1-\eta)(R-1)}\right)^{\frac{1}{\rho}}\right]}$$

In order for d_β to be increasing in y :

$$\eta < \frac{2^\rho}{\frac{1}{(R-1)} + 2^\rho} \equiv \overline{\eta}$$

$$\frac{\partial d_\beta\left(d_\alpha\right)}{\partial R} = \frac{\left[\frac{y}{2\rho}\left(\frac{\eta}{(1-\eta)}\right)^{\frac{1}{\rho}}\left(R-1\right)^{-\left(\frac{1+\rho}{\rho}\right)}\right] + \left[\frac{y}{4}\left(\frac{\eta}{(1-\eta)}\right)^{\frac{2}{\rho}}\left(R-1\right)^{-\frac{2}{\rho}}\right] - \left[\frac{y+pd_\alpha}{2}\frac{\rho-1}{\rho}\left(\frac{\eta}{(1-\eta)}\right)^{\frac{1}{\rho}}\left(R-1\right)^{-\frac{1}{\rho}}\right]}{\left[1 + \frac{1}{2}\left(\frac{\eta}{(1-\eta)}\right)^{\frac{1}{\rho}}\left(R-1\right)^{\frac{\rho-1}{\rho}}\right]^2}$$

In order for d_β to increase with R :

$$\left(\frac{\eta}{(1-\eta)}\right) > \left[\left(\frac{y + pd_\alpha}{2} - \frac{pd_\alpha}{2\rho}\right)\frac{4}{y} - \frac{4}{\rho} - \frac{4}{(R-1)y}\right]^\rho (R-1)$$

This is the lower bound on default risk that is necessary for d_β to increase with R :

$$\eta > \frac{\left[4\left(\frac{y(\rho-2)+pd_\alpha(\rho-1)}{2\rho y}-\frac{1}{(R-1)y}\right)\right]^\rho (R-1)}{1+\left[4\left(\frac{y(\rho-2)+pd_\alpha(\rho-1)}{2\rho y}-\frac{1}{(R-1)y}\right)\right]^\rho (R-1)} \equiv \underline{\eta}$$

3. Proposition 3.

The proof of Proposition 3 is similar to Propositions 1 and 2.

4. Comparative Statics in Lemma 1.

Derivation of $\frac{\partial d_\beta^*}{\partial p}$:

$$d_\beta^* = \frac{px + y\left[2 - p\left\{\frac{2R-1}{2(R-1)}\right\}\right]}{3-p}$$

$$\frac{\partial d_\beta^*}{\partial p} = \frac{\left[x - y\left\{\frac{2R-1}{2(R-1)}\right\}\right]\cdot[3-p] + 1\cdot\left[px + y\left[2 - p\left\{\frac{2R-1}{2(R-1)}\right\}\right]\right]}{[3-p]^2}$$

$$\frac{\partial d_\beta^*}{\partial p} = \frac{\left[3x - 3y\left\{\frac{2R-1}{2(R-1)}\right\}\right] + 2y}{[3-p]^2}$$

$\frac{\partial d_\beta^*}{\partial p} > 0$ as long as:

$$x > \left(\frac{2R+1}{6(R-1)}\right) y \equiv x_0$$

5. Corollary 1

For a haircut less than 1 the return rate must satisfy:

$$R > \frac{6y - 2p\,(y-x)}{4y - 2p\,(y-x)} \equiv \underline{R}$$

For a haircut greater than 0 the return rate must be:

$$R < \frac{4y - 3x - 2p\,(y-x)}{(y-x)\,(3-2p)} \equiv \overline{R}$$

Note that:

$$\frac{\partial d_{\beta}^{*}}{\partial p}=\frac{\left[3x-3y\left\{\frac{2R-1}{2(R-1)}\right\}\right]+2y}{\left[3-p\right]^{2}}$$

$$\frac{\partial d_{\alpha}^{*}}{\partial p}=\frac{2y\left[\left(\frac{R-2}{(R-1)}\right)\right]+6x}{\left[2\left(6-p\right)\right]^{2}}$$

Haircuts are increasing in p *iff:*

$$\frac{2y\left[\left(\frac{R-2}{(R-1)}\right)\right]+6x}{\left[2\left(6-p\right)\right]^{2}}>\frac{\left[3x-3y\left\{\frac{2R-1}{2(R-1)}\right\}\right]+2y}{\left[3-p\right]^{2}}$$

$$R>\frac{6y-2p\left(y-x\right)}{4y-2p\left(y-x\right)}$$

which is satisfied by the assumption that $R>\underline{R}$ as long as:

$$p<\frac{6y\left(x+3\right)-14y^{2}}{\left(7y-6\right)\left(x-y\right)}\equiv\overline{p}$$

The effects of y on the level of haircuts follow in a similar manner.