Banking Structure, Policy Responses, and Financial Fragility*

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Abstract

Events in recent years have drawn attention to the potential for fragility in the banking system along with concerns about the competitive structure of the banking sector. To examine these important issues, we study suspension policies in an environment without commitment, following Ennis and Keister (2009). In their framework, financial intermediaries are perfectly competitive and the incentives of banks perfectly align with a banking authority (BA). If the banking sector is not competitive, the objectives of the BA are less clear. If the BA only values the welfare of depositors, surprisingly, fragility is the same across the competitive structure of the banking sector. Although the monopolistic bank has positive profits that provide a buffer against runs, the BA will tend to freeze deposits later than under perfect competition and the two channels offset each other. If, instead, the BA puts some weight on the monopolist's utility, fragility is lower in the concentrated system.

Keywords: Bank Runs, Bank Size, Competitive Banks, Monopoly Bank, Suspension of Convertibility, Time Inconsistency

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1 Introduction

How should banking authorities respond to stress in the banking system? When runs on institutions start to take place, does optimal decision-making by banking regulators depend on the size or the competitive structure of the banking sector? To address these important questions we extend the framework of Ennis and Keister (2009) who study suspension policy and fragility under perfect competition. While they argue that a BA will eventually intervene and suspend banks, depositors anticipate that the BA will do so relatively late, thereby validating the decision to run on the banking system. However, in a standard framework with perfect competition, it is obvious that a BA's objective function focuses on depositors. Further, under perfect competition, the incentives of banks are perfectly aligned with social objectives.

Yet, past experiences with government-declared bank suspensions indicate that these actions can be subject to tension between large banks and banking authorities. For example, Friedman and Schwartz (1963) describe how large bankers in New York successfully lobbied for a state banking holiday while President Hoover initially resisted calls from the same group for a nation-wide holiday during the Great Depression. From this perspective, one might argue that the New York bankers could exert greater political pressure in the state of New York than across the United States. In addition, Kennedy (1973, p.67) documents the inter-play between bankers and state bank authorities across different states in 1933. In the case of Nevada, Kennedy argues that the banking system was highly concentrated which actually appeared to result in suspensions relatively quickly: "A state's governmental administration became closely tied with its financial institutions. And when those banks were imperiled, the state did all in its power to save them." In this manner, anticipation of the decision by the BA to allow banks to suspend early would have reduced the incentives for run behavior to emerge. Further, due to the size and role of such banks within the state of Nevada, the banks in some sense were "too big to fail." Thus, it appeared that monopoly power in the banking system also led to great political power. Moreover, these concerns about the power of large institutions impacted the regulatory landscape and development of the banking sector across the United States in the years prior to the Great Depression. For example, Allen and Gale (2000, p.234) write: "Distrust of power in the hands of very large institutions very early led to restrictions on the ability of banks to expand geographically or to diversify into other activities."

Following this logic, governments' objectives may not be perfectly aligned with depositors' incentives.[1] In particular, essential factors such as political power and pressure have received much attention in a number of recent cross-country studies.[2] However, there has been less attention to the

[1] Repullo (2000), for example, assumes that some government agencies such as a central bank or deposit insurance corporation have their own objectives which differ from social welfare in a political economy framework.

[2] For example, Brown and Dinc (2005) identify the significant role of political concerns in regulating the banking system. In particular, they find closures of failing banks are less likely to occur prior to elections as governments

issue in theoretical frameworks that study banking fragility. If the banking sector is not competitive, the objectives of the BA are less clear but they would certainly matter for fragility. This seems quite important in light of events during the recent financial crisis. In particular, many argued that the actions of governments were arbitrary which clouded decision-making by participants in the financial system. Thus, developing frameworks to study how the *preferences of banking regulators affect decision-making* in periods of stress seems to be an important task.

The objective of this paper is to examine the relationship between the competitive structure of the banking system, fragility, and policy responses by banking regulators. As a benchmark, we begin by looking at the conditions for run behavior to emerge in a competitive banking sector. In this regard, much of our analysis directly follows Ennis and Keister although we do consider whether the size of the banking system matters for fragility. To examine the relationship between fragility and the competitive structure of the banking system, we study how a BA adopts policy in response to a run on a monopolistic bank. As the monopolistic bank is focused on maximizing profits and not depositors' welfare, in principle, self-fulfilling runs would never occur. As soon as unanticipated redemptions begin to take place, such an institution would immediately suspend and the commitment to suspending withdrawals would be credible – unlike the claims for perfectly competitive banks in Diamond and Dybvig (1983) which were later rebuked by Ennis and Keister.

Nevertheless, if it is the sole discretion of the BA to allow suspensions, monopolistic banks may still be susceptible to runs because the commitment to suspend by the BA may not be rational. Along these lines, we derive conditions for fragility in the absence of commitment by the BA. If the BA only values the welfare of depositors, surprisingly, fragility is the same across the competitive structure of the banking sector. Although the monopolistic bank has positive profits that provide a buffer against runs, the BA will tend to freeze deposits later than under perfect competition and the two channels offset each other. If, instead, the BA puts some weight on the monopolist's utility, fragility is lower in the concentrated system because the BA will cut off access to funds for some depositors truly in need in order to protect a bank's monopoly rents.

The rest of the paper is organized as follows. To begin, section 2 outlines a benchmark model with *ex-post* efficient policy responses by a benevolent banking authority (BA) that directly follows Ennis and Keister. In section 3, we study an environment where the banking system is fully concentrated. We then compare how the suspension decisions in a monopolistic banking system differ from the decisions in a perfectly competitive banking sector. We also investigate how bank

face pressure from various interest groups that would be adversely affected. In addition, Rosas (2006) also suggests that governments vary widely in their propensity to rescue institutions and that politics at various levels plays a role. Furthermore, Faccio, Masulis, and McConnell (2006) argue that political connections are closely related to corporate bailouts. To be specific, they investigate the likelihood of government bailouts of 450 politically connected firms from 35 countries during 1997-2002. They conclude that politically connected firms are significantly more likely to be bailed out than similar nonconnected firms. In this manner, political connections impact the allocation of capital when connected companies experience economic distress.

size plays a role in each system. Next, we proceed to look at run behavior in settings where the BA lacks a technology to communicate its preferences to depositors. In Section 4, we offer concluding remarks.

2 A Competitive Banking System

We first study the bank run model of Ennis and Keister (2009), but we allow for size variation in the deposit base of the banking system. In this way, we are able to study how run behavior and the desirability of allowing bank suspensions depends on the size of the banking sector. However, our principal focus is to introduce some essential results regarding suspension policy and fragility under perfect competition that were introduced by Ennis and Keister. Our primary contributions are presented in the following section where we study fragility in a monopolistic banking system.

2.1 The Environment

There are three time periods, indexed by $t = 0, 1, 2$. There is also a continuum of *ex-ante* identical agents called depositors with mass one, $i \in [0, 1]$. In contrast to Ennis and Keister, each depositor is endowed with e units of the single consumption good in period 0 and nothing in periods 1 and 2. From this perspective, economies where depositors have larger endowments implies that banks' balance sheets would also be larger and total assets in the banking system would also be higher. Let c_t denote the amount of goods consumed at period t. Depositors' preferences over consumption in each period are:

$$u(c_1, c_2; \omega_i) = \frac{(c_1 + \omega_i c_2)^{1-\gamma}}{1-\gamma}$$

where $\gamma \in (0, 1)$. Further, ω_i is a binomial random variable that has support $\{0, 1\}$. If $\omega_i = 0$, the depositor is impatient and only derives utility from consumption in period 1. By comparison, if $\omega_i = 1$, the depositor is patient and can derive utility from consumption equally in both periods 1 and 2. Agents privately observe their type at the beginning of period 1. In addition, each depositor has a probability π of being impatient. Moreover, π is deterministic. Consequently, by the law of large numbers, π is also the aggregate proportion of impatient depositors in the population. Further, a consumption plan for depositor i is then $c^i = (c_1^i, c_2^i) \in \mathbb{R}^2_+$.

There are two kinds of constant-returns-to-scale technologies. First, there is a simple storage technology available where 1 unit of the consumption good returns 1 unit of the good in the following period. On the other hand, there is also an investment technology available which takes time to mature. From this technology, an investment of one unit of the consumption good returns $R > 1$ units of the consumption good placed in period 2. If resources from the technology are pulled in

period 1, a fraction $\tau \in (0,1)$ of the investment will be lost. That is, τ represents the liquidation cost from the investment technology because illiquid investment is not easily convertible into the consumption good.

Consumers deposit their funds at the beginning of period 0. Since the population of consumers is equal to one, each bank has e units of deposits. If e is relatively large, we refer to this bank as a "large" bank. Otherwise, the bank is called a "small" bank. In the decentralized economy, a large number of identical banks operate an intermediation technology. Each bank behaves competitively such that intermediaries act to maximize the expected utility of their depositors. In addition, there is a benevolent banking authority (BA) which has the power to suspend convertibility of deposits in period 1 if it believes doing so would improve social welfare.

Upon learning their types, each depositor has equal opportunity to contact the bank once in either period 1 or 2. Depositors who contact the bank in period 1 do so in a randomly assigned order. We follow Wallace (1988) in assuming that depositors are spatially separated and cannot trade with others in period 1. Hence, depositors must consume immediately after contacting the bank. In addition, throughout the paper, we further assume that the bank ignores the possibility of runs when determining a "simple" contract that promises a fixed period-1 payoff until the intermediary runs out of funds completely.[3] Notably, following Ennis and Keister, the BA is also subject to the sequential service constraint. That is, a BA can only freeze deposits that have not been withdrawn.

2.2 The First-best Allocation

To begin, we consider the planner's allocation. We then proceed to decentralize this optimal allocation under the planning problem. In this benchmark case, a planner can observe all depositors' types and has the objective to maximize aggregate depositor welfare. Let c_1 denote the amount paid to impatient depositors and c_2 the amount paid to patient depositors. Also, x denotes the fraction of total deposits placed into the investment technology. In addition, an allocation is a list of consumption plans, which are given by $\{(c_1^i, c_2^i)\}_{i\in[0,1]}$, and the allocation is symmetric if $(c_1^i, c_2^i) = \left(c_1^j, c_2^j\right)$ for all i, j. Notably, for the best symmetric allocation, the planner solves:

$$W_P \equiv \max_{\{c_1,c_2\}} \pi \frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma} \tag{1}$$

subject to

$$\pi c_1 = e - x, (1-\pi)c_2 = Rx,$$

[3]Ennis and Keister (2009a) assume that competitive banks offer simple contracts in this form. Such simple contracts can also be justified by the environment here. As in Freeman (1988), depositors will only accept fixed payoff contracts since the number of period-1 withdrawals is not observed by depositors until period 2. For more detailed discussion, see also Ennis and Keister (2006).

$$c_1 \geq 0,\ c_2 \geq 0, \text{ and } 0 \leq x \leq e.$$

In particular, (c_1^*, c_2^*) represents the solution to the problem. Thus, the solutions c_1^* and c_2^* are:

$$c_1^* = \frac{e}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}};\ c_2^* = \frac{eR^{\frac{1}{\gamma}}}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}, \tag{2}$$

where c_1^* and c_2^* are strictly increasing in bank size, e. In other words, compared to depositors with lower initial endowments, depositors with higher initial wealth receive strictly larger consumption in both periods. Otherwise, the deposit contract is suboptimal. Notably, a competitive bank can decentralize this optimal allocation and offer deposit contracts described above. However, the deposit contracts admit the possibility of a run equilibrium if:

Assumption 1: Existence of Bank Run Equilibrium under Perfect Competition. $c_1^* > e - \tau x^*$

The exact proof of Assumption 1 can be found in Cooper and Ross (1998). Moreover, as explained in Ennis and Keister, this assumption implies that the amount of consumption to be received by impatient depositors is greater than the per capita liquidation value of all assets in period 1. Hence, the planner's allocation provides liquidity insurance to depositors. Using (2), Assumption 1 can be reduced to the following parameter values:

$$\tau > 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}}. \tag{3}$$

This condition shows that the banking system can be illiquid in the short-run. Specifically, the size of liquidation costs is critical in determining the susceptibility of the banking system to runs. Whenever liquidation costs are significant, Assumption 1 will hold regardless of other parameters. Furthermore, the size of the bank does not impact whether Assumption 1 holds. That is, the bank run equilibrium condition is independent of the size of a bank's deposit base.

2.3 Bank Runs under Perfect Competition and a Simple Suspension Scheme

As in Diamond and Dybvig (1983), given the fact that the banking system is illiquid under Assumption 1, there can exist two equilibria. First, depositor i chooses a strategy that assigns an action to each possible realization of her preference type ω_i,

$$y_i \in \{0, 1\},$$

where $y_i = 0$ corresponds to withdrawing in period 1 and $y_i = 1$ corresponds to waiting until period 2. Also, let y denote a profile of strategies for all depositors. Then $y = \{y_i\}_{i\in[0,1]}$ is a complete profile of withdrawal strategies, and y_{-i} represents profile of strategies for depositors except i. On one-hand, the first-best allocation can be achieved as an equilibrium in which every depositor's withdrawal strategy follows $y_i(\omega_i) = \omega_i$ for all i. On the other hand, patient depositors can be driven by self-fulfilling beliefs that all other patient depositors misrepresent their type and withdraw in period 1. In this case, knowing that the bank is not able to satisfy all depositors' demand on deposits, she would find it individually rational to withdraw early in response to $y_{-i} = 0$. As a result, this strategy profile generates another equilibrium that resembles a run on the banking system. Following Ennis and Keister, the banking system is *fragile* if this type of run equilibrium exists.

As the banking system is prone to instability, Diamond and Dybvig show how a deposit freeze policy by the BA with commitment can make the first-best allocation the unique equilibrium outcome. Specifically, once the BA recognizes that a run is underway since unexpected redemptions occurred, it then suspends convertibility of deposits immediately at π withdrawals. Consequently, the remaining patient depositors must wait until period 2 to withdraw. However, the remaining impatient depositors would not receive anything. This policy response guarantees that the bank has sufficient resources and thus eliminates the bank run equilibrium. As a result, the first-best allocation can be achieved as runs would never occur. This argument is expressed in the following Proposition:

Proposition 1: Tough Suspension Policy with Commitment and Stability of the Banking System. *If deposits are frozen after π withdrawals in period 1, the banking system is not fragile.*

The rationale behind the Proposition is straightforward – patient depositors are guaranteed to receive a higher period-2 payoff than period-1 payoff. Therefore, waiting to withdraw is a strictly dominant strategy for patient individuals, and the only equilibrium has all patient depositors withdrawing in period 2. Thus, the banking system is not fragile. But, as argued by Ennis and Keister, the suspension policy put forward in Diamond and Dybvig is not *ex-post efficient.* Instead, the BA would choose an optimal freeze point as the banking crisis develops.

We proceed to study the *ex-post* desirable arrangement for a deposit freeze and the role of bank size for this suspension policy. Is there an equilibrium with $y_i = 1$ for all i? Suppose all depositors follow this strategy profile, what will be the best response of the government? First, we refer to π_s as the "freeze point" imposed by the BA. This is the fraction of depositors the bank would serve before the BA freezes deposits. Notably, the period-2 payoff critically depends on this suspension policy and will be the equal share of remaining resources left at bank. In particular, the payout

schedule in period 2 is characterized by:

$$c_2(\pi_s) = \left\{ \begin{array}{c} \frac{R}{1-\tau}\left(e-\tau x^*-\pi_s c_1^*\right) \\ \hline (1-\pi)(1-\pi_s) \\ 0 \end{array} \right\} \text{ as } \left\{ \begin{array}{c} \pi \leq \pi_s < \pi_s^U \\ \hline \pi_s = \pi_s^U \end{array} \right\}, \tag{4}$$

where $\pi_s^U \in (\pi, 1)$ is the maximum proportion of depositors that can be paid c_1^* in period 1. That is, $\pi_s^U \equiv \frac{e-\tau x^*}{c_1^*}$. The BA then seeks to maximize the welfare of depositors by solving the following problem:

$$W_{BA}\left(\pi_s\right) \equiv \max_{\{\pi_s\}} \pi_s \frac{\left(c_1^*\right)^{1-\gamma}}{1-\gamma} + (1-\pi_s)(1-\pi)\frac{\left(c_2(\pi_s)\right)^{1-\gamma}}{1-\gamma} \tag{5}$$

The objective (5) states that the BA would allow π_s depositors to be served in period 1, and each will receive c_1^*. Moreover, there are $(1-\pi_s)$ remaining depositors, and only a fraction $(1-\pi)$ would be patient. Consequently, each of the remaining patient depositors would receive an equal share of the resources left in the bank. This amount of consumption is denoted by the function $c_2\left(\pi_s\right)$ in (4). The other fraction of $(1-\pi_s)\pi$ impatient depositors will receive nothing, which leads to zero utility.

Following Ennis and Keister, under Assumption 1, the function $c_2\left(\pi_s\right)$ is strictly concave. Combined with the concavity of the utility function, this fact implies that the objective function (5) has a unique solution (the welfare-maximizing solution) denoted by π_s^M, where $\pi_s^M \in \left[\pi, \pi_s^U\right]$. Thus, the complete profile of strategies in this withdrawal game is thus $(y, \pi_s - \pi) \in \{0,1\}_{i\in[0,1]} \times [0, 1-\pi]$.

First, we offer the following Proposition which shows that when liquidity demand in the short run is small, the BA would find it optimal to declare a deposit freeze completely right at π:

Proposition 2: Credible, Tough Suspension Policy under Perfect Competition. *The optimal freeze point, π_s^M, is equal to π if π satisfies:*

$$\pi \leq 1 - \left[\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\tau} + \left(\frac{\gamma}{1-\gamma}\right) R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{\gamma}} \equiv \underline{\pi}. \tag{6}$$

If liquidity demand is small, the BA would not find it optimal to delay the suspension after identifying a run. Instead, freezing deposits completely is the optimal choice. By doing so, the BA could eliminate runs without imposing zero consumption on many depositors. This occurs because there is a sufficiently small number of truly impatient consumers in the population. Thus, the BA's commitment is credible and the first-best allocation is achieved.

