International Capital Flows, Liquidity Risk, and Monetary Policy

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Abstract

For developing countries, public sector lending to other countries has been higher than any other sector, while the corporate sector in advanced countries has issued the largest share of portfolio debt as a percentage of GDP to other countries in recent years. What are the implications of these flows for the global financial system and overall economic activity? Do they have an impact on the transmission channels of monetary policy across the different stages of development? We investigate these important issues using a micro-founded model of money and banking based on the level of financial development so that the effects of monetary policy across countries can be meaningfully studied. Notably, at high global interest rates, the capital drain away from the poor country adversely affects domestic investment. Moreover, the increase in capital inflows to the advanced economy renders monetary policy in advanced countries to be less effective. Yet, our results indicate that it is generally optimal for lower income countries to lend to the advanced world. Nevertheless, we do find that the optimal amount of lending to advanced countries critically depends on the level of financial development – if it is sufficiently weak, then public sector lending to advanced economies is not warranted. Consequently, our results indicate that governments in developing countries should carefully consider how much capital they send abroad to foreign countries.

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1 Introduction

Within the last three decades, there have been large swings in the flow of capital across countries. However, while net capital flows have garnered significant attention, much less attention

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has been devoted to the role of gross capital flows. Indeed, the implications of these different flows by foreign and domestic investors may be quite different (Forbes and Warnock, 2012; Broner et al., 2013; Avdjiev et al., 2018; Caballero and Simsek, 2020; Davis et al., 2021, among others). For example, Broner et al. (2013) notes that holding an asset in a foreign country may be necessary to obtain access to technology, hedge against income earnings or even to hedge against the possibility of default. Consequently, the effects of different types of capital flows (debt, equity, FDI) may vary significantly as well. Notably, debt flows have constituted the lion's share of aggregate capital flows over the last few decades.[1]

In addition to the types of assets traded, their sources have received even less attention. In particular, many of these flows have been "uphill" – from developing countries to advanced countries. Using data on the United States as an example, developing countries held as much as $3.2 trillion in debt securities issued by the U.S. in 2010. Moreover, in five years, holdings increased to approximately $3.8 trillion.[2]

More importantly, a breakdown of these debt flows by sector deserves some attention. In fact, Avdjiev et al. (2018) highlight that there are stark differences between the amount of private and public sector borrowing and lending by domestic and foreign individuals. As they show, in emerging markets, the public sector is the main source of funding to other countries. In fact, the public sector has steadily been increasing its holdings of external portfolio debt. By comparison, in advanced countries, flows into the corporate sector's external portfolio debt liabilities were the highest of any other sector pre-Great Financial Crisis, and have rebounded after the global financial crisis and the 2013 Taper Tantrum episodes.

Interestingly, as shown in Figure 1, among the sectors issuing portfolio debt from the United States to developing countries, liabilities issued by the corporate sector grew faster than any other sector, particularly at the turn of the 21st century and after the Taper Tantrum episodes.[3] Further, Bernanke (2005) posits that holdings by the public sector have been funded by issuing debt domestically. Indeed, Bernanke (2005) suggests that such governments were not just fiscal authorities – instead, they took on an additional role as financial intermediaries between their own citizens and participants in international capital markets from other countries.

With these observations, there are a number of important questions that deserve serious consideration. Firstly, what are the implications of "uphill" public sector flows for economic activity in developing countries, particularly when funds are raised by public sector domestic debt to hold portfolio assets in the advanced world? And what about the implications for

[1] See Avdjiev et al. (2018) for a more detailed discussion.

[2] Authors' calculations from total U.S. securities held by foreign countries as gathered from the U.S. Department of Treasury. The criterion used to classify countries was that used by the World Bank.

[3] Warnock and Warnock (2009) document that foreign official holdings of U.S. Treasury bonds increased significantly at the turn of the century and led to lower yields. Moreover, they argue that holdings of all types of U.S. bonds held by foreigners increased – Treasury, corporate, and agency debt securities. In addition, Craine and Martin (2009) provide evidence that foreign official holdings of Treasuries with maturities longer than one year increased significantly around the same time frame. Csonto and Mora (2017) document that higher foreign official holdings of U.S. Treasury securities reduce medium-term (2-3 year) yields. See also the discussion in Bernanke et al. (2011) who point out that foreign official holdings of mortgage-backed securities also increased. Finally, Goda et al. (2013) extend previous work to look at foreign holdings of U.S. corporate and municipal bonds.