It is also worth noting how fundamentals such as liquidation costs affect the scope for complete

suspension policy. Notably, $\underline{\pi}$ is an increasing function of τ. Thus, as liquidation costs are higher, there is a wider range of π in which the BA will impose suspensions immediately after identifying a run. Similar analysis applies to the return to the investment technology. However, bank size does not have any impact on the scope for complete suspensions.

Nevertheless, if liquidity demand is sufficiently large, then $\pi_s^M \in \left(\pi, \pi_s^U\right]$. Under this condition, the BA balances important trade-offs in trying to maximize social welfare. First, delaying the deposit freeze implies that a smaller number of the truly impatient will lose an opportunity to consume. The problem is the decision to delay decreases the period-2 payoff to the remaining patient depositors. Further, if depositors expect such a lenient response, this expectation could be a source of banking fragility. The key determinant is whether $c_2(\pi_s^M) > c_1^*$ holds. The following Lemma addresses this issue:

Lemma 1: Freeze Points and Fragility under Perfect Competition. *There exists a value of $\pi^T \in \left[\pi, \pi_s^U\right]$ such that for all freeze points $\pi_s \in [\pi, \pi_T)$, the banking system is not fragile.*

The Lemma shows the effectiveness of a deposit freeze policy in preventing a self-fulfilling run. Specifically, if the *ex-post* efficient deposit freeze π_s^M exceeds threshold value π^T, which is defined by the condition $c_1^* = c_2\left(\pi^T\right)$, patient depositors would find it optimal to withdraw in period 1 in response to this lenient policy. Consequently, this strategic profile makes the banking system fragile. Furthermore, substituting c_1^* and x^* in π^T yields:

$$\pi^T = \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right] - (1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau} - (1-\pi)\right]\widetilde{R}},$$

where $\widetilde{R} \equiv \left[\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-1}$. Notably, as the expression shows, the threshold freeze point (π^T) does not depend on the size of the bank.

The following Proposition summaries the main result of this section. It states that it is possible that the BA's socially optimal deposit freeze induces fragility in the banking system:

Proposition 3: Fragility under Perfect Competition. *Under optimal suspension policy by a benevolent BA, the perfectly competitive banking system is fragile if:*

$$\pi \geq \frac{1-\gamma}{\gamma}\left(\frac{R}{1-\tau} - 1\right) \equiv \widetilde{\pi}. \tag{7}$$

In particular, the fragility condition under perfect competition is independent of bank size.

The proposition highlights that if real liquidity demand is sufficiently large, the banking system tends to be fragile. Moreover, the cost of liquidation, return to investment and coefficient of risk

aversion are all crucial parameters. Specifically, if illiquid investments are not easily convertible into liquid resources or if the return to investment is sufficiently high, the BA would choose to protect the assets of the bank and a relatively early suspension would be imposed. Consequently, the banking system is not fragile and the bank run equilibrium is eliminated. Thus, the first-best allocation can be implemented. However, if depositors are sufficiently risk averse, the BA would find it optimal to freeze deposits relatively late but a self-fulfilling run could occur, leading to a fragile financial system. It is also worth noting that bank size does not affect the fragility condition. In this manner, fragility does not depend on the size of banks' balance sheets.

Moreover, bank size does not appear to impact the optimal suspension decision for the banking system under perfect competition. We offer the following example to illustrate. Suppose $\pi = 0.75$, $R = 1.2$, $\gamma = 0.5$, and $\tau = 0.3$. Under this set of parameters, $\widetilde{\pi} = 0.7143$, so it would be optimal to induce fragility by an ex-post suspension scheme. Please see Table 1 below which lists the optimal freeze points across different endowments for depositors:

e	0.8	0.9	1.0	1.1	1.2
π^T	0.9532	0.9532	0.9532	0.9532	0.9532
π_s^M	0.9535	0.9535	0.9535	0.9535	0.9535

Table 1. Bank Size, Thresholds, and Optimal Freeze Points

As can be observed, the optimal freeze point does not appear to be affected by bank size in perfectly competitive banking system. Nevertheless, we argue that the results presented here might be overturned when banking structures are taken into consideration seriously. The BA could still find it optimal to react to a crisis quickly for a highly concentrated bank that possesses monopoly power and earns profits. In other words, the BA may not only be concerned with the welfare of depositors but also the bank's profits, possibly resulting from tensions between the banking regulators and banks. To address this issue, we turn to suspension policy in a monopolistic banking system in the following section.

3 A Monopolistic Banking System

In this section, we proceed to study how a BA responds to a run on a concentrated, monopolistic banking system. As in the previous section, we also consider the role of bank size.

3.1 The Environment

The environment is close to that of the basic model. However, the banking economy is fully concentrated. Further, the monopolistic banker only consumes in period 2 and is risk-neutral:

$$\widehat{W}_B = v\left(\psi\right) = \psi,$$

where ψ denotes the bank's profits. Moreover, as there is only one bank available, an agency problem arises naturally: the monopolistic bank exerts its market power in the deposit sector. Nevertheless, individuals face a portfolio decision in which they can directly invest in storage technology or invest in the bank in the form of deposits. Therefore, the monopolistic institution must offer sufficient incentives to attract these deposits. In other words, the bank faces a participation constraint among depositors. For tractability, as in Matsuoka (2013), individuals only have access to a storage technology. Consequently, individuals are always willing to deposit as long as they can be guaranteed to derive an expected utility higher than the autarky level:

$$\widehat{W}_D \geq \pi \frac{e^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{e^{1-\gamma}}{1-\gamma} = \frac{e^{1-\gamma}}{1-\gamma}.$$

3.2 A Monopoly Bank's Asset Allocation

We start our analysis with the monopoly bank's asset allocation problem. To begin, let $\widehat{c}_1$ denote the amount paid to depositors who withdraw in period 1 and $\widehat{c}_2$ the amount given to depositors who return in period 2. Further, $\widehat{x}$ denotes the fraction of deposits received that are allocated to the investment technology. As discussed above, the bank seeks to maximize profits so that it will only offer sufficient returns where the participation constraint binds. Therefore, the monopoly bank determines the optimal investment level and endowment dependent contracts $(\widehat{c}_1, \widehat{c}_2)$ by solving:

$$\widehat{W}_B = v(\psi) \equiv \max_{\{\widehat{c}_1, \widehat{c}_2, \widehat{x}\}} R\widehat{x} - (1-\pi)\widehat{c}_2 \tag{8}$$

subject to:

$$\pi\widehat{c}_1 = e - \widehat{x},$$

$$\pi\frac{(\widehat{c}_1)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(\widehat{c}_2)^{1-\gamma}}{1-\gamma} = \frac{e^{1-\gamma}}{1-\gamma},$$

and also usual non-negative requirements on $\widehat{x}$, $\widehat{c}_1$, and $\widehat{c}_2$.

Let $(\widehat{c}_1^*, \widehat{c}_2^*)$ denote the solution to this problem. As in the case of perfect competition, the bank offers a risk-sharing contract to depositors. However, under perfect competition, the risk-sharing assignment has an upper bound due to the limited resources of banks. Instead, the monopolistic

institution offers a level of risk-sharing which has a lower bound, depending on the expected utility from autarky. Hence, the expressions for $\widehat{c}_1^*$ and $\widehat{c}_2^*$ are:

$$\widehat{c}_1^* = \left[\frac{e^{1-\gamma}}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} \text{ and } \widehat{c}_2^* = \left[\frac{e^{1-\gamma}R^{\frac{1-\gamma}{\gamma}}}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}. \tag{9}$$

Clearly, in order for the possibility of a bank run to arise, the following assumption must be satisfied:

Assumption 2: Existence of Bank Run Equilibrium in a Monopolistic Banking System. $\widehat{c}_1^* > e - \tau\widehat{x}^*$

Using (9), Assumption 2 can be shown to be equivalent to the following condition:

$$\tau > \frac{1 - \widetilde{R}^{\frac{1}{1-\gamma}}}{1 - \pi\widetilde{R}^{\frac{1}{1-\gamma}}}. \tag{10}$$

Similar to (3), the condition implies that if liquidation costs are high, Assumption 2 will hold. Hence, the bank's allocation could have a run equilibrium. As in Matsuoka (2013), there are different conditions for the existence of a run equilibrium across banking economies. For example, a monopoly bank can always satisfy more early redemptions than a bank in a competitive banking sector because the monopolistic bank provides a relatively lower level of risk-sharing. Thus, our result mirrors Matsuoka who demonstrates that the parameter space in which a run equilibrium is possible under perfect competition is larger than that for a monopoly bank.

Notably, as can be observed, condition (3) under perfectly competition is independent of liquidity demand. By comparison, $\widetilde{R} \equiv \left[\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-1}$, so the run equilibrium condition in a monopolistic system does depend on the degree of liquidity risk. In this manner, there exists a minimum value of size of the liquidity shock, π^*, below which only competitive banks are susceptible to runs, while deposit contracts offered by monopoly bank are run-proof. We formalize this result in the following corollary.

Corollary 1: Comparing Conditions for Run Equilibria across Banking Systems. *Suppose τ, R and γ satisfy condition (3) and $\overline{\tau}$ and $\underline{\tau}$ are defined in the appendix. Further, suppose that $\tau \in (\underline{\tau}, \overline{\tau})$ where $\overline{\tau} > \underline{\tau}$ if:*

$$\max\left\{\frac{1 - \widetilde{R}^{\frac{1}{1-\gamma}}}{1 - \pi\widetilde{R}^{\frac{1}{1-\gamma}}}, \frac{R^{\frac{1-\gamma}{\gamma}} - 1}{R^{\frac{1-\gamma}{\gamma}} - \gamma}\right\} < 1 - R^{-\frac{1}{\gamma}}.$$

Under these conditions, there exists a unique value, $\pi^* \in (0,1)$ such that for all $\pi \in (0, \pi^*)$, a run

equilibrium occurs under perfectly competitive banks, but not in a monopolistic banking system. In particular, the condition is independent of bank size.

The proof of existence and uniqueness of π^* is provided in the appendix. However, we should be clear that the above arguments do not take optimal suspension policy into account. For example, condition (6) shows that if $\pi \leq \underline{\pi}$, the BA would invoke a credible, tough suspension policy in a competitive banking system. We have yet to consider this possibility in a monopolistic bank. Thus, such arguments are only valid in the absence of intervention by a BA.

Notably, in seeking to maximize profits, the monopoly bank allocates more assets to the investment technology than perfectly competitive banks. To see this point, we define the difference between investments as Δ_x and verify that $\Delta_x > 0$ where:

$$\Delta_x \equiv \pi e \widetilde{R} \left(1 - \widetilde{R}^{\frac{\gamma}{1-\gamma}}\right).$$

Proposition 4: Comparing the Level of Investment across Banking Structures and the Role of Bank Size. *The amount of assets allocated to the investment technology in a monopolistic banking system is higher than that under perfect competition. Further, the difference is increasing in bank size.*

Similar to what Matsuoka (2013) shows, the proposition demonstrates that investment assets in a monopolistic banking sector are strictly larger than those under perfect competition. This is due to the fact that the monopoly bank is focused on maximizing profits. However, in contrast to Matsuoka, we find that the difference also depends on bank size. In this manner, the excessively high level of investment in the illiquid asset is increasing in bank size – the larger the balance sheet, the greater the degree of inefficiency.

To continue, while bank size does not appear to be a factor in the different conditions for a run equilibrium to emerge, it does impact the different levels of solvency across banks. We can examine the solvency of each type of intermediary by comparing the assets and liabilities of each bank. For example, consider the balance sheets of each bank if they are able to hold all of their illiquid investments until maturity:

Hold to Maturity Prices, Perfect Competition

Assets	Liabilities
Investment $= Rx^*$	Deposits $= c_1^*$
Storage $= e - x^*$	
	Equity $= E_S$

Hold to Maturity Prices, Monopoly Bank

Assets	Liabilities
Investment $= R\widehat{x}^*$	Deposits $= \widehat{c}_1^*$
Storage $= e - \widehat{x}^*$	
	Equity $= \widehat{E}_S$

Hence, from the balance sheet, a competitive bank's equity is denoted by:

$$E \equiv (R-1)\, x^* - c_1^* + e.$$

$\widehat{E}$ has an analogous expression. Next, the difference in equity is defined as: $\Delta_E \equiv \widehat{E} - E$. Consequently, simple algebra shows that the difference in equity is positive where:

$$\Delta_E = e\widetilde{R}\left(1 - \widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\left[\pi\,(R-1)+1\right].$$

Therefore, it is easy to see that $\frac{\partial \Delta_E}{\partial e} > 0$. This implies that the monopoly bank becomes more solvent relative to competitive banks as bank size increases.

We now turn to the relative degrees of illiquidity by looking at the liquidation values across the different banks:

Liquidation Prices, Perfect Competition

Assets	Liabilities
Investment $= (1-\tau)\, x^*$	Deposits $= c_1^*$
Storage $= e - x^*$	
	Liquidation Value $= L$

Liquidation Prices, Monopoly Bank

Assets	Liabilities
Investment $= (1-\tau)\, \widehat{x}^*$	Deposits $= \widehat{c}_1^*$
Storage $= e - \widehat{x}^*$	
	Liquidation Value $= \widehat{L}$

The liquidation value of a perfectly competitive bank is:

$$L \equiv e - \tau x^* - c_1^*.$$

According to Assumption 1, $e - \tau x^* < c_1^*$, so $L < 0$. Thus, the competitive bank is illiquid. By comparison, at the monopoly bank:

$$\widehat{L} \equiv e - \tau \widehat{x}^* - \widehat{c}_1^*.$$

Similarly, from Assumption 2, $\widehat{L} < 0$. This shows that the monopolistic bank is also illiquid. In turn, the difference in the liquidation values is $\Delta_L \equiv \widehat{L} - L$, which is positive. Specifically, simple algebra yields:

$$\Delta_L = e\widetilde{R}\left(1 - \widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)(1 - \tau\pi).$$

Therefore, it is easy to verify that $\frac{\partial \Delta_L}{\partial e} > 0$. Though it is still illiquid, the liquidity differential is augmented as bank size increases.

While the monopoly bank is more solvent, it is also less illiquid than a perfectly competitive bank because it offers lower returns in the first period. We make these comparisons formally in the

following Lemma:

Lemma 2: Differences in Equity and Liquidation Values, Bank Size Effects, and Banking Stability. *Let $\underline{R}$ and $\overline{R}$ be as defined in the appendix.*

(i) $\Delta_E > 0$ *and* $\frac{\partial \Delta_E}{\partial e} > 0$.

(ii) $\Delta_L > 0$ *and* $\frac{\partial \Delta_L}{\partial e} > 0$.

(iii) $\frac{\partial \Delta_E}{\partial e} > \frac{\partial \Delta_L}{\partial e}$.

(iv) Δ_L *is strictly* $\left(\begin{array}{c} \textit{increasing} \\ \textit{constant} \\ \textit{decreasing} \end{array} \right)$ *in* R *as* $R \in \left\{ \begin{array}{c} (1, \underline{R}) \\ \underline{R} \\ (\underline{R}, \overline{R}) \end{array} \right\}$ *if* $\pi \in \left(0, \frac{(1-\gamma)^{\frac{1}{\gamma}} - (1-\tau)}{(1-\gamma)^{\frac{1}{\gamma}} \tau}\right)$.

As explained, simple algebra shows that the monopoly bank becomes more solvent relative to its competitive counterpart as bank size increases. Similar results are shown for liquidity. That is, a large monopolistic bank is less illiquid than a large perfectly competitive bank. Moreover, the relative difference between solvency and illiquidity is increasing in bank size. In this manner, the larger the balance sheet, the greater the difference in solvency and illiquidity across the competitive structure of the banking system.

In addition to effects by the size of bank's balance sheet, the investment return, R, exhibits a non-monotone relationship with respect to the differences in liquidation values across structures. To be specific, if $1 < R \leq \underline{R}$, the liquidity differential is increasing in asset returns. By comparison, once R exceeds threshold $\underline{R}$ and becomes relatively large, the difference in the liquidation value begins to decrease. Thus, a given difference in the liquidation value across competitive structures can occur at two different bank sizes.

Discussion. Similar to the two equilibria generated by the first-best allocation under perfect competition, two types of equilibria can also arise under the monopoly bank's asset allocation. The intuition is the same. On one-hand, the profit maximization allocation can be achieved as a Nash equilibrium outcome if all depositors follow $y_i(\omega_i) = \omega_i$ for all i. On the other, self-fulfilling runs can occur due to a wave of economy-wide pessimistic expectations.

If the monopoly bank could choose independently when it can suspend, the bank would not face the same commitment problem as the BA does under perfect competition. Simply put, it values its own profits and as soon as they are threatened, then they would want to suspend and it would be individually optimal to do so. From this perspective, run equilibria would not take place in a fully concentrated banking system.

Yet, the authority to allow suspensions is determined by the BA. And, the BA may not find it optimal to freeze deposits completely right at π. Similar to what is shown for perfectly competitive banks, the BA's inability to commit can lead to an inefficient equilibrium where all depositors run on the bank. Therefore, how does the BA propose a desirable ex-post suspension policy in a monopolistic banking system? Is the process of decision-making by a banking regulator exactly the same under both structures? We address these important questions in the following subsection.

3.3 Optimal Simple Suspension Policies and Equilibria

We first consider a situation in which all depositors follow the strategy profile of $y_i = 1$. What will be the BA's best response? To decide when to declare a deposit freeze, the BA may face a trade-off of balancing the welfare of different groups of agents. Notably, in a monopolistic banking sector, the monopolistic bank values its positive profits. In turn, it is also possible that a BA may take the monopoly bank's welfare into account. For example, as discussed in the introduction, the government's social objectives may not be perfectly aligned with depositors. To refrain, Kennedy provides anecdotal evidence that banking regulators respond to pressure from large banks: "A state's governmental administration became closely tied with its financial institutions." Also, Allen and Gale describe the potential powerful role of large banks in the political system. Further, Brown and Dinc and Rosas also highlight the potential for banks to exert political pressure.