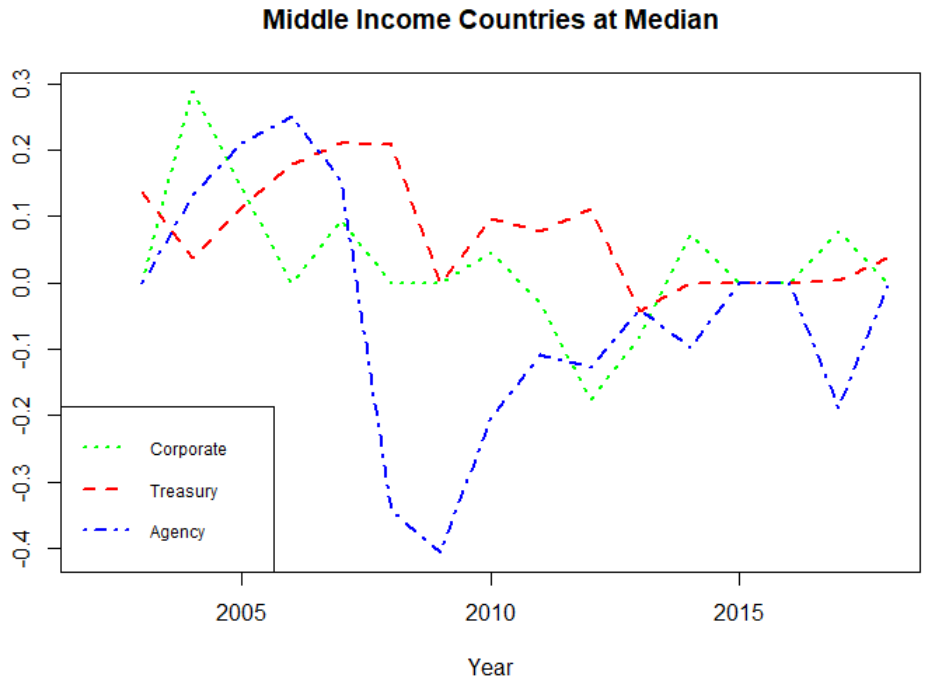


Figure 1: Median growth rate of foreign holdings of US long-term debt securities by middle income countries. The growth rate is calculated by taking the first difference of the values in log-levels.

global economic activity? Secondly, how do these "uphill" portfolio asset holdings impact the transmission channels of monetary policy in developing and advanced countries? In particular, do they render monetary policy more or less effective? Also, what does this mean for the spillover effects of monetary policy? That is, how does monetary policy in advanced countries impact economic activity in the developing world? Finally, is it optimal for developing countries to fund foreign portfolio holdings by issuing debt domestically? What are the implications for the effects of monetary policy in lower income countries?

In this paper, we develop a monetary general equilibrium model where the public sector of a developing economy holds portfolio debt from the corporate sector of the developed world to study these important issues. In so doing, we are able to look at the questions raised above but also to study optimal international lending (borrowing) policy and monetary policy in developing (advanced) countries. Notably, the two countries in our model differ fundamentally in the size of the capital stock and the level of financial development.

Since we are particularly interested in understanding how "uphill" portfolio debt holdings impact the transmission channels of monetary policy across countries, it is necessary that our model includes important *information frictions* that endogenously generate a transactions role for money in each country. These information frictions impede investment activity, which allow us to establish microeconomic foundations for money in each country as a medium of exchange so that the effects of monetary policy can be meaningfully studied.

Further, it is understood that monetary policy works by affecting activity in the banking system – thus, the model should also include the role of the banking system in the process of

mobilizing savings and capital flows across countries which is missing in existing work such as Aoki et al. (2009). To do so, we build on the seminal work of Diamond and Dybvig (1983) which demonstrates that financial intermediaries (i.e., banks) perform important risk pooling functions for individuals exposed to idiosyncratic liquidity risk.[4] In a setting where money is dominated in rate of return by interest-bearing assets, greater exposure to liquidity risk impedes banks' ability to allocate resources to assets with the highest returns. This is a key feature of the banking system, and thus serves as a proxy for the level of financial development in both countries.

What else is needed for a rigorous model of capital flows across countries? We view that it is important to include sovereign debt. As previously mentioned, Bernanke (2005) argues that domestically issued sovereign debt by developing countries is important in understanding public sector flows to advanced economies. Thus, we construct a model in which a government of a low income country mobilizes the savings of its citizens to lend in international capital markets. Finally, we include a corporate sector and a financial system in the capital-rich high income country. The corporate sector is represented by entrepreneurs whose investment projects are so large that they desire to obtain more funding than they can acquire from their local banks. Thus, this creates demand for funding from abroad.