In light of these aspects, a BA's social welfare function may include bankers' utility. In this manner, if the BA values a bank's welfare, then a bank's profits are no longer considered a pure loss to the economy. That is, the social welfare function is likely to place some weight on a bank's utility. Thus, the aggregate social welfare may be described by:

$$\widehat{W}_{BA} \equiv \widehat{W}_D + \delta \widehat{W}_B,$$

where $\delta \geq 0$ measures the relative importance of the monopoly bank's welfare. There have also been similar representations for social welfare in other recent work. For example, Kashyap, Tsomocos, and Vardoulakis (2014) assign weights for different agents in a social welfare function, including bankers. In particular, we suggest that δ is a proxy for the political or bargaining power of the monopolistic banking system. By comparison, if δ is equal to zero, then the BA's interests are perfectly aligned with depositors.

We now consider the role of market power and bank size in determining how a BA may respond to run behavior in the banking system. To begin, let $\widehat{\pi}_s$ denote the "freeze point," in which a fraction $\widehat{\pi}_s$ would be served in period 1, each receiving $\widehat{c}_1^*$. Further, we assume that the monopoly bank would not react to runs strategically in order to protect its profits. That is, as we describe below, the monopolistic bank continues to pay the same returns it promised in period 0 as long

as it is able. This captures the observations in a number of studies that claim that concentrated banking systems are more stable because banks are protected by their buffer of positive profits. For example, the bank continues to pay $\widehat{c}_2^*$ as the period-2 payoff if it earns non-negative profits. As profits are first used to mitigate the effects of runs, the bank still has sufficient resources to honor its pre-specified redemptions.

In addition, bank profits are derived through excess revenues after making payments to patient depositors in period 2 and are calculated by:

$$\psi\left(\widehat{\pi}_s\right) = \left\{ \begin{array}{c} \frac{\left(e-\tau\widehat{x}^*-\widehat{\pi}_s\widehat{c}_1^*\right)R}{1-\tau} - (1-\pi)\left(1-\widehat{\pi}_s\right)\widehat{c}_2^* \\ 0 \end{array} \right\} \text{ as } \left\{ \begin{array}{c} \pi \leq \widehat{\pi}_s < \widehat{\pi}_{ZP} \\ \text{otherwise} \end{array} \right\},$$

where $\widehat{\pi}_{ZP} \equiv \frac{(e-\tau\widehat{x}^*)R-(1-\tau)(1-\pi)\widehat{c}_2^*}{\widehat{c}_1^*R-(1-\tau)(1-\pi)\widehat{c}_2^*}$ represents the zero profit threshold, which is defined by the condition $\psi\left(\widehat{\pi}_{ZP}\right) = 0$. Notably, under Assumption 2, it is obvious that $\widehat{\pi}_{ZP}$ is less than one. Moreover, if profits are totally exhausted, all depositors withdrawing in period 2 receive an equal share of the resources left in the bank until no resources are available. Hence, $\widehat{c}_2\left(\widehat{\pi}_s\right)$ is characterized as follows:

$$\widehat{c}_2\left(\widehat{\pi}_s\right) = \left\{ \begin{array}{c} \widehat{c}_2^* \\ \frac{\left(e-\tau\widehat{x}^*-\widehat{\pi}_s\widehat{c}_1^*\right)\frac{R}{1-\tau}}{(1-\pi)\left(1-\widehat{\pi}_s\right)} \\ 0 \end{array} \right\} \text{ as } \left\{ \begin{array}{c} \pi \leq \widehat{\pi}_s < \widehat{\pi}_{ZP} \\ \widehat{\pi}_{ZP} \leq \widehat{\pi}_s < \widehat{\pi}_s^U \\ \widehat{\pi}_s = \widehat{\pi}_s^U \end{array} \right\}, \tag{11}$$

where $\widehat{\pi}_s^U \equiv \frac{e-\tau\widehat{x}^*}{\widehat{c}_1^*}$ denotes the maximum proportion of depositors that could be paid $\widehat{c}_1^*$ in period 1. Notably, it can be easily verified that an increase in bank size proportionally augments payoffs in both period 1 and 2.

It is worth pointing out that payout schedule (11) differs from (4) under perfect competition, where $c_2\left(\pi_s\right)$ in (4) is contingent on any additional withdrawals after π. In what follows, Figure 1 clearly depicts such differences in addition to how bank size affects respective payments in period 2, where $e_2 > e_1$:

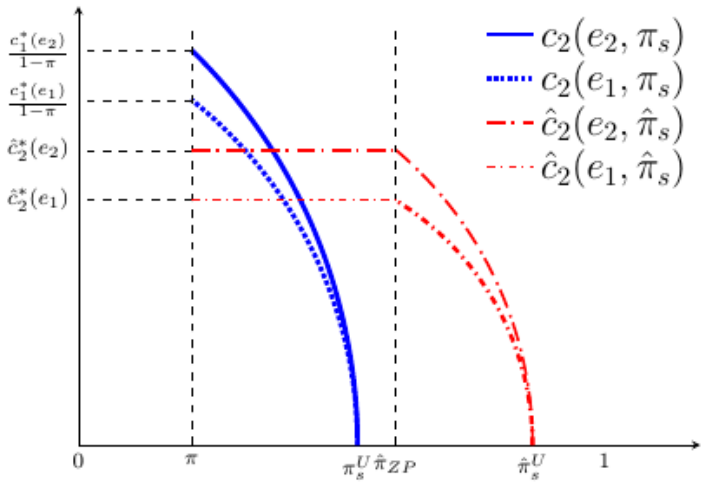

Figure 1. Period-2 Payoffs after a Deposit Freeze

From an ex-post perspective, as long as only a few of additional period-1 withdrawals are allowed, patient depositors would continue to receive higher payments under perfect competition. However, as is well known in the Diamond and Dybvig framework, every early unanticipated redemption imposes a negative externality on the remaining depositors. Once a sufficiently large number of additional withdrawals are permitted, payments under a monopolistic system would be higher than perfect competition.

In comparison to perfectly competitive banks in Diamond and Dybvig (1983) and Ennis and Keister (2009), payments by a monopolistic institution do not depend on the extent of early withdrawal demand as long as $\pi \leq \widehat{\pi}_{ZP}$. Yet, even along the range from $\widehat{\pi}_{ZP}$ to $\widehat{\pi}_S^U$, the monopolist can still make payments but a competitive bank would be unable to do so. That is, it is easy to show that $\pi_s^U < \widehat{\pi}_{ZP}$.

In what follows, we investigate how the relative importance of the bank's welfare and bank size affect the optimal suspension policy and fragility of the banking system. Notably, when faced with run behavior, the BA would react optimally and know that depositors' strategies will correspond to an equilibrium generated by such optimal policy.

3.3.1 Suspension Policy When the BA is Completely Benevolent

We first assume that the BA is completely benevolent. That is, the BA only maximizes depositors' utility. As in Ennis and Keister (2010), this assumption also implies that the BA will need to intervene when faced with a run and a possible chance for an improvement in resource allocation. In order to focus on the role of one particular source of financial fragility, we further assume that

depositors have complete information about the government's preferences. That is, they are aware that the BA only values their expected utility. As we will argue later, uncertainty about the BA's preferences would create another channel for banking instability.

In what follows, we first characterize conditions in which the BA has sufficient incentives to carry out a complete deposit freeze right after identifying a run. To do so, the BA solves the following problem:

$$\widehat{W}_{BA}\left(\widehat{\pi}_s\right) \equiv \max_{\{\widehat{\pi}_s\}} \begin{array}{c} \widehat{\pi}_s \frac{\left(\widehat{c}_1^*\right)^{1-\gamma}}{1-\gamma} + \left[\left(1-\widehat{\pi}_s\right)\left(1-\pi\right)\frac{\left(\widehat{c}_2^*\right)^{1-\gamma}}{1-\gamma}\right] I_{\psi>0} \\ + \left[\left(1-\widehat{\pi}_s\right)\left(1-\pi\right)\frac{\left(\widehat{c}_2(\widehat{\pi}_s)\right)^{1-\gamma}}{1-\gamma}\right] I_{\psi=0} \end{array} \tag{12}$$

The objective function (12) states that a fraction $\widehat{\pi}_s$ of depositors can be paid $\widehat{c}_1^*$ in period 1. Of the remaining $(1-\widehat{\pi}_s)$ depositors, only a fraction $(1-\pi)$ will be patient and return to withdraw their funds in period 2. The other π will be impatient and receive nothing. The indicator function $I_{\psi>0}$ binds such that $I_{\psi>0}=1$ and $I_{\psi=0}=0$ if $\pi \leq \widehat{\pi}_s < \widehat{\pi}_{ZP}$ where the bank is still able to pay $\widehat{c}_2^*$ to patient depositors in period 2. Otherwise the bank would have lost its income cushion from positive profits in which case $I_{\psi=0}=1$ while $I_{\psi>0}$ takes value 0. Further, under this situation, the monopolist's period-2 payoff would be contingent on $\widehat{\pi}_s$. To begin, we first look at the situation where optimal suspension occurs before the bank depletes its profits. Similarly, we show that when liquidity demand in period 1 is relatively small, the BA would optimally freeze deposits completely right at π. The following proposition characterizes this result:

Proposition 5: Credible, Tough Suspension Policy in a Monopolistic Banking System under a Benevolent BA. *Suppose $\delta = 0$. The BA would declare a deposit freeze completely at π if and only if:*

$$\pi \leq 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}} \equiv \underline{\widehat{\pi}}^*. \tag{13}$$

Once the BA recognizes that a run is underway, freezing deposits immediately or not depends critically on the magnitude of early liquidity demand. First, note that the marginal impact of allowing a small number of individuals to run is constant as long as $\widehat{\pi}_s < \widehat{\pi}_{ZP}$. This takes place because the bank's payments to depositors are independent of the extent of run behavior for a relatively small extent of unanticipated early withdrawals – the monopolist is protected by its cushion of positive profits. If the proportion of truly impatient individuals is relatively small, there would be a net welfare loss from allowing excessive redemptions – there are relatively more patient individuals in the population. And, a patient depositor would only receive $\widehat{c}_1^*$ but would lose more income than if they would wait to withdraw as long as the bank earns positive profits. Hence, the BA allows the bank to suspend as soon as run behavior is detected. However, if there is truly a

relatively high number of impatient individuals, denying consumption to them would generate a welfare loss. In this case, the BA may impose a somewhat weaker suspension policy and not allow banks to suspend until they run out of profits. We turn to this possibility below.

Further, it is interesting to compare condition (13) with that in Proposition 2. Recall that the condition (6) is the necessary and sufficient condition for the BA to impose a credible, tough suspension policy (they suspend immediately after identifying a run) in a perfectly competitive banking system. The notable difference between these two conditions is that the liquidation cost does not affect condition (13) under a monopoly bank. In particular, when liquidation costs are high, there exist parameter values where condition (6) holds, but not condition (13). In these cases, there are intermediate values of π such that a deposit freeze will be imposed after identifying a run under perfect competition but not in a monopolistic setting. Consequently, policies to address stability in the banking system clearly depend on the competitive structure of the banking system. Nevertheless, bank size does not appear to matter in either structure. We formalize this result in the following corollary:

Corollary 2: The Role of Banking Competition for Credible, Tough Suspension Policy under a Benevolent BA. *There exists $\widetilde{\tau} \in (0,1)$ such that for all $\tau \geq \widetilde{\tau}$, condition (6) in Proposition 2 holds, but condition (13) in Proposition 5 does not.*

We proceed to study the efficient deposit freeze under a monopolistic system if liquidity demand is relatively large. This leads to a relatively late suspension policy where the bank exhausts all profits. Does the suspension scheme at this stage have a destabilizing effect on the banking system? Notably, $\widehat{c}_2(\widehat{\pi}_s)$ is strictly concave if $\widehat{\pi}_{ZP} \leq \widehat{\pi}_s \leq \widehat{\pi}_s^U$. Combined with the concavity of the utility function, this implies that the objective function has a unique solution denoted by $\widehat{\pi}_s^M$. Specifically, the efficient freeze $\widehat{\pi}_s^M$ may be equal to $\widehat{\pi}_{ZP}$. We now offer the following Proposition:

Proposition 6: Credible, Relatively Tough Suspension Policy in a Monopolistic Banking System under a Benevolent BA. *Suppose that $\pi \in \left(\max\left\{\pi^*, \underline{\widehat{\pi}}^*\right\}, \underline{\widehat{\pi}} \equiv 1 - \frac{\gamma-\tau}{\gamma(1-\tau)R^{\frac{1-\gamma}{\gamma}}} \right]$ and $\underline{\widehat{\pi}} < \widetilde{\pi}$. In this range of liquidity risk, the BA would find it optimal to declare a suspension at $\widehat{\pi}_{ZP}$.*

First, note that if π exceeds π^*, then a bank run equilibrium exists under both the perfectly competitive and monopolistic banking systems. Further, as π exceeds $\underline{\widehat{\pi}}^*$, the BA would not impose a complete suspension policy but it would if $\pi < \underline{\widehat{\pi}}^*$. In this case, patient depositors have no incentives to withdraw early since $\widehat{c}_2(\widehat{\pi}_{ZP}) > \widehat{c}_1^*$. Thus, the banking system is not fragile. Alternatively, the BA may impose a weaker suspension policy if $\pi > \underline{\widehat{\pi}}$.

Once again, the threshold value of π is the crucial parameter that determines the stability of the banking system. As $\widehat{\pi}^T$ refers to the value of π where $\widehat{c}_1^* = \widehat{c}_2\left(\widehat{\pi}^T\right)$, we now make a comparison

between the threshold freezing points across banking systems. Straightforward algebra shows that $\Delta_T \equiv \widehat{\pi}^T - \pi^T > 0$, and we can write this difference as

$$\Delta_T = \frac{R\left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}} - 1\right]}{\left[\frac{R}{1-\tau} - (1-\pi)\right]\widetilde{R}}.$$

We can also study the amount of consumption that can be delivered to all consumers in period 1 by liquidating investment without causing a run. Allen and Gale (2000) refer to this level of consumption as a bank's "buffer." Specifically, the buffer under perfect competition can be expressed as:

$$b(\pi) = (1-\tau)\left[x - \frac{\left(1-\pi^T\right)(1-\pi)c_1^*}{R}\right].$$

First, note that π^T represents the threshold level of liquidity risk in which $c_1^* = c_2\left(\pi^T\right)$. In order to avoid causing a run, the bank must give the patient consumers at least c_1^* in period 2. If the BA imposes a suspension of payments at π^T, a bank with a remaining fraction $\left(1-\pi^T\right)(1-\pi)$ of patient depositors must leave at least $\frac{\left(1-\pi^T\right)(1-\pi)c_1^*}{R}$ units of assets in the investment technology to satisfy the patient depositors in period 2. Then, the amount of investment that can be liquidated in period 1 is $x - \frac{\left(1-\pi^T\right)(1-\pi)c_1^*}{R}$. Nevertheless, liquidating this amount only provides $(1-\tau)\left[x - \frac{\left(1-\pi^T\right)(1-\pi)c_1^*}{R}\right]$ units of consumption in period 1 because of the liquidation costs. We let $b(\pi)$ represent this buffer level of consumption. Equivalently, $b(\pi)$ can also reduced to:

$$b(\pi) = \left(\pi^T - \pi\right)c_1^*.$$

The difference ($\pi^T - \pi$) denotes the number of additional unanticipated redemptions that a bank can sustain without a run occurring. Since payments to depositors are increasing in the size of the deposit base, the bank's buffer is also increasing in the size of the bank.

By comparison, the monopoly bank's buffer is:

$$\widehat{b}(\pi) = \left(\widehat{\pi}^T - \pi\right)\widehat{c}_1^*.$$

It is also easy to show that the buffer held by the monopoly bank is larger than those owned by

competitive banks. Similarly, we define $\Delta_b \equiv \widehat{b}(\pi) - b(\pi)$ to be the buffer differential where:

$$\Delta_b = e \left(\begin{array}{c} \left\{ \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}\right)\right]-(1-\pi)\widetilde{R}^{\frac{1}{1-\gamma}}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}^{\frac{1}{1-\gamma}}} - \pi \right\} \widetilde{R}^{\frac{1}{1-\gamma}} \\ - \left\{ \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-(1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} - \pi \right\} \widetilde{R} \end{array} \right).$$

The following proposition characterizes these relationships.

Proposition 7: Credible, Weak Suspension Policy under a Benevolent BA. *Suppose that* $\pi \in \left(\underline{\widehat{\pi}}, 1\right)$ *. Under this condition, the threshold freeze point under monopoly is strictly larger than that under perfect competition. However,* Δ_T *is independent of bank size. In addition, the monopoly bank holds a higher buffer level than competitive banks. Further, the buffer differential* (Δ_b) *is monotonically increasing in bank size.*

The proposition establishes a crucial comparison that shows the threshold freeze point in a fully concentrated banking system is strictly larger than that under perfect competition. Further, such difference sheds light on the prediction that a concentrated banking sector has a larger scope for maintaining banking stability. As pointed out by Allen and Gale (2000, p.269), this apparent advantage under a monopolistic banking system is due to the fact that the bank holds profits that could be liquidated to satisfy more unexpected redemptions. In addition, Δ_T is monotonically increasing in R.

We proceed to see if the desirable *ex-post* deposit freeze induces fragility in the banking system. Similarly, if a suspension occurs after $\widehat{\pi}^T$, the banking system is fragile. Hence, in these settings, the BA's policy could not rule out depositors' ex-ante incentives to run on the bank. Proposition 8 summaries the major results for this case:

Proposition 8: Fragility across Banking Structures under a Benevolent BA. *Suppose* $\delta = 0$ *and* $\pi > \widetilde{\pi}$. *Under these conditions, the monopoly banking is fragile. The parameter space associated with fragility does not depend on the competitive structure of the banking system.*

The result above gives conditions under which a self-fulfilling run could arise. It is worth noting that the fragility condition associated with liquidity demand is the same as that under perfect competition. It demonstrates that if the proportion of impatient depositors is sufficiently large, the banking system tends to be fragile. Hence, if the BA's social objective remains focused exclusively on depositors' utility, surprisingly, the fragility condition across banking structures is the same. Although the monopolistic bank has positive profits that provide a buffer against runs, the BA will tend to freeze deposits later than under perfect competition and the two channels offset each other.

Otherwise, as we have shown previously in Proposition 5 (for example), suspension policy clearly depends on the competitive structure of the banking system.