How do outbound flows of savings affect activity in a small open developing country? Our work in Section 3 suggests that the impact of sending more funds abroad depends on the level of the world real interest rate. At a relatively high world interest rate, high returns on external debt holdings enable the public sector to borrow more funds domestically – such activity will crowd out domestic capital formation. Thus, in this setting, pursuing investment returns abroad comes at the cost of lower economic activity domestically. By comparison, lending more abroad at low world interest rates would increase real activity. Finally, consistent with the available evidence on the effects of money growth in developing countries, we find that monetary policy crowds out capital accumulation and lowers real economic activity. This takes place in our framework because the increase in seigniorage revenue allows the government to borrow more domestically which crowds out capital formation.

What do we learn about the role of foreign flows in developed economies? To address such issues, we proceed in Section 4 to look at activity in a small open developed economy with a corporate sector that issues bonds to the rest of the world in order to obtain funding. Global market conditions that lower the world interest rate will raise demand for external funding by entrepreneurs. In turn, the increase in funding from abroad will allow more domestic funds to be devoted to capital accumulation. Thus, real economic activity in the developed nation will be higher if funding from abroad is available at a lower interest rate.

How does monetary policy in the advanced country affect worldwide interest rates? To

[4] Ghossoub and Reed (2010) discuss how individuals in developing countries have greater exposure to liquidity risk – this interrupts their savings plans and forces people to hold more liquid assets. Consequently, there is a greater reliance on cash in poor countries than high income economies. However, they look at banking system activity in a closed economy without sovereign debt or a separate corporate sector. Hence, there is no room for financial flows across countries at different stages of development as in our analysis or different sectors of the economy.

address this important question, in Section 5 we look at two large economies: one where the advanced country issues portfolio debt to the public sector of the developing country. If the money growth rate in the advanced country is higher, there is less demand by entrepreneurs for funding from the low income country such that gross inflows fall. In turn, the interest rate on government bonds issued by the low income country to domestic banks will also be lower and will contribute to an increase in economic activity across both countries. Thus, in contrast to Banerjee et al. 2016; Miranda-Agrippino et al. 2020, among others, we argue that the spillover effects from monetary expansion in the advanced country operate through an *interest rate channel with government budget constraint effects*. That is, monetary policy in the advanced country primarily affects government revenues in the developing country by affecting rates on debt issued in the advanced country and thereby the incentives of capital to flow "uphill".

Conversely, if the central bank in the advanced country lowers the money growth rate, the availability of funding from abroad would mitigate the effects of monetary tightening. In this manner, our framework serves as an example of how the effects of monetary policy can be weakened by capital flows. This is an important contribution of our work – we provide a rigorous model that explains how gross capital inflows may impact the transmission channels of monetary policy in advanced countries.

Is it optimal for governments in developing countries to fund foreign portfolio holdings by issuing debt domestically? Our analysis indicates that the advanced country benefits from high capital inflows as it supports investment activity in the corporate sector. By comparison, there is an intermediate level of capital outflows that maximizes welfare in the developing country. Moreover, we find that the optimal amount of public sector lending to advanced countries critically depends on the level of financial development – if the level of development is weak, raising funds in this way to hold portfolio assets in the advanced country is less likely to be warranted. *Yet, our results do indicate that it can indeed be optimal for lower income countries to lend to the advanced world – lower income countries tend to have less investment opportunities than advanced countries so lending to the advanced world can help increase returns earned by depositors in the banking system.*

As a final analysis, we test the predictions of our model empirically. In particular, for a small open developing economy, the relationship between the cost of borrowing and the level of official "uphill" flows to the corporate sector depends on global interest rates. However, for large open developing economies, the relationship depends on how much funds are being lent initially. Moreover, for small open economies, the effects of monetary policy on the domestic interest rates would be weaker as capital outflows increase. By contrast, the efficacy of monetary policy is weaker in large open developing economies only at low levels of public flows to the advanced world. Once the level of capital outflows is relatively large, then the efficacy of monetary policy becomes stronger. We find strong evidence in support of these predictions using cross-country data on developing countries that hold corporate securities issued by the United States between 2002 and 2018.