Comparison of Welfare and the Size of Suspensions Across the Competitive Structure of the Banking System. To further reinforce the above points, we provide comparisons of welfare and the size of suspensions for both economies. Under the same set of parameters from Table 1 with $e = 1.1$, the panel (a) of Figure 2 presents the welfare of perfectly competitive and monopolistic banking systems as a function of π.

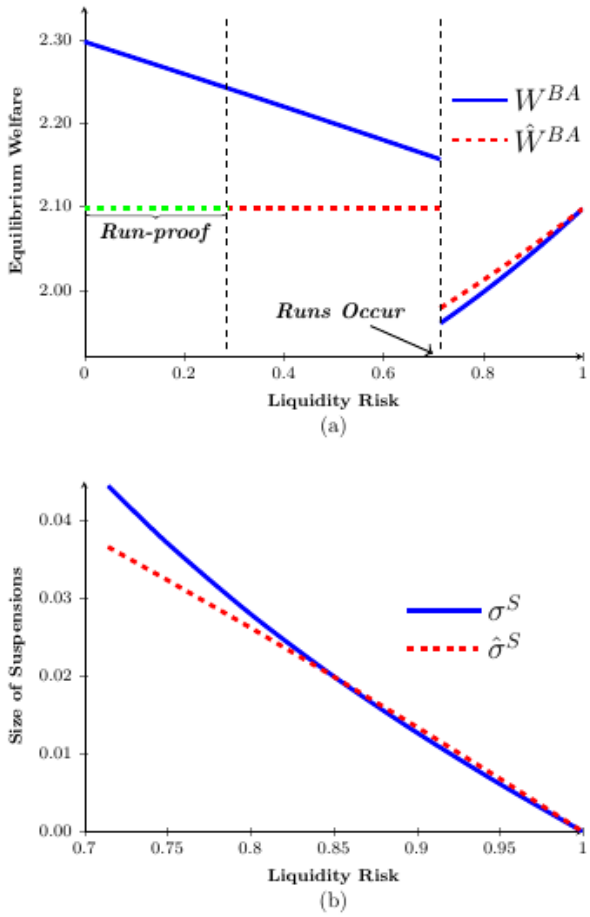

Figure 2. Equilibrium Welfare and Size of Suspensions across Structures

When liquidity demand is relatively low, the BA's suspension policy eliminates run equilibria in both market settings. Once π exceeds $\max\{\pi^*, \widehat{\underline{\pi}}^*\}$, the BA relaxes the suspension policy somewhat in the monopolistic system. Since the bank does not exhaust its profits over the range from $\widehat{\pi}_{ZP}$ to $\widetilde{\pi}$, welfare is unaffected by the run behavior. As can be observed in the figure, when $\pi < 0.7143$, welfare is always lower under monopoly since depositors only receive a fixed level

of expected utility. Once π exceeds $\widetilde{\pi}$, both banking systems are fragile. Though the fragility condition is the same, that does not mean both banking systems will have the same optimal freeze point or suspension policy. Indeed, welfare under a fragile monopolistic system is higher than welfare under a fragile perfectly competitive banking system. This occurs because payments to early depositors are lower in the monopolistic sector so suspending deposits and shutting out some truly impatient depositors is less impactful than in a perfectly competitive sector.

On the other hand, the size of suspensions in equilibrium, as shown in the panel (b) of Figure 2, can be larger under either economy, depending on the liquidity demand. In particular, the size of suspensions is larger under a monopoly bank if and only if π is sufficiently large.

3.3.2 Suspension Policy When the BA Values Bank's Welfare

As previously discussed, the BA's welfare function may include bankers' welfare. Consequently, the BA's optimal suspension policy during times of crisis may depend on the relative importance of depositors' expected utility relative to bankers' incentives. In the BA's welfare function, we consider that the relative weight of the bank's welfare is measured by δ. We view that the higher the value of δ, the greater the bargaining or political power for the bank. Again, if $\delta = 0$, the incentives of the BA and depositors are perfectly aligned. In general, in the presence of $\delta > 0$, the optimal suspension policy is affected by the trade-off between depositors' and banker's welfare effects. We start with a case in which the relative weight of the bank's welfare is independent of any additional withdrawals after π and then return to the more general case where the relative weight depends on $\widehat{\pi}_s$.

Withdrawal Independent Weight. Assume that δ is independent of $\widehat{\pi}_s$. In determining its suspension policy, the BA seeks to maximize aggregate welfare by solving:

$$\widehat{W}_{BA}\left(\widehat{\pi}_s\right) \equiv \max_{\{\widehat{\pi}_s\}} \begin{array}{l} \widehat{\pi}_s \frac{\left(\widehat{c}_1^*\right)^{1-\gamma}}{1-\gamma} + \left[\left(1-\widehat{\pi}_s\right)\left(1-\pi\right)\frac{\left(\widehat{c}_2^*\right)^{1-\gamma}}{1-\gamma} + \delta\left[\psi\left(\widehat{\pi}_s\right)\right]\right] I_{\psi>0} \\ \qquad + \left[\left(1-\widehat{\pi}_s\right)\left(1-\pi\right)\frac{\left(\widehat{c}_2(\widehat{\pi}_s)\right)^{1-\gamma}}{1-\gamma}\right] I_{\psi=0} \end{array} \tag{14}$$

We begin by considering the conditions under which the BA imposes a tough deposit freeze. In the case of a completely benevolent BA, the BA imposes immediate suspensions after identifying run behavior when liquidity demand in period 1 is small. However, now the decision also depends on the bank's bargaining power and bank size. The following proposition summarizes these results.

Proposition 9: Credible, Tough Suspension Policy if the Bank Has Bargaining Power. *Suppose $\delta > 0$. Under this condition, $\widehat{\pi}_s^M = \pi$ if and only if:*

$$\pi \leq 1 - \left[\frac{\frac{1}{1-\gamma}\frac{1}{e^\gamma}\frac{1}{R} - \delta \widetilde{R}^{\frac{\gamma}{1-\gamma}} \frac{1}{1-\tau}}{R^{\frac{1-\gamma}{\gamma}} \left(\frac{1}{1-\gamma}\frac{1}{e^\gamma}\frac{1}{R} - \delta \widetilde{R}^{\frac{\gamma}{1-\gamma}} \right)} \right] \equiv \underline{\widehat{\pi}}^{**}.$$

Again, the proposition demonstrates that the BA could uniquely implement the monopoly bank's profit maximization allocation if there are only a few impatient depositors. Interestingly, both bank size and the relative importance of the bank's profit can affect the BA's incentive to freeze deposits completely at π. Straightforward algebra shows that if the values of these two factors are higher, the parameter space associated with an immediate suspension is also larger:

Lemma 3. $\frac{\partial \underline{\widehat{\pi}}^{**}}{\partial \delta} > 0$ *and* $\frac{\partial \underline{\widehat{\pi}}^{**}}{\partial e} > 0$.

Bank size affects suspension policy because the larger a bank's balance sheet, the more illiquid investment it makes and the larger the bank's profits. Though the bank can satisfy some deposit withdrawals, doing so would lower its profits which can affect the BA's incentives.

While Proposition 9 shows that tough suspension policy can be dependent on the level of liquidity risk, we can alternatively express how the bargaining power affects the decision to immediately suspend:

Proposition 10: The Role of Bargaining Power in Credible, Tough Suspension Policy. *Suppose $\pi > \max\{\pi^*, \underline{\widehat{\pi}}^*\}$. There exists a value $\overline{\delta} > 0$ such that $\pi_s^M = \pi$ holds if*

$$\delta > \frac{1 - (1-\pi)R^{\frac{1-\gamma}{\gamma}}}{(1-\gamma)\, e^\gamma R^{\frac{1}{\gamma}} \widetilde{R}^{\frac{\gamma}{1-\gamma}} \left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}} \frac{1}{1-\tau} - (1-\pi) \right]} \equiv \overline{\delta}. \tag{15}$$

Furthermore, the parameter space where a deposit freeze is imposed immediately at π is larger if the bank size is larger, as $\overline{\delta}$ is decreasing in e.

As before, our interest is to determine whether an *ex-post* efficient suspension can rule out bank run equilibria. Yet, if the bargaining power of the monopoly bank is relatively weak, the fragility condition is the same as if the BA were completely benevolent:

Proposition 11: Fragility and Bank Bargaining Power. *Suppose $0 < \delta < \overline{\delta}$ and $\pi \geq \widetilde{\pi}$. Under the ex-post efficient deposit freeze by the BA, the monopolistic banking sector is fragile.*

The intuition behind the proposition is straightforward. Both the BA's preferences for the bank's welfare and bank size are involved in determining the fragility condition. If liquidity demand is substantial, the BA's primary preference is to emphasize depositors' welfare. As a result, the banking system tends to be fragile. Instead, if the BA places sufficient weight on the monopoly bank's welfare, an early deposit freeze would be declared and the profit maximization allocation could be uniquely implemented. However, bank size plays an important role. If bank size is large enough, there is greater scope for the condition in Proposition 10 to hold. Thus, policies to address fragility in the banking system clearly depend on the competitive structure and bank size.

We are now starting to see how analyzing decision-making during a crisis can be difficult. If the BA's incentives are perfectly aligned with depositors, suspensions generally take place later in concentrated banking systems than competitive systems because the monopolistic bank is protected by a cushion from profits. That is, as Corollary 2 shows, there are intermediate values of π such that a deposit freeze will be imposed after identifying a run under perfect competition but not in a monopolistic setting. Yet, the fragility condition is the same regardless of the competitive structure. However, larger banks may be able to exert more pressure pressure on the BA so that it allows the bank to suspend *earlier* in concentrated systems.

Withdrawal Dependent Weight. These additional channels may shed light on criticisms that decision-making during a crisis can seem arbitrary – it might be difficult in practice to completely understand the incentives of banking authorities. An alternative interpretation is that they can evolve during the stages of a crisis. For example, in the early stages of a crisis there may not appear to be much concern and the BA may want banks to remain open. However, as the crisis unfolds and banks respond more to redemption demand, the BA may place greater emphasis on bankers' welfare. Hence, it is worth considering that δ may not be constant and is instead a function of how many depositors have run on the banking system so that δ instead is a function of $\widehat{\pi}_s$. Therefore, we simply assume that $\delta\left(\widehat{\pi}_s\right)$ takes the following functional form $\delta\left(\widehat{\pi}_s\right) = \widetilde{\delta}\left[1 + \left(\widehat{\pi}_s - \pi\right)^2\right]$, where $\widetilde{\delta}$ is the initial value of the weight. As $\delta\left(\widehat{\pi}_s\right)$ matters only when the bank earns profits, we then focus on the BA's optimization problem in which $I_{\psi>0}$ binds:

$$\widehat{W}_{BA}\left(\widehat{\pi}_s\right) \equiv \max_{\{\widehat{\pi}_s\}} \widehat{\pi}_s \frac{\left(\widehat{c}_1^*\right)^{1-\gamma}}{1-\gamma} + \left[\left(1-\widehat{\pi}_s\right)\left(1-\pi\right)\frac{\left(\widehat{c}_2^*\right)^{1-\gamma}}{1-\gamma} + \delta\left(\widehat{\pi}_s\right)\psi\left(\widehat{\pi}_s\right)\right] I_{\psi>0}$$

As a crisis develops, the BA values the bank's profits at a higher weight. However, deposits are frozen at π if condition (15) holds since the effective weight at π is just equal to $\widetilde{\delta}$. Yet, as the bank satisfies more withdrawal requests upon knowing that a run is underway, the BA may impose a suspension before profits are completely exhausted, in which case the run equilibrium is ruled out costlessly and the banking system is not fragile:

Proposition 12: Withdrawal Dependent Weight, Suspension Policy, and Financial Fragility. *Suppose* $\pi \in [\widetilde{\pi}, 1)$ *and* $\delta\left(\widehat{\pi}_{ZP}\right) = \frac{\overline{\delta}}{1+(\widehat{\pi}_{ZP}-\pi)^2} \equiv \underline{\delta}$*. The banking system is*

(i) fragile if and only if $\widetilde{\delta} \in (0, \underline{\delta}]$ *as* $\widehat{\pi}_s^M \in \left[\widehat{\pi}^T, \widehat{\pi}_s^U\right]$*;*

(ii) not fragile if and only if $\widetilde{\delta} \in (\underline{\delta}, \infty)$ *as* $\widehat{\pi}_s^M \in [\pi, \widehat{\pi}_{ZP}]$.

The intuition is straightforward. If liquidity demand is significant and the relative weight on bank profits is relatively low, the run equilibrium still exists under optimal intervention. To demonstrate precisely the relationship among the BA's preferences, the effective weight on bank's welfare, and suspension policy, we construct the following example. Please see Table 2 below where we list various predictions for the model under different levels of liquidity risk. Moreover, we adopt the a set of parameters in which case $R = 1.2$, $\gamma = 0.5$, $\tau = 0.3$, $e = 1.1$ and $\widetilde{\delta} = 0.88$.

π	0.525	0.550	0.575	0.600	0.625	0.650	0.675	0.700	0.725
$\overline{\delta}$	0.87148	0.89669	0.91974	0.94083	0.96013	0.97782	0.99402	1.00887	1.02247
$\underline{\delta}$	0.71397	0.74867	0.78202	0.81401	0.84463	0.87384	0.90162	0.92793	0.95271
$\widehat{\pi}^T$	0.99560	0.99548	0.99542	0.99541	0.99544	0.99551	0.99563	0.99578	0.99597
$\widehat{\pi}_s^M$	0.52500	0.85868	0.89571	0.92916	0.96002	0.98892	0.99557	0.99576	0.99598
$\delta(\widehat{\pi}_s^M)$	0.88000	0.96385	0.97051	0.97534	0.97877	0.98108	0	0	0
$\psi(\widehat{\pi}_s^M)$	0.44396	0.13423	0.10191	0.07036	0.03887	0.00700	0	0	0
$\widehat{W}_{BA}$	1.82405	1.88849	1.90090	1.91572	1.93296	1.95252	1.96378	1.97304	1.98246

Table 2. Effective Weights, Cutoff Weights, and Financial Fragility when $\widetilde{\delta} = 0.88$

These exercises show allocations from an ex-post perspective in which a run is underway. When liquidity demand is sufficiently small ($\pi = 0.525$), the importance of bank welfare dominates that of depositors' as the initial value, $\widetilde{\delta}$, has already exceeded the upper-cutoff $\overline{\delta}$. In turn, a suspension is imposed immediately without incurring too much social costs. It is also clear that, as the proportion of truly impatient depositors is higher, both the upper- and lower-cutoff weights increase. In these cases, keeping $\widetilde{\delta}$ unchanged, the BA allows more withdrawals while the bank's profits shrink. Interestingly, if π is sufficiently high, the effective weights that are imposed on profits are thus invalid ex-post. Finally, a run equilibrium is not eliminated while the banking system becomes fragile if the proportion of impatient depositors is sufficiently large.

On the other hand, we again see how bank size affects the fragility condition. In particular, $\underline{\delta}$ is decreasing in bank size, implying that parameter space where Proposition 12 applies is also decreasing in e. Hence, large banking systems are less likely to be fragile since the BA places greater weight on profits if they are large and concentrated. The BA thus is more prone to respond

to the needs of the banking sector. In this manner, we argue that the fragility conditions in a monopolistic banking system sharply differ from a perfectly competitive system. All that matters for fragility under perfectly competitive systems is the extent of liquidity risk. By comparison, factors such as bank size are also important if the BA's incentives evolve as a crisis deepens.

It is also important to point out that, in the model, appointing a BA that is more inclined to preserve banks' welfare rather than being completely benevolent might help eliminate run equilibria. Consequently, depositors do not panic and suspensions would not need to be imposed. This implication is similar in spirit to the literature on the optimal monetary policy, i.e., Rogoff (1985). That is, it is plausible that society should appoint a BA with very different objectives from depositors so that the system is less susceptible to fragility.

3.3.3 "Type" of the BA, Weight Uncertainty, and Equilibria

We have previously explained that it may be difficult to pin down how a BA will respond to stress in the system. For example, bank size does not play a role in fragility of the banking system under perfect competition but it does in a monopolistic banking system if the BA also responds to pressure from bankers. Interestingly, if the BA imposes a large weight on the bank's welfare, depositors should have less incentive to run: an early suspension announcement would be expected and the banking system remains solvent. However, it can also be complicated if depositors do not know in practice how much they contribute to the BA's goals or what "type" of policy maker they are facing. In this manner, we identify an important banking policy problem of uncertainty about the BA's "type." This kind of uncertainty about a policy maker's "type" was formally discussed in Barro (1986) for the reputation of monetary authorities to tackle inflation. In the spirit of Barro's example, we explain why a run equilibrium is not completely eliminated even if the BA may have strong incentive to freeze deposits very early ex-post. That is, the BA may not be able to communicate its preferences effectively.

To begin, suppose that the BA places a weight, δ, on bank profits. In particular, with probability λ, $\delta_0 \geq \overline{\delta}$, where $\overline{\delta}$ is defined by condition (15). In this case, the BA is a "tough" type, in which it highly values bank welfare. Thus, it would allow banks to suspend redemptions before profits are depleted. In contrast, with probability $1-\lambda$, $\delta_1 < \overline{\delta}$, the BA behaves in a "weak" manner, leading to a late freeze if π is relatively large ($\pi \in \left(\underline{\widehat{\pi}}, 1\right)$).

The period-2 payoff is now type-dependent. Specifically, patient depositors are still guaranteed to receive a fixed period-2 payoff, $\widehat{c}_{2S}(\widehat{\pi}^S) = \widehat{c}_2^*$, if the BA is "tough." The problem faced by a "weak" type BA whose major objective is to mitigate the loss of depositors' welfare is analogous. That is, in comparison to the allocation that would occur if the BA is tough, the period-2 consumption under a weak BA will be contingent on every additional withdrawal in which

$\widehat{c}_{2W}(\widehat{\pi}^W) = \frac{\left(e-\tau\widehat{x}^*-\widehat{\pi}^W\widehat{c}_1^*\right)\frac{R}{1-\tau}}{(1-\pi)\left(1-\widehat{\pi}^W\right)}$, where $\widehat{\pi}^W > \widehat{\pi}_{ZP}$. In what follows, for notational convenience, we define the expected utility to depositors, $\varepsilon\left(\widetilde{c}_2(\lambda,\widehat{\pi}^W)\right)$, from waiting to withdraw as:

$$\varepsilon\left(\widetilde{c}_2(\lambda,\widehat{\pi}^W)\right) \equiv \lambda u\left(\widehat{c}_2^*\right) + (1-\lambda)\, u\left(\widehat{c}_{2W}(\widehat{\pi}^W)\right),$$

where $\widehat{\pi}^W$ is chosen by solving the objective (14) where $I_{\psi=0}$ binds, depending on the weight of the BA on bank profits. Notably, if $u\left(\widehat{c}_1^*\right) < \varepsilon\left(\widetilde{c}_2(\lambda,\widehat{\pi}^W)\right)$, depositors would choose not to run on the bank. By comparison, a patient depositor i follows strategy profile $y_i\left(1\right) = 0$ if and only if:

$$u\left(\widehat{c}_1^*\right) \geq \varepsilon\left(\widetilde{c}_2(\lambda,\widehat{\pi}^W)\right),$$

where the run condition holds with equality at threshold λ^T. We then formalize results in the following proposition.

Proposition 13: Weight Uncertainty and Fragility. *Under the simple suspension scheme and weight uncertainty by the different "type" of the BA, the threshold probability λ^T in which $u\left(\widehat{c}_1^*\right) = \varepsilon\left(\widetilde{c}_2(\lambda^T,\widehat{\pi}^W)\right)$ holds is independent of bank size, and the banking system is:*

(i) fragile if and only if $\pi \in (\widetilde{\pi},1)$ and $\lambda \in \left[0,\lambda^T\right]$;

(ii) not fragile if and only if $\lambda \in \left(\lambda^T,1\right]$ for all values of $\pi \in (0,1)$;

(iii) not fragile if and only if $\pi \in (0,\widetilde{\pi}]$ for all values of $\lambda \in [0,1]$.

Specifically, in case (i), there is a large number of truly impatient depositors in the banking system. If depositors anticipate that the probability that the BA is a tough type is sufficiently low, they have a strong incentive to run since they have a strong belief that they will receive a lower payment from waiting to withdraw, $\widehat{c}_{2W}(\widehat{\pi}^W)$, than the payoff from joining the run, $\widehat{c}_1^*$. By comparison, case (ii) shows that if depositors believe that there is a high probability that a BA will impose a suspension in a "tough" manner, they will not find it optimal to withdraw early as an early suspension will occur. Further, the last case demonstrates that if the proportion of depositors with a real need to consume early is small enough, depositors will always withdraw according to their preference types regardless of the "type" of the BA.

Thus, in the presence of incomplete information over the BA's type, *depositors' expectations truly determine whether the banking system is fragile or not (not the actual preferences of the BA) – by definition, the banking system is fragile if a positive measure of patient depositors run on the system* and behave in a way that is inconsistent with their privately observed preferences over the timing of consumption. *By comparison, the preferences of the BA matter for the actual*

suspension policy implemented to address the fragility of the banking system. If the BA is truly tough, but cannot communicate that it would impose a suspension early, one problem is that some truly impatient individuals will be shut off from access to their funds. That is, in the presence of incomplete information regarding the BA's type, run behavior may be observed even though early suspensions would eventually take place.

To see this possibility in a more concrete manner, we adopt the same set of parameters as in our previous examples but focus on cases where real withdrawal demand is sufficiently large. Please see the allocation below where we examine the decision to run uncertainty where $e = 1.1$:

π	0.725	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
$\widehat{c}_1^*$	0.9883	0.9977	1.0073	1.0170	1.0269	1.0369	1.0470	1.0573	1.0677	1.0783
$\overline{\delta}$	1.0225	1.0349	1.0463	1.0568	1.0663	1.0750	1.0829	1.0900	1.0965	1.1024
$\widehat{\pi}^W$	0.9960	0.9962	0.9965	0.9968	0.9971	0.9974	0.9978	0.9982	0.9986	0.9990
$u(\widehat{c}_{2W}(\widehat{\pi}^W))$	1.9736	1.9504	1.9291	1.9096	1.8916	1.8751	1.8597	1.8455	1.8323	1.8200
$u\left(\widehat{c}_2^*\right)$	2.3859	2.3973	2.4087	2.4203	2.4320	2.4438	2.4557	2.4678	2.4799	2.4922
λ^T	0.0356	0.1060	0.1630	0.2102	0.2499	0.2839	0.3133	0.3390	0.3618	0.3821
$\varepsilon(\widetilde{c}_2(\lambda,\widehat{\pi}^W))$	1.9883	1.9977	2.0073	2.0169	2.0267	2.0365	2.0465	2.0565	2.0666	2.0768

Table 3: Allocation under Uncertainty over δ

The table above shows how the decision to run depends on the depositors' beliefs over the probability that the BA will take a "tough" stance. Further, the threshold probability needs not to be significantly high. On one hand, at low values of $\lambda < \lambda^T$, a run is part of a rational expectations equilibrium. In these cases, there will be a small percentage of time where depositors run but the BA is actually tough and suspensions take place very early. As previously mentioned, these situations are costly because some truly impatient individuals lose access to their funds. Another problem is that patient depositors earn less income than if they would have waited and eventually earned higher returns. Nevertheless, the allocation can be immune to vulnerability. At larger values of $\lambda > \lambda^T$, patient depositors do not run since the expected utility from waiting is higher, though the cutoff $\overline{\delta}$ needs to be relatively high too. Yet, it is possible that the BA is truly weak, but the system remains stable because depositors incorrectly perceive that the BA is tough when in fact suspensions would incur rather late.

4 Concluding Remarks

Events in recent years have drawn attention to the potential for fragility in the banking system. As a result, it is critically important to try to understand the various factors which affect decision-making during a financial crisis. In turn, the objective of this paper is to study fragility across

the competitive structure of the banking sector. To do so, we extend the framework of Ennis and Keister (2009) who show that a banking authority (BA) may not be able to commit to suspending payments which prevent fragility because such suspensions are not ex-post efficient. In their framework, financial intermediaries are perfectly competitive and the incentives of banks perfectly align with a BA. Yet, past experiences with government-declared bank suspensions indicate these actions can be subject to tension between large banks and banking authorities – thus, the preferences of banking regulators can significantly affect decision-making in periods of stress. In order to consider the conditions for fragility across the competitive structure of the banking system, we study run behavior in both competitive and monopolistic systems. We also allow for size variation of the deposit base to study how banking authorities respond to stress in the banking sector depending on bank size. While the scope for bank failures in a monopolistic system is smaller than perfectly competitive banks, the relationship to suspensions is less clear as a banking authority may intervene in the interests of the monopolistic banking industry – cutting off access to funds for depositors truly in need in order to protect a bank's monopoly rents. This motivation for the banking authority does not occur in perfectly competitive systems where banks inherently do not earn any profits.

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Appendix

Proof of Proposition 2. We investigate whether deposits are completely frozen right at π under an ex-post efficient suspension policy. The derivative of the objective function is given by:

$$\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} = \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{[c_2(\pi_s)]^{1-\gamma}}{1-\gamma} + (1-\pi_s)(1-\pi)[c_2(\pi_s)]^{-\gamma}\frac{\partial c_2(\pi_s)}{\partial \pi_s},$$

where

$$\begin{aligned}
\frac{\partial c_2(\pi_s)}{\partial \pi_s} &= \left(\frac{R}{1-\tau}\right)\frac{(1-\pi)(e-\tau x^*-\pi_s c_1^*)-(1-\pi)(c_1^*-\pi_s c_1^*)}{(1-\pi)^2(1-\pi_s)^2} \\
&= \left(\frac{R}{1-\tau}\right)\frac{(1-\pi)[e-\tau x^*-\pi_s c_1^*-c_1^*+\pi_s c_1^*]}{(1-\pi)^2(1-\pi_s)^2} \\
&= \left(\frac{R}{1-\tau}\right)\frac{(e-\tau x^*-c_1^*)}{(1-\pi)(1-\pi_s)^2}
\end{aligned}$$

As a result:

$$\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} = \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{[c_2(\pi_s)]^{1-\gamma}}{1-\gamma} + [c_2(\pi_s)]^{-\gamma}\left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^*-c_1^*}{1-\pi_s}\right).$$

We next evaluate this derivative at π:

$$\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s}\,|_{\pi_s=\pi} = \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{[c_2(\pi)]^{1-\gamma}}{1-\gamma} + [c_2(\pi)]^{-\gamma}\left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^*-c_1^*}{1-\pi}\right).$$

Note that:

$$\begin{aligned}
\left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^*-c_1^*}{1-\pi}\right) &= \left(\frac{R}{1-\tau}\right)\left[\frac{e-\tau x^*-\pi c_1^*-c_1^*+\pi c_1^*}{1-\pi}\right] \\
&= \left(\frac{R}{1-\tau}\right)\left[\frac{(e-\tau x^*-\pi c_1^*)-(1-\pi)c_1^*}{1-\pi}\right] \\
&= \left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^*-\pi c_1^*}{1-\pi}\right)-\left(\frac{R}{1-\tau}\right)\left(\frac{(1-\pi)c_1^*}{1-\pi}\right) \\
&= \left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^*-\pi c_1^*}{1-\pi}\right)-\left(\frac{R}{1-\tau}\right)c_1^*
\end{aligned}$$

Therefore:

$$\begin{aligned}\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} \quad | \quad _{\pi_s=\pi} &= \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{[c_2(\pi)]^{1-\gamma}}{1-\gamma} + \\ & [c_2(\pi)]^{-\gamma}\left(\left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^*-\pi c_1^*}{1-\pi}\right) - \left(\frac{R}{1-\tau}\right)c_1^*\right) \\ &= \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{[c_2(\pi)]^{1-\gamma}}{1-\gamma} + \\ & [c_2(\pi)]^{-\gamma}\left(\frac{\left(\frac{R}{1-\tau}\right)(e-\tau x^*-\pi c_1^*)}{1-\pi} - \left(\frac{R}{1-\tau}\right)c_1^*\right)\end{aligned}$$

Recall the expression for $c_2(\pi_s)$:

$$c_2(\pi_s) = \frac{\frac{R}{1-\tau}(e-\tau x^*-\pi_s c_1^*)}{(1-\pi)(1-\pi_s)}$$

Since we are interested in $\pi_s = \pi$, we next look at $c_2(\pi)$:

$$\begin{aligned}c_2(\pi) &= \frac{\frac{R}{1-\tau}(e-\tau x^*-\pi c_1^*)}{(1-\pi)^2} = \frac{\left(\frac{R}{1-\tau}\right)[e-\tau(e-\pi c_1^*)-\pi c_1^*]}{(1-\pi)^2} \\ &= \left(\frac{1}{(1-\pi)}\right)\left\{\frac{\left(\frac{R}{1-\tau}\right)[e-\tau(e-\pi c_1^*)-\pi c_1^*]}{1-\pi}\right\} \\ &= \left(\frac{1}{(1-\pi)}\right)\left\{\frac{\left(\frac{R}{1-\tau}\right)[(e-\pi c_1^*)-\tau(e-\pi c_1^*)]}{1-\pi}\right\} \\ &= \left(\frac{1}{(1-\pi)}\right)\left\{\frac{\left(\frac{R}{1-\tau}\right)(1-\tau)(e-\pi c_1^*)}{1-\pi}\right\} \\ &= \left(\frac{1}{(1-\pi)}\right)\left\{\frac{\frac{R}{1-\tau}(1-\tau)x^*}{1-\pi}\right\} \\ &= \left(\frac{1}{(1-\pi)}\right)\left\{\frac{Rx^*}{1-\pi}\right\} = \frac{c_2^*}{1-\pi}\end{aligned}$$

Since

$$c_2^* = R^{\frac{1}{\gamma}}c_1^*$$

Therefore,

$$c_2(\pi) = \frac{R^{\frac{1}{\gamma}}}{1-\pi}c_1^*$$

Further

$$\begin{aligned}\frac{\frac{R}{1-\tau}\left(e-\tau x^{*}-\pi c_{1}^{*}\right)}{1-\pi} &= (1-\pi)\frac{\frac{R}{1-\tau}\left(e-\tau x^{*}-\pi c_{1}^{*}\right)}{(1-\pi)^{2}} \\ &= (1-\pi)\,c_{2}(\pi) \\ &= (1-\pi)\frac{c_{2}^{*}}{1-\pi} \\ &= (1-\pi)\frac{R^{\frac{1}{\gamma}}c_{1}^{*}}{1-\pi}\end{aligned}$$

Upon substitution of $c_2(\pi)$ and $\frac{\left(\frac{R}{1-\tau}\right)\left(e-\tau x^{*}-\pi c_{1}^{*}\right)}{1-\pi}$ into $\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s}\mid_{\pi_s=\pi}$:

$$\begin{aligned}\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} \mid_{\pi_s=\pi} &= \frac{\left(c_{1}^{*}\right)^{1-\gamma}}{1-\gamma}-(1-\pi)\frac{\left[c_{2}(\pi)\right]^{1-\gamma}}{1-\gamma}+ \\ &\quad \left[c_{2}(\pi)\right]^{-\gamma}\left(\frac{\left(\frac{R}{1-\tau}\right)\left(e-\tau x^{*}-\pi c_{1}^{*}\right)}{1-\pi}-\left(\frac{R}{1-\tau}\right)c_{1}^{*}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} \mid_{\pi_s=\pi} &= \frac{\left(c_{1}^{*}\right)^{1-\gamma}}{1-\gamma}-(1-\pi)\frac{\left[c_{2}(\pi)\right]^{1-\gamma}}{1-\gamma}+\left[c_{2}(\pi)\right]^{-\gamma}\left[(1-\pi)\frac{R^{\frac{1}{\gamma}}}{1-\pi}c_{1}^{*}-\frac{R}{1-\tau}c_{1}^{*}\right] \\ &= \frac{\left(c_{1}^{*}\right)^{1-\gamma}}{1-\gamma}-(1-\pi)\frac{\left[\frac{R^{\frac{1}{\gamma}}}{1-\pi}c_{1}^{*}\right]^{1-\gamma}}{1-\gamma}+\left[\frac{R^{\frac{1}{\gamma}}}{1-\pi}c_{1}^{*}\right]^{-\gamma}c_{1}^{*}\left[R^{\frac{1}{\gamma}}-\frac{R}{1-\tau}\right] \\ &= \frac{\left(c_{1}^{*}\right)^{1-\gamma}}{1-\gamma}-\frac{1}{1-\gamma}(1-\pi)^{\gamma}R^{\frac{1-\gamma}{\gamma}}\left(c_{1}^{*}\right)^{1-\gamma}+\left(c_{1}^{*}\right)^{1-\gamma}(1-\pi)^{\gamma}R^{-1}\left[R^{\frac{1}{\gamma}}-\frac{R}{1-\tau}\right] \\ &= \left(c_{1}^{*}\right)^{1-\gamma}\left\{\frac{1}{1-\gamma}-\frac{1}{1-\gamma}(1-\pi)^{\gamma}R^{\frac{1-\gamma}{\gamma}}+(1-\pi)^{\gamma}\left[R^{\frac{1}{\gamma}-1}-\frac{1}{1-\tau}\right]\right\} \\ &= \left(c_{1}^{*}\right)^{1-\gamma}\left\{\frac{1-(1-\pi)^{\gamma}R^{\frac{1-\gamma}{\gamma}}}{1-\gamma}+\frac{(1-\pi)^{\gamma}}{R}\left[R^{\frac{1}{\gamma}}-\frac{R}{1-\tau}\right]\right\}\end{aligned}$$

In order for the BA to find it optimal to suspend immediately, we must have:

$$\begin{aligned}\frac{1}{1-\gamma}-\frac{1}{1-\gamma}(1-\pi)^{\gamma}R^{\frac{1-\gamma}{\gamma}}+(1-\pi)^{\gamma}\left[R^{\frac{1}{\gamma}-1}-\frac{1}{1-\tau}\right] &\leq 0 \\ \frac{1}{1-\gamma}-(1-\pi)^{\gamma}\left\{\frac{1}{1-\gamma}R^{\frac{1-\gamma}{\gamma}}-\left[R^{\frac{1}{\gamma}-1}-\frac{1}{1-\tau}\right]\right\} &\leq 0\end{aligned}$$

$$\frac{1}{1-\gamma}\leq(1-\pi)^{\gamma}\left\{\frac{1}{1-\gamma}R^{\frac{1-\gamma}{\gamma}}-\left[R^{\frac{1}{\gamma}-1}-\frac{1}{1-\tau}\right]\right\}$$

$$\begin{aligned}
\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\gamma}R^{\frac{1-\gamma}{\gamma}} - \left(R^{\frac{1}{\gamma}-1} - \frac{1}{1-\tau}\right)} &\leq (1-\pi)^{\gamma} \\
\left[\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\gamma}R^{\frac{1-\gamma}{\gamma}} - R^{\frac{1}{\gamma}-1} + \frac{1}{1-\tau}}\right]^{\frac{1}{\gamma}} &\leq 1-\pi \\
\pi \leq 1 - \left[\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\gamma}R^{\frac{1-\gamma}{\gamma}} - R^{\frac{1}{\gamma}-1} + \frac{1}{1-\tau}}\right]^{\frac{1}{\gamma}} &\equiv \underline{\pi}
\end{aligned}$$

The final expression shows that deposits would be frozen completely right at π if and only if $\pi \leq \underline{\pi}$.