At this point, we would like to contrast our contribution relative to other recent research in

the literature on international capital flows. This is presented in the following section. Finally, the Appendix provides a summary of the derivations of key results in our manuscript.

2 Related Literature

While many papers on international capital flows such as Aoki et al. (2009) continue to emphasize how inbound financial flows to developing countries affect economic activity in such countries, an emerging literature studies the role of the developing world as a source of funding for advanced nations as we do. For example, Mendoza et al. (2007) look at financial flows from developing to advanced economies, but they are primarily interested in private flows rather than flows of official savings as in our framework.[5] From this perspective, they do not consider that governments can act as financial intermediaries as Bernanke (2005) emphasizes. By comparison, we view that private citizens in developing countries may not have the same degree of access to international capital markets as these papers portray.[6]

In comparison to other recent work on the role of capital flows across countries at different stages of economic and financial development, Jeanne (2012) constructs a model of a small open economy in which there are capital controls in place. By inducing forced savings in the private sector in terms of holdings of domestically issued sovereign debt, the government can cause the country's real exchange rate to fall. However, Jeanne (2012) does not consider monetary exchange so there is no attention devoted to monetary policy and how its effects depend on a country's flow of capital which are central features of our model. The closest paper to our work in this regard is Bacchetta et al. (2013), who model a "semi-open economy" where the central bank has access to international capital markets, but the private sector does not. They look at the optimal policy and welfare implications of the developing country under this framework. However, they do not speak to how public sector flows of capital can impact the transmission channels of monetary policy in the advanced country.

Further, Bacchetta and Benhima (2015) study an overlapping generations economy with credit-constrained firms in developing countries. In contrast to our work, they focus on the impact of corporate savings coming from lower income countries instead of corporate borrowings from the advanced world. Moreover, they do not study monetary economies so they cannot address how capital flows can impact the transmission channels of monetary policy across countries.

As it relates to monetary policy, there is a large literature that looks at the spillover effects of monetary policy in an advanced country on a developing country. In particular, monetary policy in advanced countries have been shown to spillover into developing countries via the exchange rate channel (Bruno and Shin, 2015b; Banerjee et al., 2016; Rey, 2016; Akinci and Queralto, 2019; Miranda-Agrippino and Rey, 2020), balance sheet channel (Dedola et al., 2013; Banerjee et al., 2016; Morelli et al., 2021), and risk-taking channel (Borio and Zhu, 2012;

[5] Caballero et al. (2008) serves as another example in this regard.

[6] There is a large literature that provides justification for private agents' limited access to international capital markets (see, for example, Bianchi and Mendoza, 2011; Bianchi, 2011; Korinek, 2018; Jeanne and Korinek, 2019).

Miranda-Agrippino and Rey, 2012; Bruno and Shin, 2015a; Gertler and Karadi, 2015).[7] In contrast to these works, we find that monetary policy from the advanced country can affect the developing country through the interest rate channel which results in government budget constraint effects that filter through the developing economy. More importantly, we show how public sector portfolio flows from developing to advanced countries can render monetary policy in advanced countries less effective in a rigorous model of money and banking, which is currently not explored in the literature.[8]

Our paper is also closely related to a number of contributions in recent years which have argued that the range of options available to central banks is limited when interest rates are at the zero lower bound (see, for example, Ireland 2005; Devereux et al. 2011; Kocherlakota 2015; Acharya and Dogra 2018). Ireland (2005) demonstrates that a real balance effect can emerge from a change in monetary policy in a model with population growth. In a similar manner, our model produces a real balance effect because seigniorage taxes are not borne by all of the demographic groups in an overlapping generations setup – seigniorage revenues are redistributed to all of the young, but the costs of the inflation tax only fall upon a segment of the old.

Our work is also connected to the literature on public debt as private liquidity as in Woodford (1990).[9] In that framework, the introduction of government debt can raise returns for agents that are currently saving which allows them to transfer more income to periods where they are borrowers. Consequently, public debt can help relax private liquidity constraints. We also demonstrate that introducing domestically issued sovereign debt can deliver higher expected returns for savers. However, in our setup, there are three important differences. First, higher returns occur because the government lends to borrowers in the corporate sector of the more advanced country – that is, public savings is used for productive public investment. Moreover, government debt is financed by seigniorage rather than lump-sum taxes in our model. In addition, our analysis is conducted in an open economy setting.

3 A Small Open Low Income Country

In this section, we concentrate on a small open low income economy that extends capital outflows at a fixed world real interest rate. High and low income countries differ in the level of capital stock and the level of financial development. Also, the low income country has greater dependence on seigniorage and domestic bond issuance as sources of revenue.

[7] See Rey (2016) for a more detailed discussion.

[8] Bernanke et al. (2004), Warnock and Warnock (2009), and Beltran et al. (2010) through empirical work have shown that official inflows of capital into the US can reduce yields. However, what such flows mean for the effectiveness of monetary policy has not been explored theoretically in a rigorous model of money and banking, to the best of our knowledge.

[9] See also Farhi and Tirole (2011).

3.1 Environment

We consider a discrete-time economy consisting of an infinite sequence of two-period lived overlapping generations, with an initial old generation. Each generation is identical in size and contains a continuum of agents with unit mass. Time is indexed by $t = 0, 1, ...\infty$.

Further, the low income country is separated into two geographically separated islands. Even though the population resides in two separate locations, there is a single consumption good that is available on both islands.

In the low income country, there are four groups of agents: workers (depositors), firms, financial intermediaries (banks), and a government which also acts as the hybrid fiscal-monetary authority. Each worker is endowed with one unit of labor which they supply inelastically when young and is retired when old.

A single final good is produced using a Cobb-Douglas production function $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$ with capital (K_t) and labor (L_t) as inputs, and A as the level of productivity. Capital is rented by firms who also hire workers. Due to constant returns to scale, $y_t = Ak_t^{\alpha}$, where $k_t \equiv \frac{K_t}{L_t}$. Obviously, the production technology satisfies standard Inada conditions. Workers care only about old-age consumption. Let c_t be consumption per person at time t. Then, the lifetime utility of workers is given by

$$U(c_t) = ln(c_t).$$

In the absence of financial intermediation, agents do not have access to financial markets. For instance, one can assume that high transactions costs prevent agents from investing in credit and money markets (see Ghossoub et al. 2012). Moreover, financial intermediaries arise endogenously in this environment to insure workers against idiosyncratic risk. Thus, all income earned when young is saved and stored in the bank as long as they receive a higher expected utility than in the absence of financial intermediation.

There are four assets available in the low income country. First, there is a homogenous consumption investment good. Second, agents have access to the locally-issued fiat currency. We denote the per worker monetary base as m_t. In addition, the local government issues bonds that are held by domestic banks. The fourth asset is the amount of bonds issued by the rest of the world. As the country is a low income economy, it takes the demand for funds from abroad, $\hat{b}_t^d$, as given.

We now describe the timing of activity at each date. At the beginning of time, the generation of old agents at each location is endowed with the aggregate money and capital stocks. Since the total population equals one, these values also represent the aggregate values. Furthermore, one unit of investment by a young agent at time t becomes one unit of capital at time $t + 1$.

Next, at the beginning of each period, trade takes place in factor and asset markets. At this stage, workers get paid and deposit all of their income in the bank. After workers have deposited their funds, each young agent faces a probability π of being relocated to the other island.[10] This idiosyncratic shock implies that at the end of each period a fraction of the young

[10]Private information serves as the primary trade friction in the economy. Although each island is character-

population is relocated to the other island. Therefore, there is no aggregate uncertainty in the economy.

As in Townsend (1987), there are *information frictions* – private information and limited communication across the two different islands – generate a transactions role for money. As individuals cannot communicate information about their asset holdings on the other island, privately issued liabilities by depositors will not circulate. That is, individuals cannot write contracts as a means of payment. Such transactions must be quid pro quo. Further, capital cannot be transported across islands. Thus, following Schreft and Smith (1997, 1998), relocated agents must carry fiat currency to the other island to purchase consumption. Moreover, currency cannot be counterfeited, which makes it acceptable in both regions. In this manner, the effects of monetary policy in each country can be meaningfully studied because the transactions role for money is endogenous in the model. Consequently, π also serves as an index of frictions in the financial sector in the country which are relatively high compared to advanced economies.

Capital controls prevent banks from accessing international capital markets.[11] The government, on the other hand, has access to the global bond market. Further, as the low income country is a small open economy, it takes the world interest rate (r^*) as given.