■

Proof of Proposition 3. The proof follows the proof of Proposition 2 of this paper and also that of Proposition 2 in Ennis and Keister (2009). By definition, the banking system is fragile if the ex-post efficient deposit freeze π_s^M exceeds the threshold freeze point π^T. Since once $\pi_s^M > \pi^T$, then $c_1^* > c_2(\pi_s^M)$ in the sense that patient depositors would not find it rational to wait to withdraw. Therefore, we examine whether the BA has an incentive to freeze deposits after π^T. We start by differentiating the objective function with respect to π_s:

$$\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} = \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{[c_2(\pi_s)]^{1-\gamma}}{1-\gamma} + [c_2(\pi_s)]^{-\gamma}\left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^* - c_1^*}{1-\pi_s}\right)$$

This is the same equation as that provided in the proof of Proposition 2. Now we are interested in evaluating this derivative at π^T, where π^T is defined by condition $c_1^* = c_2\left(\pi^T\right)$:

$$\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s}\,|_{\pi_s=\pi^T} = \frac{(c_1^*)^{1-\gamma}}{1-\gamma} - (1-\pi)\frac{(c_1^*)^{1-\gamma}}{1-\gamma} + (c_1^*)^{-\gamma}\left(\frac{R}{1-\tau}\right)\left(\frac{e-\tau x^* - c_1^*}{1-\pi^T}\right).$$

Note that, as we define in the paper, $\widetilde{R} = \frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}$, and $c_1^* = \frac{e}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}} = e\widetilde{R}$. In addition,

$$\begin{aligned}
\pi^T &= \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right] - (1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau} - (1-\pi)\right]\widetilde{R}} \\
&= \frac{\frac{R}{1-\tau}\left[e-\tau\left(e-\pi e\widetilde{R}\right)\right] - (1-\pi)e\widetilde{R}}{\left[\frac{R}{1-\tau} - (1-\pi)\right]e\widetilde{R}} \\
&= \frac{\frac{R}{1-\tau}\left[e-\tau\left(e-\pi c_1^*\right)\right] - (1-\pi)c_1^*}{\left[\frac{R}{1-\tau} - (1-\pi)\right]c_1^*}.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\frac{R}{1-\tau}\frac{e-\tau x^*-c_1^*}{1-\pi^T} &= \left(\frac{R}{1-\tau}\right)\left(\frac{1}{\frac{-\frac{R}{1-\tau}\left\{e-c_1^*-\tau x^*\right\}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]c_1^*}}\right)(e-\tau x^*-c_1^*) \\
&= \left(\frac{R}{1-\tau}\right)\frac{\left[e-\tau x^*-c_1^*\right]\left[\frac{R}{1-\tau}-(1-\pi)\right]c_1^*}{-\frac{R}{1-\tau}\left\{e-c_1^*-\tau x^*\right\}} \\
&= \left[(1-\pi)-\frac{R}{1-\tau}\right]c_1^*
\end{aligned}$$

Upon substitution of $\frac{R}{1-\tau}\frac{e-\tau x^*-c_1^*}{1-\pi^T}$ into $\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s}$:

$$\begin{aligned}
\frac{\partial W_{BA}(\pi_s)}{\partial \pi_s} \quad \Big| \quad _{\pi_s=\pi^T} &= \pi\frac{(c_1^*)^{1-\gamma}}{1-\gamma}-(c_1^*)^{1-\gamma}\left[\frac{R}{1-\tau}-(1-\pi)\right] \\
&= \frac{(c_1^*)^{1-\gamma}}{1-\gamma}\left\{\pi-(1-\gamma)\left[\frac{R}{1-\tau}-(1-\pi)\right]\right\}.
\end{aligned}$$

From the final expression, deposits would not be frozen before π^T if and only if the following condition holds:

$$\begin{aligned}
\pi-(1-\gamma)\left[\frac{R}{1-\tau}-(1-\pi)\right] &\geq 0 \\
\pi-(1-\gamma)\frac{R}{1-\tau}+(1-\gamma)(1-\pi) &\geq 0 \\
\pi-(1-\gamma)\pi+(1-\gamma)\left(1-\frac{R}{1-\tau}\right) &\geq 0 \\
\gamma\pi &\geq (1-\gamma)\left(\frac{R}{1-\tau}-1\right) \\
\pi &\geq \frac{1-\gamma}{\gamma}\left(\frac{R}{1-\tau}-1\right)\equiv\widetilde{\pi}.
\end{aligned}$$

■

Proof of Corollary 1. In order to find conditions required in the Corollary, we begin by seeking to find conditions in which there exists a unique value $\pi^* \in (0,1)$ such that both conditions $c_1^* > e-\tau x^*$ and $\widehat{c}_1^* = e-\tau\widehat{x}^*$ hold. First, note that if $\tau > 1-\frac{1}{R^{\frac{1-\gamma}{\gamma}}}$, then $c_1^* > e-\tau x^*$. We next look to determine conditions in which there exists a unique value $\pi^* \in (0,1)$ such that $\widehat{c}_1^* = e-\tau\widehat{x}^*$. To begin, we have:

$$\widehat{c}_1^* = e-\tau\widehat{x}^*,$$

Substituting $\widehat{c}_1^* = e\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}$ yields:

$$e\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} = e\left(1-\left\{\tau-\tau\pi\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}\right\}\right).$$

We cancel out e on both sides of the equation and define $\mu_1(\pi) \equiv \left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}$ and $\mu_2(\pi) \equiv 1-\tau+\tau\pi\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}$.
In addition, it is also easy to show that:

$$\begin{aligned}
\frac{\partial\mu_1(\pi)}{\partial\pi} &= \frac{1}{1-\gamma}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{2-\gamma}{1-\gamma}}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)>0\\
\frac{\partial^2\mu_1(\pi)}{\partial\pi^2} &= \frac{1}{1-\gamma}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)^2\frac{2-\gamma}{1-\gamma}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{3-2\gamma}{1-\gamma}}>0\\
\frac{\partial\mu_2(\pi)}{\partial\pi} &= \tau\left\{\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}+\pi\frac{1}{1-\gamma}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{2-\gamma}{1-\gamma}}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)\right\}>0
\end{aligned}$$

and

$$\frac{\partial^2\mu_2(\pi)}{\partial\pi^2} = \tau\left\{\begin{array}{c}\frac{2}{1-\gamma}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{2-\gamma}{1-\gamma}}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)\\ +\pi\frac{1}{1-\gamma}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)^2\frac{2-\gamma}{1-\gamma}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{3-2\gamma}{1-\gamma}}\end{array}\right\}>0.$$

Furthermore, we have:

$$\begin{aligned}
\lim_{\pi\to0}\mu_1(\pi) &= \lim_{\pi\to0}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} = \left(\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\right)^{\frac{1}{1-\gamma}}\\
\lim_{\pi\to0}\mu_2(\pi) &= \lim_{\pi\to0}\left\{1-\tau+\tau\pi\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}\right\} = 1-\tau\\
\lim_{\pi\to1}\mu_1(\pi) &= \lim_{\pi\to1}\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} = 1
\end{aligned}$$

and

$$\lim_{\pi\to 1}\mu_2(\pi)=\lim_{\pi\to 1}\left\{1-\tau+\tau\pi\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}\right\}=1.$$

Subsequently, it is helpful to see the following properties of the functions $\mu_1(\pi)$ and $\mu_2(\pi)$:

$$\lim_{\pi\to 1}\frac{\partial\mu_1(\pi)}{\partial\pi}=\frac{1}{1-\gamma}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)$$

and

$$\lim_{\pi\to 1}\frac{\partial\mu_2(\pi)}{\partial\pi}=\tau\left[1+\frac{1}{1-\gamma}\left(R^{\frac{1-\gamma}{\gamma}}-1\right)\right].$$

Hence, to guarantee that there is an intersection $\pi^*\in(0,1)$ of the two functions, we need both of the following conditions to be satisfied: (a) $\lim_{\pi\to 0}\mu_2(\pi)>\lim_{\pi\to 0}\mu_1(\pi)$ and (b) $\lim_{\pi\to 1}\frac{\partial\mu_2(\pi)}{\partial\pi}>\lim_{\pi\to 1}\frac{\partial\mu_1(\pi)}{\partial\pi}$. By property (b), $\mu_1(\pi)$ lies above $\mu_2(\pi)$ as π is arbitrarily close to 1. Thus, by the mean value theorem, there must be a $\pi^*\in(0,1)$ where $\mu_1(\pi)=\mu_2(\pi)$ implying that a run equilibrium occurs in both types of financial systems. Moreover, since $\lim_{\pi\to 0}\mu_2(\pi)>\lim_{\pi\to 0}\mu_1(\pi)$, then $e-\tau\widehat{x}^*>\widehat{c}_1^*$ so that a run equilibrium does not occur in a monopolistic banking system for small values of $\pi,\pi\in(0,\pi^*)$.

First, suppose $\tau>\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}}$. Note that for $\lim_{\pi\to 0}\mu_2(\pi)>\lim_{\pi\to 0}\mu_1(\pi)$ to hold, we must have $\tau<1-R^{-\frac{1}{\gamma}}\equiv\overline{\tau}$. Moreover, suppose $\tau>\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}$, then $\lim_{\pi\to 1}\frac{\partial\mu_2(\pi)}{\partial\pi}>\lim_{\pi\to 1}\frac{\partial\mu_1(\pi)}{\partial\pi}$ is satisfied. Also define $\underline{\tau}\equiv\max\left\{\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}},\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}\right\}$. Hence, $\overline{\tau}>\underline{\tau}$ if and only if $\max\left\{\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}},\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}\right\}<1-R^{-\frac{1}{\gamma}}$. Therefore, under conditions (a) and (b) above, $\pi^*\in(0,1)$ exists. Further, since both $\mu_1(\pi)$ and $\mu_2(\pi)$ are convex in π, π^* is unique. That is, if $\tau\in(\underline{\tau},\overline{\tau})$ and $\pi\in(0,\pi^*)$, a run equilibrium exists under the perfectly competitive banking system while deposit contracts offered by the monopolistic bank are run-proof.

Next, we show that the parameter space that $\max\left\{\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}},\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}\right\}<1-R^{-\frac{1}{\gamma}}$ is non-empty. To begin, suppose that:

$$\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}}<\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma},$$

Upon substitution of $\widetilde{R}=\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-1}$, we then have:

$$\frac{1-\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-\frac{1}{1-\gamma}}}{1-\pi\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-\frac{1}{1-\gamma}}}<\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}$$

To simplify our analysis, we assume that $\gamma = \frac{1}{2}$, the above condition can be reduced:

$$\begin{aligned}
\frac{1-\left[\pi+(1-\pi)R\right]^{-2}}{1-\pi\left[\pi+(1-\pi)R\right]^{-2}} &< \frac{R-1}{R-\frac{1}{2}} \\
\frac{1-\frac{1}{\left[\pi+(1-\pi)R\right]^2}}{1-\frac{\pi}{\left[\pi+(1-\pi)R\right]^2}} &< \frac{R-1}{R-\frac{1}{2}} \\
\frac{\left[\pi+(1-\pi)R\right]^2-1}{\left[\pi+(1-\pi)R\right]^2-\pi} &< \frac{R-1}{R-\frac{1}{2}}
\end{aligned}$$

To proceed, we seek to obtain conditions in which $\left[\pi+(1-\pi)R\right]^2-1 < R-1$ holds:

$$\begin{aligned}
\left[\pi+(1-\pi)R\right]^2 &< R \\
\pi &< \frac{R-R^{\frac{1}{2}}}{R-1} = \frac{1-R^{-\frac{1}{2}}}{1-R^{-1}}.
\end{aligned}$$

Moreover, we next look at the relationship between $\left[\pi+(1-\pi)R\right]^2-\pi$ and $R-\frac{1}{2}$:

$$\begin{aligned}
\lim_{\pi\to 0}\ \left[\pi+(1-\pi)R\right]^2-\pi &= R^2 > R-\frac{1}{2} \\
\lim_{\pi\to 1}\ \left[\pi+(1-\pi)R\right]^2-\pi &= 0 < R-\frac{1}{2}
\end{aligned}$$

which obviously holds since $R > 1$. Thus, there is a parameter space that $\frac{\left[\pi+(1-\pi)R\right]^2-1}{\left[\pi+(1-\pi)R\right]^2-\pi} < \frac{R-1}{R-\frac{1}{2}}$. On the other hand, $\frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma} < 1-R^{-\frac{1}{\gamma}}$ can reduce to:

$$\begin{aligned}
\frac{\left(1-R^{-\frac{1}{\gamma}}\right)\left(R^{\frac{1-\gamma}{\gamma}}-\gamma\right)}{R^{\frac{1-\gamma}{\gamma}}-\gamma} &> \frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma} \\
\left(1-R^{-\frac{1}{\gamma}}\right)\left(R^{\frac{1-\gamma}{\gamma}}-\gamma\right) &> R^{\frac{1-\gamma}{\gamma}}-1 \\
\left(R^{\frac{1}{\gamma}-1}-\gamma\right)-\left(R^{-1}-\gamma R^{-\frac{1}{\gamma}}\right) &> R^{\frac{1}{\gamma}-1}-1 \\
\left(R^{-\frac{1}{\gamma}}-1\right)\gamma-R^{-1} &> -1 \\
R^{-1}-\left(R^{-\frac{1}{\gamma}}-1\right)\gamma &< 1 \\
\gamma &< \frac{1-R^{-1}}{1-R^{-\frac{1}{\gamma}}}.
\end{aligned}$$

Again, upon substituting $\gamma = \frac{1}{2}$, we see that the above condition holds:

$$\begin{aligned} \frac{1}{2} &< \frac{1-R^{-1}}{1-R^{-2}} = \frac{1-R^{-1}}{(1-R^{-1})(1+R^{-1})} = \frac{1}{1+R^{-1}} \\ \frac{1}{2} &< \frac{1}{1+R^{-1}} \\ R^{-1} &< 1. \end{aligned}$$

We next show that the parameter space that $\max\left\{\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}}, \frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}\right\} = \frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}} < 1 - R^{-\frac{1}{\gamma}}$ can also hold. To begin, this implies that

$$\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}} > \frac{R^{\frac{1-\gamma}{\gamma}}-1}{R^{\frac{1-\gamma}{\gamma}}-\gamma}$$

As demonstrated above, there exists a parameter space where $\frac{[\pi+(1-\pi)R]^2-1}{[\pi+(1-\pi)R]^2-\pi} > \frac{R-1}{R-\frac{1}{2}}$. We next see if $\frac{1-\widetilde{R}^{\frac{1}{1-\gamma}}}{1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}} < 1 - R^{-\frac{1}{\gamma}}$ can hold, which is simplified to:

$$\frac{1-\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-\frac{1}{1-\gamma}}}{1-\pi\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]^{-\frac{1}{1-\gamma}}} < 1 - R^{-\frac{1}{\gamma}}$$

Again, imposing $\gamma = \frac{1}{2}$ yields:

$$\frac{[\pi+(1-\pi)R]^2-1}{[\pi+(1-\pi)R]^2-\pi} < 1 - R^{-2}.$$

It is helpful to derive some properties of $\frac{[\pi+(1-\pi)R]^2-1}{[\pi+(1-\pi)R]^2-\pi}$:

$$\begin{aligned} \lim_{\pi\to 0}\frac{[\pi+(1-\pi)R]^2-1}{[\pi+(1-\pi)R]^2-\pi} &= \frac{R^2-1}{R^2-\pi} \\ \lim_{\pi\to 1}\frac{[\pi+(1-\pi)R]^2-1}{[\pi+(1-\pi)R]^2-\pi} &= 0 \end{aligned}$$

In this manner, the parameter set is non-empty if:

$$0 < 1 - R^{-2},$$

which holds since $R > 1$.

We next seek to verify if it is possible that a run equilibrium occurs in a monopolistic banking

system, but not a perfectly competitive banking system. Note that in order for a run equilibrium to be possible in a monopolistic system, we must have $\tau > 1 - R^{-\frac{1}{\gamma}} \equiv \overline{\tau}$. Yet, in order to rule out the possibility of a run equilibrium under perfect competition we must have $\tau < 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}}$. Combining the properties together:

$$1 - R^{-\frac{1}{\gamma}} < 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}}$$

which reduces to:

$$R^{-1} > 1$$

which is a contradiction since $R > 1$.

To summarize, there exists a unique value $\pi^* \in (0, 1)$ under which a run occurs with a perfectly competitive bank while run does not appear with a monopolistic bank, which completes the proof of Corollary 1.

■

Proof of Lemma 2. We focus on the demonstration of (iv) in the Lemma. To begin, to see how a difference in the liquidation value across competitive structures is affected by asset returns, we first need Assumption 2 to hold so that the monopolistic bank is susceptible to a self-fulfilling run. Specifically, Assumption 2 holds if:

$$\begin{aligned}
\widehat{c}_1^* &> e - \tau \widehat{x}^* \\
e\left[\frac{1}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} &> e - \tau\left(e - \pi e\left[\frac{1}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}\right) \\
\left[\frac{1}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} - \tau\pi\left[\frac{1}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} &> 1 - \tau \\
(1 - \tau\pi)\left[\frac{1}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} &> 1 - \tau \\
\left(\frac{1-\tau}{1-\tau\pi}\right)^{1-\gamma}\left[\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}\right] &< 1
\end{aligned}$$

$$R < \left[\frac{\left(\frac{1-\tau}{1-\tau\pi}\right)^{\gamma-1} - \pi}{(1-\pi)}\right]^{\frac{\gamma}{1-\gamma}} \equiv \overline{R}.$$

Thus, investment return has an upper bound in which deposit contracts offered by the monopoly bank are not run-proof, in which it also leads to satisfaction of Assumption 1. To proceed, differ-

entiating Δ_L with respect to R yields:

$$\frac{\partial \Delta_L}{\partial R} = e\left(1-\tau\pi\right)\widetilde{R}^2\left(1-\pi\right)\frac{1-\gamma}{\gamma}R^{\frac{1-2\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\widetilde{R}^{\frac{\gamma}{1-\gamma}}-1\right).$$

Simple algebra shows that $\frac{\partial \Delta_L}{\partial R} > 0$ if:

$$\begin{aligned}
\frac{1}{1-\gamma}\widetilde{R}^{\frac{\gamma}{1-\gamma}}-1 &> 0 \\
\frac{1}{1-\gamma}\left(\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right)^{\frac{\gamma}{1-\gamma}}-1 &> 0 \\
\left(\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right)^{\frac{\gamma}{1-\gamma}} &> 1-\gamma \\
\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right](1-\gamma)^{\frac{1-\gamma}{\gamma}} &< 1
\end{aligned}$$

$$R < \left[\frac{(1-\gamma)^{-\frac{1-\gamma}{\gamma}}-\pi}{1-\pi}\right]^{\frac{\gamma}{1-\gamma}} \equiv \underline{R}.$$

In addition, $\overline{R} > \underline{R}$ if:

$$\begin{aligned}
\left[\frac{\left(\frac{1-\tau}{1-\tau\pi}\right)^{\gamma-1}-\pi}{1-\pi}\right]^{\frac{\gamma}{1-\gamma}} &> \left[\frac{(1-\gamma)^{-\frac{1-\gamma}{\gamma}}-\pi}{(1-\pi)}\right]^{\frac{\gamma}{1-\gamma}} \\
\left(\frac{1-\tau}{1-\tau\pi}\right)^{\gamma-1} &> (1-\gamma)^{\frac{\gamma-1}{\gamma}}
\end{aligned}$$

since $\gamma - 1 < 0$, we have

$$\begin{aligned}
\frac{1-\tau}{1-\tau\pi} &< (1-\gamma)^{\frac{1}{\gamma}} \\
\tau\pi &< 1-\frac{1-\tau}{(1-\gamma)^{\frac{1}{\gamma}}} \\
\pi &< \frac{(1-\gamma)^{\frac{1}{\gamma}}-(1-\tau)}{(1-\gamma)^{\frac{1}{\gamma}}\tau}
\end{aligned}$$

That is, π is such that $\pi \in \left(0, \frac{(1-\gamma)^{\frac{1}{\gamma}}-(1-\tau)}{(1-\gamma)^{\frac{1}{\gamma}}\tau}\right)$. Consequently, Δ_L is strictly increasing as $R \in (1, \underline{R})$. On the other hand, $\frac{\partial \Delta_L}{\partial R} < 0$ implying that Δ_L is strictly decreasing as $R \in \left(\underline{R}, \overline{R}\right)$. Notably, we argue that Δ_L can be viewed as one critical measures for the potential for a run to emerge across banking systems. Hence, we demonstrate that an increase in investment return R

may exhibit non-monotone relationship with bank's vulnerability to runs across structures.