We next turn to the government budget constraint for the low income country. As explained by Bernanke (2005), the government obtains reserves by issuing debt domestically (b_t^s) and using the funds to acquire bonds from the global bond market. To do so, the government of the low income country has to repay previously issued debt (b_{t-1}^s) with interest, (R_{t-1} being the interest rate at which it has to repay local holders of its debt), as well as issue funds to the rest of the world at the world interest rate r^*. The government budget constraint is satisfied when:[12]

$$\hat{b}_t^d + R_{t-1} b_{t-1}^s = \frac{M_t - M_{t-1}}{P_t} + r^* \hat{b}_{t-1}^d + b_t^s \tag{1}$$

The first two terms on the left-hand side represent expenses for the government - lending funds to the rest of the world and payments on previously issued domestic debt.

By comparison, everything on the right-hand side is a form of revenue for the low income

ized by complete information, communication across islands is not possible. Consequently, private liabilities do not circulate. The friction of the "relocation shock" is just a metaphor to represent different information sets across individuals which lead to the micro-founded medium exchange role for money in our framework.

[11] Alternatively, one may view the lack of foreign asset holdings as a consequence of home bias on the part of private investors. In particular, Lau et al. (2010) find that home bias tends to be a lot higher in developing countries than in advanced countries.

[12] Governments in developing economies can experience difficulties raising tax revenue because of the role of informal markets and the size of the underground economy. As such, governments in developing countries tend to rely more on seigniorage than advanced countries. For example, Enste and Schneider (2002) estimate that the size of the shadow economy is largest in developing countries. In particular, they estimate the range as a percentage of GDP lies between 35 and 44%. Furthermore, based on their numbers, the average size of the shadow economy among the top 10 holders of U.S. long-term corporate securities that are developing economies may be as high as 48%. As a result of the size of the shadow economy, it can be difficult to raise revenue through standard forms of taxation. Hence, we want to draw attention to the role of seigniorage for government revenue in developing economies and build a model consistent with an important role for monetary policy in the developing world. Thus, we choose to omit taxes in our analysis.

government. Thus, the first term shows seigniorage revenue for the government.[13] The second term presents the returns for the developing country from lending abroad. Finally, the last term lists the revenue of the government from issuing new debt. In this manner, we show how the low income government also takes on the role of a financial intermediary.

Additionally, the government follows a fixed money growth rule. The aggregate nominal stock of cash in period $t+1$ can be expressed as:

$$M_{t+1} = \sigma M_t \tag{2}$$

where σ is the gross rate of money creation and is exogenously given. In particular, the government implements an inflation targeting regime such that the constant money growth rule pins down the long-run inflation target, $\sigma = \frac{P_{t+1}}{P_t}$. We assume that government bonds cannot be liquidated before maturity (for example, a secondary market for bonds does not exist (see, for example, Ghossoub and Reed, 2017).

3.2 Trade (Low Income Country)

3.2.1 Trade in Factor Markets

Due to perfect competition, factor inputs are paid their marginal products. The rental rate and wage rates in period t are:

$$r_t = \alpha A k_t^{\alpha-1} \tag{3}$$

$$w_t = (1-\alpha) A k_t^{\alpha} \tag{4}$$

3.2.2 The Bank's Problem in the Low Income Country

The banking sector is also perfectly competitive. Thus, banks exhaust all income in trying to attract deposits. Consequently, banks choose portfolios to maximize the expected utility of each depositor. Since banks provide insurance against the liquidity shocks, each young depositor at time t will put all of his income in the bank rather than holding assets directly.[14] The bank uses the funds to acquire a real amount of money balances, m_t, an amount of domestically issued sovereign debt, b^d, and investment in capital, i_t. The bank's balance sheet constraint is

[13] As we have emphasized, we want to draw attention to the role of seigniorage in developing economies. Interestingly, Aisen and Veiga (2008) report that during the 1990s, seigniorage was responsible for 14.65% of government revenue in their sample of developing countries. By comparison, the number was much lower at 1.64% for advanced countries. Notably, the average fraction was around 11.6% among the top 10 holders of long-term U.S. corporate securities that are developing economies. In stark contrast, seigniorage revenue contributes to just 2% of government revenue in the United States.

[14] Individuals in our model only derive utility from consumption in the second period of their lives. In addition, as the banking system is perfectly competitive, banks do not earn any profits. As a result, the bank's problem amounts to maximizing welfare across depositors. That is, the problem is equivalent to maximizing a given depositor's expected utility. Because the bank has the ability to pool risks across a large number of depositors, participation in the banking system provides depositors with higher expected utility than they could achieve on an individual basis. In sum, the bank's problem in each country is to maximize the household problem at the aggregate (pool of households) level.

expressed by:

$$m_t + b_t^d + i_t \leq w_t. \tag{5}$$

Depending on the bank's money holdings and the inflation rate, the return to relocated agents satisfies:[15]

$$\pi r_t^m w_t \leq m_t \left(\frac{P_t}{P_{t+1}} \right) \tag{6}$$

In particular, the bank needs to hold enough cash reserves to meet the anticipated demand for liquidity by movers in time $t+1$. As money is dominated in rate of return, banks will not bring real money balances between periods. In addition, the return paid to agents who are not relocated must satisfy:

$$(1-\pi) r_t^n w_t \leq R_t b_t^d + r_{t+1} i_t \tag{7}$$

In particular, the bank's total payments to non-movers are paid out of its returns on government bonds and capital, respectively. Therefore, a bank will choose values of r_t^m, r_t^n, m_t, b_t^d, and i_t in order to solve the following problem:

$$\max_{r_t^m, r_t^n, m_t, b_t^d, i_t} \pi ln(r_t^m w_t) + (1-\pi) ln\left(r_t^n w_t\right) \tag{8}$$

subject to (5), (6), and (7).

The bank's optimal choice of the reserve to deposit ratio yields the following result for its money holdings in equilibrium:

$$m_t = \pi w_t \tag{9}$$

Since all primary asset holdings are intermediated in equilibrium, the money market clears when the demand by banks for cash reserves is equal to the supply of money at each date.[16] Similarly, the period $t+1$ per capita capital stock, k_{t+1}, must equal i_t.

$$k_{t+1} = w_t - m_t - b_t \tag{10}$$

Additionally, an absence of arbitrage opportunities requires that the interest rate on bonds issued by the government of the low income country to its domestic banks is equal to the rate of return on capital investments domestically. Specifically,

$$R_t = r_{t+1}. \tag{11}$$

[15] This is where the cash-in-advance constraint is imposed in the small, open economy banking system. The left-hand side of equation (6) lists total payments to all short-term depositors (movers) on the basis of their deposits which are equal to wages. The right-hand side shows how banks provide such returns — it depends on their holdings of total money balances. These money holdings are distributed across all movers and expressed in period $t+1$ prices to account for the fact that inflation lowers the return to holding money. This takes place because consumption occurs after relocation in the following period. Again, this reflects the cash-in-advance constraint — relocated agents transact with individuals who cannot verify their asset holdings. Hence, money is required to finance consumption in such transactions.

[16] To see the role of the liquidity shock, first note that in (9) that a bank's holdings of money balances are directly proportional to the probability of the liquidity shock. As previously emphasized, this occurs because relocated agents must use money to finance consumption.

3.3 General Equilibrium

For the low income country, equations (3), (4), (2), (5), (9), (11), and (1) characterize the full set of equilibrium conditions.

3.3.1 Steady State Analysis

Equilibrium in the banking sector of the low income country is characterized by a set of non-negative quantities, (m, b^d, k) and prices $(\frac{P_t}{P_{t+1}}, R, r)$ that clear the money market, the domestic bond market, and the domestic capital market respectively. The money market clears when:

$$m = \pi w(k) \tag{12}$$

Also, imposing steady state on the evolution of money, the rate of money growth is equal to the inflation rate:

$$\sigma = \frac{P_{t+1}}{P_t} \tag{13}$$

Substituting the demand for capital derived from (3) into the bank balance sheet constraint (5) and taking note of the no arbitrage condition (11), the demand for bonds is:

$$b^d(R) = (1-\alpha)(1-\pi)A\left[\frac{\alpha A}{R}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{\alpha A}{R}\right]^{\frac{1}{1-\alpha}} \tag{14}$$

Again, the first term is total deposits net of money demand while the second reflects the demand for capital in the low income country. Once funding to capital demand and money demand is satisfied, the remainder of total deposits is used to hold bonds issued by the government of the low income country.

Imposing steady-state on the government budget constraint, (2) while taking advantage of bond market-clearing condition $\left(b^d(R) = b^s\right)$yields:

$$(R-1)b^d(R) = \frac{(\sigma-1)m(R)}{\sigma} + (r^*-1)\hat{b}^d \tag{15}$$

where the demand for real money balances depend on the real interest rate by virtue of its dependence on the demand for capital.