■

Proof of Proposition 5. We investigate when deposits are completely frozen credibly right at π if the BA is completely benevolent. The derivative of the objective function is given by:

$$\frac{\partial \widehat{W}_{BA}\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s} \mid_{\widehat{\pi}_s=\pi} = \frac{1}{1-\gamma} e^{1-\gamma} \widetilde{R} \left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]$$

Define $\underline{\widehat{\pi}}^* \equiv 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}}$. This final expression shows that $\frac{\partial \widehat{W}_{BA}(\widehat{\pi}_s)}{\partial \widehat{\pi}_s} \mid_{\widehat{\pi}_s=\pi} \leq 0$ holds and hence deposits would be frozen completely right at π, if and only if the following condition satisfies:

$$\begin{aligned} 1-(1-\pi)R^{\frac{1-\gamma}{\gamma}} &\leq 0 \\ \frac{1}{R^{\frac{1-\gamma}{\gamma}}} &\leq (1-\pi) \end{aligned}$$

$$\pi \leq 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}} \equiv \underline{\widehat{\pi}}^*.$$

■

Proof of Corollary 2. Note that from condition (6) in Proposition 2:

$$\pi \leq 1 - \left[\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\gamma}R^{\frac{1-\gamma}{\gamma}} - R^{\frac{1}{\gamma}-1} + \frac{1}{1-\tau}}\right]^{\frac{1}{\gamma}}$$

and condition (13) in Proposition 5:

$$\pi \leq 1 - \frac{1}{R^{\frac{1-\gamma}{\gamma}}}$$

It is straightforward to derive that there exists parameter space such that condition (6) in Proposi-

tion 2 holds, but condition (13) in Proposition 5 does not:

$$\begin{aligned}
1-\left[\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\tau}+\left(\frac{\gamma}{1-\gamma}\right)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{\gamma}} &\leq 1-\frac{1}{R^{\frac{1-\gamma}{\gamma}}} \\
\left[\frac{\frac{1}{1-\gamma}}{\frac{1}{1-\tau}+\left(\frac{\gamma}{1-\gamma}\right)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{\gamma}} &\geq \frac{1}{R^{\frac{1-\gamma}{\gamma}}} \\
(1-\gamma)^{-1} &\geq R^{\gamma-1}\left[(1-\tau)^{-1}+\gamma(1-\gamma)^{-1}R^{\frac{1-\gamma}{\gamma}}\right] \\
(1-\gamma)^{-1} &\geq R^{\gamma-1}(1-\tau)^{-1}+\gamma(1-\gamma)^{-1}R^{\frac{1}{\gamma}+\gamma-2} \\
\frac{(1-\gamma)^{-1}-\gamma(1-\gamma)^{-1}R^{\frac{1}{\gamma}+\gamma-2}}{R^{\gamma-1}} &\geq (1-\tau)^{-1} \\
\frac{R^{\gamma-1}}{(1-\gamma)^{-1}-\gamma(1-\gamma)^{-1}R^{\frac{1}{\gamma}+\gamma-2}} &\geq 1-\tau \\
\tau &\geq 1-\frac{(1-\gamma)R^{\gamma-1}}{1-\gamma R^{\frac{1}{\gamma}+\gamma-2}} \equiv \widetilde{\tau}.
\end{aligned}$$

■

Proof of Proposition 6. To begin, we evaluate the first derivative of the BA's objective function at $\widehat{\pi}_{ZP}$:

$$W'(\widehat{\pi}_{ZP}) = u\left(\widehat{c}_1^*\right) - (1-\pi)\,u\left[\widehat{c}_2\left(\widehat{\pi}_{ZP}\right)\right] - u'\left[\widehat{c}_2\left(\widehat{\pi}_{ZP}\right)\right]\frac{R}{1-\tau}\frac{\widehat{c}_1^* - (e-\tau\widehat{x}^*)}{1-\widehat{\pi}_{ZP}}$$

As $\widehat{c}_2\left(\widehat{\pi}_{ZP}\right) = \widehat{c}_2^*$:

$$W'(\widehat{\pi}_{ZP}) = u\left(\widehat{c}_1^*\right) - (1-\pi)\,u\left(\widehat{c}_2^*\right) - u'\left(\widehat{c}_2^*\right)\frac{R}{1-\tau}\frac{\widehat{c}_1^* - (e-\tau\widehat{x}^*)}{1-\widehat{\pi}_{ZP}}$$

Further, note that:

$$\psi\left(\widehat{\pi}_s\right) = \left\{ \begin{array}{c} \frac{\left(e-\tau\widehat{x}^*-\widehat{\pi}_s\widehat{c}_1^*\right)R}{1-\tau} - (1-\pi)\left(1-\widehat{\pi}_s\right)\widehat{c}_2^* \\ 0 \end{array} \right\} \text{ as } \left\{ \begin{array}{c} \pi \leq \widehat{\pi}_s < \widehat{\pi}_{ZP} \\ otherwise \end{array} \right\},$$

where $\widehat{\pi}_{ZP} \equiv \frac{(e-\tau\widehat{x}^*)R-(1-\tau)(1-\pi)\widehat{c}_2^*}{\widehat{c}_1^*R-(1-\tau)(1-\pi)\widehat{c}_2^*}$ represents the zero profit threshold, which is defined by the condition $\psi\left(\widehat{\pi}_{ZP}\right) = 0$. We next prove that $\widehat{\pi}_{ZP} \in (0,1)$. Note that if the bank earns positive profits, the following condition must hold:

$$\left(e-\tau\widehat{x}^*-\widehat{\pi}_s\widehat{c}_1^*\right)R > (1-\tau)(1-\pi)\left(1-\widehat{\pi}_s\right)\widehat{c}_2^*.$$

We proceed to consider if $(e-\tau\widehat{x}^*)\,R-(1-\tau)\,(1-\pi)\widehat{c}_2^*>0$. By positive profits, we have:

$$(e-\tau\widehat{x}^*)\,R>R\widehat{\pi}_s\widehat{c}_1^*+(1-\tau)\,(1-\pi)\,(1-\widehat{\pi}_s)\,\widehat{c}_2^*.$$

From Assumption 2, a run equilibrium exists in a monopolistic banking system if and only if:

$$\widehat{c}_1^*>e-\tau\widehat{x}^*.$$

Since the bank earns positive profits, we have:

$$\begin{aligned}(e-\tau\widehat{x}^*)\,R &> R\widehat{\pi}_s\,(e-\tau\widehat{x}^*)+(1-\tau)\,(1-\pi)\,(1-\widehat{\pi}_s)\,\widehat{c}_2^*\\ (e-\tau\widehat{x}^*)\,R\,(1-\widehat{\pi}_s) &> (1-\tau)\,(1-\pi)\,(1-\widehat{\pi}_s)\,\widehat{c}_2^*\\ (e-\tau\widehat{x}^*)\,R &> (1-\tau)\,(1-\pi)\widehat{c}_2^*,\end{aligned}$$

which proves that the numerator of $\widehat{\pi}_{ZP}$ is positive. Further, from previous steps, we show that with Assumption 2:

$$\widehat{c}_1^*R>(e-\tau\widehat{x}^*)\,R>(1-\tau)\,(1-\pi)\widehat{c}_2^*,$$

which suggests that the denominator of $\widehat{\pi}_{ZP}$ is positive too. We proceed to check if $\widehat{\pi}_{ZP}<1$, implying that:

$$\begin{aligned}(e-\tau\widehat{x}^*)\,R-(1-\tau)\,(1-\pi)\widehat{c}_2^* &< \widehat{c}_1^*R-(1-\tau)\,(1-\pi)\widehat{c}_2^*\\ (e-\tau\widehat{x}^*)\,R &< \widehat{c}_1^*R\\ e-\tau\widehat{x}^* &< \widehat{c}_1^*,\end{aligned}$$

which is consistent with Assumption 2 and completes the proof that $\widehat{\pi}_{ZP}\in(0,1)$.
Next, $\widehat{\pi}_{ZP}$ can be rewritten as:

$$\begin{aligned}\widehat{\pi}_{ZP} &\equiv \frac{(e-\tau\widehat{x}^*)\,R-(1-\tau)\,(1-\pi)\widehat{c}_2^*}{\widehat{c}_1^*R-(1-\tau)\,(1-\pi)\widehat{c}_2^*}\\ &= \frac{-\left[\widehat{c}_1^*-(e-\tau\widehat{x}^*)\right]R+\left[\widehat{c}_1^*R-(1-\tau)\,(1-\pi)\widehat{c}_2^*\right]}{\widehat{c}_1^*R-(1-\tau)\,(1-\pi)\widehat{c}_2^*}\\ &= -\frac{\left[\widehat{c}_1^*-(e-\tau\widehat{x}^*)\right]R}{\widehat{c}_1^*R-(1-\tau)\,(1-\pi)\widehat{c}_2^*}+1.\end{aligned}$$

Hence,

$$1-\widehat{\pi}_{ZP}=\frac{\left[\widehat{c}_1^*-(e-\tau\widehat{x}^*)\right]R}{\widehat{c}_1^*R-(1-\tau)\,(1-\pi)\widehat{c}_2^*}$$

Upon substitution for $(1-\widehat{\pi}_{ZP})$, we have:

$$\begin{aligned} W'(\widehat{\pi}_{ZP}) &= u\left(\widehat{c}_1^*\right) - (1-\pi)\, u\left(\widehat{c}_2^*\right) - u'\left(\widehat{c}_2^*\right) \frac{R}{1-\tau} \frac{\widehat{c}_1^* - (e - \tau\widehat{x}^*)}{\left(\frac{\left[\widehat{c}_1^* - (e-\tau\widehat{x}^*)\right]R}{\widehat{c}_1^* R - (1-\tau)(1-\pi)\widehat{c}_2^*}\right)} \\ &= u\left(\widehat{c}_1^*\right) - (1-\pi)\, u\left(\widehat{c}_2^*\right) - u'\left(\widehat{c}_2^*\right) \left(\frac{\widehat{c}_1^* R - (1-\tau)\,(1-\pi)\widehat{c}_2^*}{1-\tau}\right) \end{aligned}$$

Also note that $\widehat{c}_2^* = R^{\frac{1}{\gamma}}\widehat{c}_1^*$:

$$\begin{aligned} W'(\widehat{\pi}_{ZP}) &= u\left(\widehat{c}_1^*\right) - (1-\pi)\, u\left(\widehat{c}_2^*\right) - u'\left(\widehat{c}_2^*\right) \left(\frac{\widehat{c}_1^* R - (1-\tau)\,(1-\pi)\widehat{c}_2^*}{1-\tau}\right) \\ &= u\left(\widehat{c}_1^*\right) - (1-\pi)\, u\left(R^{\frac{1}{\gamma}}\widehat{c}_1^*\right) - u'\left(R^{\frac{1}{\gamma}}\widehat{c}_1^*\right) \left(\frac{\widehat{c}_1^* R - (1-\tau)\,(1-\pi) R^{\frac{1}{\gamma}}\widehat{c}_1^*}{1-\tau}\right) \\ &= u\left(\widehat{c}_1^*\right) - (1-\pi)\, u\left(R^{\frac{1}{\gamma}}\widehat{c}_1^*\right) - u'\left(R^{\frac{1}{\gamma}}\widehat{c}_1^*\right) \widehat{c}_1^* R \left(\frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau}\right) \\ &= \frac{1}{1-\gamma}\left(\widehat{c}_1^*\right)^{1-\gamma} - \frac{1-\pi}{1-\gamma} R^{\frac{1-\gamma}{\gamma}} \left(\widehat{c}_1^*\right)^{1-\gamma} - \left[R^{-1}\left(\widehat{c}_1^*\right)^{-\gamma}\right] \widehat{c}_1^* R \left(\frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau}\right) \\ &= \frac{1}{1-\gamma}\left(\widehat{c}_1^*\right)^{1-\gamma} - \frac{1-\pi}{1-\gamma}\left(R^{\frac{1}{\gamma}}\widehat{c}_1^*\right)^{1-\gamma} - \left(\widehat{c}_1^*\right)^{1-\gamma} \left(\frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau}\right) \\ &= \frac{1}{1-\gamma}\left(\widehat{c}_1^*\right)^{1-\gamma} - \frac{1-\pi}{1-\gamma} R^{\frac{1-\gamma}{\gamma}} \left(\widehat{c}_1^*\right)^{1-\gamma} - \left(\widehat{c}_1^*\right)^{1-\gamma} \left(\frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau}\right) \\ &= \left(\widehat{c}_1^*\right)^{1-\gamma} \left[\frac{1-(1-\pi)\, R^{\frac{1-\gamma}{\gamma}}}{1-\gamma} - \frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau}\right] \end{aligned}$$

If $W'(\widehat{\pi}_{ZP}) \leq 0$, then we have:

$$\frac{1-(1-\pi)\, R^{\frac{1-\gamma}{\gamma}}}{1-\gamma} - \frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau} \leq 0$$

$$\frac{1-(1-\pi)\, R^{\frac{1-\gamma}{\gamma}}}{1-\gamma} \leq \frac{1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}}{1-\tau}$$

$$\begin{aligned} \left[1-(1-\pi)\, R^{\frac{1-\gamma}{\gamma}}\right](1-\tau) &\leq (1-\gamma)\left[1-(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}}\right] \\ (1-\tau) - (1-\tau)\,(1-\pi)\, R^{\frac{1-\gamma}{\gamma}} &\leq (1-\gamma) - (1-\gamma)\,(1-\tau)\,(1-\pi) R^{\frac{1-\gamma}{\gamma}} \end{aligned}$$

$$\gamma - \tau \leq \gamma\,(1-\tau)\,(1-\pi)\, R^{\frac{1-\gamma}{\gamma}}$$

$$\frac{\gamma-\tau}{\gamma\left(1-\tau\right)R^{\frac{1-\gamma}{\gamma}}} \leq 1-\pi$$

$$\pi \leq 1-\frac{\gamma-\tau}{\gamma\left(1-\tau\right)R^{\frac{1-\gamma}{\gamma}}} \equiv \widehat{\underline{\pi}}.$$

That is, suppose that $\pi \in \left(\max\left\{\pi^*, \widehat{\underline{\pi}}^*\right\}, \widehat{\underline{\pi}}\right]$ and $\widehat{\underline{\pi}} < \widetilde{\pi}$. That is, if π is greater than $\max\left\{\pi^*, \widehat{\underline{\pi}}^*\right\}$, a run equilibrium exists with a monopoly bank. In addition, liquidity risk is relatively high which leads the BA to allow more withdrawals beyond π. It is worth noting that if the proportion of impatient depositors is sufficiently large (i.e. $\pi \geq \widetilde{\pi}$), the BA tends to impose a freeze relatively late in which a run equilibrium is the unique outcome. Yet, as long as π is not too large, the BA would find it optimal to declare a credible suspension at $\widehat{\pi}_{ZP}$ without inducing financial fragility. Due to the complexity of solving for π^*, we are unable to proceed analytically to prove that there exists a non-empty parameter space for the proposition. Therefore, we show that such parameters exist through some numerical examples. These are based on the following set of parameters: $\gamma = 0.5$, $R = 1.15$, $\tau = 0.3$, and $e = 1$. In this exercise, π^* is equal to 0 as Assumption 2 necessarily holds for all values of π. Moreover, $\widehat{\underline{\pi}}^* = 0.1304$, $\widehat{\underline{\pi}} = 0.5031$, and $\widetilde{\pi} = 0.6429$. Thus, we have $\max\left\{\pi^*, \widehat{\underline{\pi}}^*\right\} = \widehat{\underline{\pi}}^*$. That is, if $\pi \in \left(\pi^* = 0.1304, \widehat{\underline{\pi}} = 0.5031\right]$, the BA will freeze deposits at $\widehat{\pi}_{ZP}$. Another example uses the same set of parameter values except that $\tau = 0.24$. Under these parameters, $\pi^* = 0.3$, $\widehat{\underline{\pi}}^* = 0.1304$, $\widehat{\underline{\pi}} = 0.405$, and $\widetilde{\pi} = 0.5132$. This example illustrates the case in which $\pi^* > \widehat{\underline{\pi}}^*$. Therefore, if $\pi \in \left(\pi^* = 0.3, \widehat{\underline{\pi}} = 0.405\right]$, the BA will freeze deposits at $\widehat{\pi}_{ZP}$.

■

Proof of Proposition 7. We start with the demonstration that $\Delta_T > 0$. The banking system is said to be more resistant to runs and hence less susceptible to runs if it admits a larger threshold freeze point. This implies that the banking system is able to meet more liquidity demand in the short run. To begin, under both banking structures, the expressions for π^T and $\widehat{\pi}^T$ are given by $\pi^T = \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-(1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}}$ and $\widehat{\pi}^T = \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}\right)\right]-(1-\pi)\widetilde{R}^{\frac{1}{1-\gamma}}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}^{\frac{1}{1-\gamma}}}$, respectively.

Define $\Delta_T \equiv \widehat{\pi}^T - \pi^T$. In particular,

$$\begin{aligned}
\Delta_T &= \frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}\right)\right]\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-(1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} - \left(\frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-(1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}}\right) \\
&= \frac{\frac{R}{1-\tau}\left\{\left[1-\tau\left(1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}\right)\right]\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]\right\}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} \\
&= \frac{\frac{R}{1-\tau}\left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-\tau\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}+\tau\pi\widetilde{R}-1+\tau-\tau\pi\widetilde{R}\right]}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} \\
&= \frac{\frac{R}{1-\tau}\left[(1-\tau)\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-(1-\tau)\right]}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} \\
&= \frac{\frac{R}{1-\tau}(1-\tau)\left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-1\right]}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} \\
&= \frac{R\left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-1\right]}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}} > 0,
\end{aligned}$$

as $\widetilde{R} \in (0,1)$.

Subsequently, we show that the monopoly bank holds a higher buffer level than competitive banks and the buffer differential (Δ_b) is strictly increasing in bank size. Specifically, under the perfectly competitive banking system, we have the expression for the buffer:

$$b(\pi) = \left(\pi^T - \pi\right) c_1^*.$$

Under the monopolistic banking system, the buffer is represented by:

$$\widehat{b}(\pi) = \left(\widehat{\pi}^T - \pi\right) \widehat{c}_1^*.$$

We then define the buffer differential as $\Delta_b \equiv \widehat{b}(\pi) - b(\pi)$:

$$\Delta_b = \left(\widehat{\pi}^T - \pi\right) \widehat{c}_1^* - \left(\pi^T - \pi\right) c_1^*.$$

Next, we prove that $\Delta_b > 0$. To see more clearly, assume that $\Delta_b > 0$, which is equivalent to:

$$\left(\widehat{\pi}^T - \pi\right)\widehat{c}_1^* > \left(\pi^T - \pi\right)c_1^*$$

$$\Rightarrow \frac{\widehat{\pi}^T - \pi}{\pi^T - \pi} > \frac{c_1^*}{\widehat{c}_1^*} = \frac{\frac{e}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}}{e\left[\frac{1}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}}} = \frac{\widetilde{R}}{\left(\widetilde{R}\right)^{\frac{1}{1-\gamma}}} = \left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}$$

$$\Rightarrow \frac{\widehat{\pi}^T - \pi^T + \pi^T - \pi}{\pi^T - \pi} > \left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}$$

$$\Rightarrow \frac{\widehat{\pi}^T - \pi^T}{\pi^T - \pi} + 1 > \left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}$$

$$\Rightarrow \frac{\frac{R\left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}-1\right]}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}}}{\frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-(1-\pi)\widetilde{R}-\pi\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}}} + 1 > \left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}}$$

$$\Rightarrow \frac{R\left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}} - 1\right]}{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right] - (1-\pi)\widetilde{R} - \pi\left[\frac{R}{1-\tau} - (1-\pi)\right]\widetilde{R}} > \left[\left(\frac{1}{\widetilde{R}}\right)^{\frac{\gamma}{1-\gamma}} - 1\right]$$

$$\Rightarrow \frac{R}{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right] - (1-\pi)\widetilde{R} - \pi\left[\frac{R}{1-\tau} - (1-\pi)\right]\widetilde{R}} > 1$$

The last equation shows that, if $\Delta_b > 0$ holds, the following condition must be satisfied:

$$R > \frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right] - \left\{(1-\pi) + \pi\left[\frac{R}{1-\tau} - (1-\pi)\right]\right\}\widetilde{R}.$$

To see if the above condition holds, we reduce the above condition:

$$\begin{aligned}
R &> \frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-\left\{(1-\pi)+\pi\left[\frac{R}{1-\tau}-(1-\pi)\right]\right\}\widetilde{R} \\
&> \frac{R\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]}{1-\tau}-\frac{\widetilde{R}\left\{(1-\tau)(1-\pi)+\pi\left[R-(1-\tau)(1-\pi)\right]\right\}}{1-\tau} \\
&> \frac{R\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-\widetilde{R}\left\{(1-\tau)(1-\pi)+\pi\left[R-(1-\tau)(1-\pi)\right]\right\}}{1-\tau} \\
&> \frac{R(1-\tau)+\tau\pi R\widetilde{R}-\widetilde{R}(1-\tau)(1-\pi)-\pi\widetilde{R}\left[R-(1-\tau)(1-\pi)\right]}{1-\tau} \\
&> \frac{R(1-\tau)+\tau\pi R\widetilde{R}-\pi\widetilde{R}R+\pi\widetilde{R}(1-\tau)(1-\pi)-\widetilde{R}(1-\tau)(1-\pi)}{1-\tau} \\
&> \frac{(1-\tau)R-(1-\tau)\pi R\widetilde{R}-\widetilde{R}(1-\tau)(1-\pi)^2}{1-\tau} \\
&> R-\pi R\widetilde{R}-\widetilde{R}(1-\pi)^2 \\
0 &> -\left[\pi R\widetilde{R}+\widetilde{R}(1-\pi)^2\right],
\end{aligned}$$

which necessarily holds and also completes the proof that $\Delta_b > 0$ holds. In addition, Δ_b can be written as:

$$\Delta_b = e\left(\begin{array}{c}\left\{\frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}\right)\right]-(1-\pi)\widetilde{R}^{\frac{1}{1-\gamma}}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}^{\frac{1}{1-\gamma}}}-\pi\right\}\widetilde{R}^{\frac{1}{1-\gamma}} \\ -\left\{\frac{\frac{R}{1-\tau}\left[1-\tau\left(1-\pi\widetilde{R}\right)\right]-(1-\pi)\widetilde{R}}{\left[\frac{R}{1-\tau}-(1-\pi)\right]\widetilde{R}}-\pi\right\}\widetilde{R}\end{array}\right) > 0.$$

It is straightforward to show that the buffer differential Δ_b is strictly increasing in bank size.

■

Proof of Proposition 8. The proof follows that of Proposition 3 closely.

■

Proof of Proposition 9. We investigate when deposits are completely frozen right at π if the BA follows an *ex-post* efficient suspension policy. The derivative of the objective function is given by:

$$\frac{\partial\widehat{W}_{BA}(\widehat{\pi}_s)}{\partial\widehat{\pi}_s} = \frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]+\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[(1-\pi)-\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\right].$$

This final expression shows that $\frac{\partial\widehat{W}_{BA}(\widehat{\pi}_s)}{\partial\widehat{\pi}_s}\ |_{\widehat{\pi}_s=\pi} \leq 0$ holds, and hence deposits would be frozen

completely right at π, if and only if the following condition satisfies:

$$\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]+\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[(1-\pi)-\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\right]\leq 0$$

$$\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}+\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}(1-\pi)-\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\leq 0$$

$$\frac{\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}-\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}}{R^{\frac{1-\gamma}{\gamma}}\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}-\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}}\leq 1-\pi$$

$$\begin{aligned}
\pi &\leq 1-\frac{\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}-\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}}{R^{\frac{1-\gamma}{\gamma}}\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}-\delta eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}}\\
\pi &\leq 1-\frac{\frac{1}{1-\gamma}e^{-\gamma}\widetilde{R}-\delta\widetilde{R}^{\frac{1}{1-\gamma}}R\frac{1}{1-\tau}}{R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}e^{-\gamma}\widetilde{R}-\delta R\widetilde{R}^{\frac{1}{1-\gamma}}\right)}\\
\pi &\leq 1-\left[\frac{\frac{1}{1-\gamma}e^{-\gamma}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}}{R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}e^{-\gamma}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)}\right]\equiv\underline{\widehat{\pi}}^{**}.
\end{aligned}$$

Hence, the BA would find it optimal to freeze deposits right at π after identifying a run.

■

Proof of Lemma 3. Note that $\underline{\widehat{\pi}}^{**}\equiv 1-\left[\frac{\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}}{R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)}\right]$. Thus, we have:

$$\begin{aligned}
\frac{\partial\underline{\widehat{\pi}}^{**}}{\partial\delta} &= \frac{\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)-\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}\right)R^{\frac{1-\gamma}{\gamma}}\widetilde{R}^{\frac{\gamma}{1-\gamma}}}{\left[R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\right]^2}\\
&= \frac{R^{\frac{1-\gamma}{\gamma}}\widetilde{R}^{\frac{\gamma}{1-\gamma}}\left[\frac{1}{1-\tau}\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}-\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}+\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}\right]}{\left[R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\right]^2}\\
&= \frac{R^{\frac{1-\gamma}{\gamma}}\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}\left(\frac{1}{1-\tau}-1\right)}{\left[R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\right]^2}>0.
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \widehat{\underline{\pi}}^{**}}{\partial e} &= \frac{\gamma \frac{1}{1-\gamma}\frac{1}{R}e^{-\gamma-1}R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)-\gamma\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}\right)R^{\frac{1-\gamma}{\gamma}}\frac{1}{1-\gamma}\frac{1}{R}e^{-\gamma-1}}{\left[R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\right]^{2}} \\
&= \frac{\gamma \frac{1}{1-\gamma}\frac{1}{R}e^{-\gamma-1}R^{\frac{1-\gamma}{\gamma}}\left[\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}-\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}+\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\frac{1}{1-\tau}\right]}{\left[R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\right]^{2}} \\
&= \frac{\gamma \frac{1}{1-\gamma}\frac{1}{R}e^{-\gamma-1}R^{\frac{1-\gamma}{\gamma}}\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\left(\frac{1}{1-\tau}-1\right)}{\left[R^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{1-\gamma}\frac{1}{e^{\gamma}}\frac{1}{R}-\delta\widetilde{R}^{\frac{\gamma}{1-\gamma}}\right)\right]^{2}} > 0
\end{aligned}$$

■

Proof of Proposition 10. The proof is similar to that of Proposition 8. Differentiating the objective function with respect to $\widehat{\pi}_s$ and rearranging gives:

$$\frac{\partial \widehat{W}_{BA}\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s} = \frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]+\delta e R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[(1-\pi)-\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\right].$$

Deposits are completely frozen at π if $\frac{\partial \widehat{W}_{BA}(\widehat{\pi}_s)}{\partial \widehat{\pi}_s}\mid_{\widehat{\pi}_s=\pi} \leq 0$, which depends on the following condition:

$$\begin{aligned}
\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]+\delta e R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[(1-\pi)-\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\right] &\leq 0 \\
\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right] &\leq \delta e R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]
\end{aligned}$$

$$\begin{aligned}
\delta &\geq \frac{\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]}{eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]} \\
\delta &\geq \frac{1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}}{(1-\gamma)\,e^{\gamma}R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{\gamma}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]} \equiv \overline{\delta}.
\end{aligned}$$

■

Proof of Proposition 11. This follows from the discussion in the text.

■

Proof of Proposition 12. The proposition demonstrates that if the initial value of the weight on the monopoly bank's welfare is moderate, the effective weight would become sufficiently large as extra withdrawals are allowed. Thus, the BA would impose a suspension before the bank exhausts its profits. To examine this situation, we first look at the first derivative of the objective function which is given by:

$$\widehat{W}_{BA}\left(\widehat{\pi}_s\right) \equiv \max_{\{\widehat{\pi}_s\}} \widehat{\pi}_s \frac{\left(\widehat{c}_1^*\right)^{1-\gamma}}{1-\gamma} + \left[\left(1-\widehat{\pi}_s\right)\left(1-\pi\right)\frac{\left(\widehat{c}_2^*\right)^{1-\gamma}}{1-\gamma} + \delta\left(\widehat{\pi}_s\right)\psi\left(\widehat{\pi}_s\right)\right] I_{\psi>0},$$

Differentiating with respect to $\widehat{\pi}_s$:

$$\frac{\partial \widehat{W}_{BA}\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s} = \frac{\left(\widehat{c}_1^*\right)^{1-\gamma}}{1-\gamma} - \left(1-\pi\right)\frac{\left(\widehat{c}_2^*\right)^{1-\gamma}}{1-\gamma} + \frac{\partial \delta\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s}\psi\left(\widehat{\pi}_s\right) + \delta\left(\widehat{\pi}_s\right)\frac{\partial \psi\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s}$$

Note that $\widehat{c}_1^* = \left[\frac{e^{1-\gamma}}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} = e\widetilde{R}^{\frac{1}{1-\gamma}}$, $\widehat{c}_2^* = \left[\frac{e^{1-\gamma}R^{\frac{1-\gamma}{\gamma}}}{\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{1-\gamma}} = eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}$, and $\psi\left(\widehat{\pi}_s\right) = \frac{\left(e-\tau\widehat{x}^*-\widehat{\pi}_s\widehat{c}_1^*\right)R}{1-\tau} - \left(1-\pi\right)\left(1-\widehat{\pi}_s\right)\widehat{c}_2^*$ as $\pi \leq \widehat{\pi}_s < \widehat{\pi}_{ZP}$. Upon substitution, we have:

$$\begin{aligned}
\frac{\partial \widehat{W}_{BA}\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s} &= \frac{1}{1-\gamma}\left(e\widetilde{R}^{\frac{1}{1-\gamma}}\right)^{1-\gamma} - \frac{1}{1-\gamma}\left(1-\pi\right)\left(eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\right)^{1-\gamma} \\
&\quad + \frac{\partial \delta\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s}\left\{\left[e-\tau\left(e-\pi e\widetilde{R}^{\frac{1}{1-\gamma}}\right)-\widehat{\pi}_s e\widetilde{R}^{\frac{1}{1-\gamma}}\right]\frac{R}{1-\tau} - \left(1-\pi\right)\left(1-\widehat{\pi}_s\right)eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\right\} \\
&\quad + \delta\left(\widehat{\pi}_s\right)\left[-\frac{R}{1-\tau}e\widetilde{R}^{\frac{1}{1-\gamma}} + \left(1-\pi\right)eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\right] \\
&= \frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R} - \frac{1}{1-\gamma}\left(1-\pi\right)e^{1-\gamma}R^{\frac{1-\gamma}{\gamma}}\widetilde{R} \\
&\quad + \frac{\partial \delta\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s}e\left\{\left[1-\tau\left(1-\pi\widetilde{R}^{\frac{1}{1-\gamma}}\right)-\widehat{\pi}_s\widetilde{R}^{\frac{1}{1-\gamma}}\right]\frac{R}{1-\tau} - \left(1-\pi\right)\left(1-\widehat{\pi}_s\right)R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\right\} \\
&\quad + \delta\left(\widehat{\pi}_s\right)\left[\left(1-\pi\right)eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}} - \frac{R}{1-\tau}e\widetilde{R}^{\frac{1}{1-\gamma}}\right] \\
&= \frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-\left(1-\pi\right)R^{\frac{1-\gamma}{\gamma}}\right] \\
&\quad + \frac{\partial \delta\left(\widehat{\pi}_s\right)}{\partial \widehat{\pi}_s}e\left\{\left[1-\tau+\left(\pi\tau-\widehat{\pi}_s\right)\widetilde{R}^{\frac{1}{1-\gamma}}\right]\frac{R}{1-\tau} - \left(1-\pi\right)\left(1-\widehat{\pi}_s\right)R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\right\} \\
&\quad + \delta\left(\widehat{\pi}_s\right)\left\{eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[\left(1-\pi\right) - \frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\right]\right\}
\end{aligned}$$

Note that $\delta\left(\widehat{\pi}_s\right) = \widetilde{\delta}\left[1+\left(\widehat{\pi}_s-\pi\right)^2\right]$, and hence $\frac{\partial \delta(\widehat{\pi}_s)}{\partial \widehat{\pi}_s} = 2\widetilde{\delta}\left(\widehat{\pi}_s-\pi\right)$. We evaluate the first derivative at π to determine the condition where a suspension will not occur once a run is first

identified. The exact derivation is shown under Proposition 10 in which case a deposit freeze is imposed at π if and only if:

$$\widetilde{\delta} < \frac{1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}}{(1-\gamma)\,e^{\gamma}R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{\gamma}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]} \equiv \overline{\delta}.$$

Subsequently, to see if a suspension is imposed before the bank runs out of profits, we need to check if:

$$\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]+\widetilde{\delta}\left[1+\left(\widehat{\pi}_{ZP}-\pi\right)^2\right]\left\{eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[(1-\pi)-\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}\right]\right\}<0$$

$$\begin{aligned}\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right] &< \widetilde{\delta}\left[1+\left(\widehat{\pi}_{ZP}-\pi\right)^2\right]\left\{eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]\right\}\\ \widetilde{\delta} &> \frac{\frac{1}{1-\gamma}e^{1-\gamma}\widetilde{R}\left[1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]}{\left[1+\left(\widehat{\pi}_{ZP}-\pi\right)^2\right]eR^{\frac{1}{\gamma}}\widetilde{R}^{\frac{1}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]}\end{aligned}$$

$$\widetilde{\delta} > \frac{1-(1-\pi)R^{\frac{1-\gamma}{\gamma}}}{\left[1+\left(\widehat{\pi}_{ZP}-\pi\right)^2\right](1-\gamma)\,e^{\gamma}R^{\frac{1}{\gamma}}\widetilde{R}^{\frac{\gamma}{1-\gamma}}\left[\frac{1}{R^{\frac{1-\gamma}{\gamma}}}\frac{1}{1-\tau}-(1-\pi)\right]} = \frac{\overline{\delta}}{1+\left(\widehat{\pi}_{ZP}-\pi\right)^2} \equiv \underline{\delta},$$

where $\widehat{\pi}_{ZP} \equiv \frac{(e-\tau\widehat{x}^*)R-(1-\tau)(1-\pi)\widehat{c}_2^*}{\widehat{c}_1^*R-(1-\tau)(1-\pi)\widehat{c}_2^*}$. As $\widehat{\pi}_{ZP} > \pi$, it is trivial to show that $\underline{\delta} < \overline{\delta}$.

■

Proof of Proposition 13. The proof follows from the discussion in the text.

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