Recall that only the government has access to global bond markets. Therefore, it has monopoly power over the supply of bonds to domestic banks. Consequently, it can manipulate the domestic interest rate by setting bond supply (see, for example, Bacchetta et al. 2013). For example, the government can put upward pressure on the domestic interest rate by increasing its supply of bonds to local banks. As the debt obligations to local banks increase, the need to raise revenue becomes greater in order to satisfy the budget constraint. This will of course have implications for the money market and economic activity.

It is clear that the government budget constraint (15) is non-linear in the domestic real interest rate. Consequently, our framework demonstrates that the government's participation

in the global bond market is likely to make the interaction between the domestic bond market, the money market and economic activity much less discernible. Nonetheless, we can obtain analytical solutions for all variables in the special case where the factor market is equally capital and labor-intensive. Thus, the equilibrium interest rate on domestically issued debt is summarized as follows:

Lemma 1 *Suppose that* $\alpha = 1/2$ *and* $\underline{r^*} < r^* < \bar{r}^*$. *Further, consider that* $\pi < \sigma[2-\sigma]$.[17] *Under these conditions, the steady state real interest rate on government debt in the low income country,* $R(r^*, \hat{b}^d, A, \pi, \sigma)$ *is given by*:

$$R^* = R(r^*, \hat{b}^d, A, \pi, \sigma) \tag{16}$$

The upper bound on r^* in Lemma 1 is such that the government does not fund its external asset holdings by solely printing money or using revenue from its previous external asset holdings. In particular, if global rates are too high, then instead of issung domestic debt to raise funds, it would only print money or reinvest revenues from its previous asset holdings. Since it is the only supplier of government bonds, the domestic bond market would be non existent.. On the other hand, the lower bound on r^* in Lemma 1 ensures that the government is able to repay previously issued domestic debt from its returns on external asset holdings. In particular, if global rates are too low, then the revenue from holdings of foreign assets would be so low that the returns are not enough to repay previously issued domestic debt. Thus, there would be no need to raise funds domestically by issuing debt to local banks, rendering the domestic bond market obsolete.

These conditions on r^* in Lemma 1 gives multiple solutions for R. While there are multiple solutions to the quadratic equation, however, there are not multiple equilibria. Notably, if $\pi < \sigma[2-\sigma]$, the lower nominal interest rate would be less than one and hence bonds would be dominated by money.[18] In particular, though the level of financial development is lower than that of advanced countries, it cannot be too low such that banks are not able to allocate scarce resources to assets with the highest returns.[19] Thus, the real interest rate that clears the domestic bond market also depends on the level of financial development.

3.4 Steady-State Equilibrium

In this section, we focus on the *steady-state equilibrium* in the context of the environment where a small open low income economy lends in the international capital market.

Definition 1 *(Steady-state equilibrium).* A *non-trivial, steady-state equilibrium* (SSE) is

[17] As explained in the introduction, one of the central assumptions in our framework is that the level of financial development is lower in developing countries than advanced economies.

[18] See appendix for details.

[19] This, of course, is a key feature of the financial system. See King and Levine (1993a,b), among others.

satisfied under the following conditions:

(G-1) banks maximize the expected utility of each depositor (8);
(G-2) the government of the low income country borrows domestically at the prevailing domestic real interest rate and lends internationally at the prevailing world real interest rate;
(G-3) in the international bond market, there is a unique real interest rate (r^*) to satisfy (G-2);
(G-4) in the bond market of the low income country, there is only one valid real interest rate (16) that clears the market.
(G-5) in both bond markets, the nominal interest rate is greater than 1.

Theorem 1 *(Existence) Suppose that Lemma 1 holds and there exists r^* such that $I^* > 1$. Then there exists a unique steady-state equilibrium interest rate in the international bond market and a unique equilibrium real interest rate for the domestic bond market.*

3.5 Comparative Statics

We now consider how an increase in capital outflows by the government of the low income country will affect domestic economic activity. We obtain the following:

Proposition 1 *(Effect of an increase in capital outflows on the domestic real interest rate and domestic economic activity.) Suppose that a SSE exists and $r^* < 1$.[20] Under this condition, an increase in the steady state capital outflows from the low income country to the rest of the world leads to a decrease in the domestic real interest rate $\left(\frac{\partial R}{\partial \hat{b}^d} < 0\right)$ and an increase in economic activity in the low income country. However, suppose that $r^* > 1$. Instead, under this condition, an increase in the steady state capital outflows leads to an increase in the domestic real interest rate $\left(\frac{\partial R}{\partial \hat{b}^d} > 0\right)$ and a decrease economic activity in the low income country.*[21]

Proposition 1 states that when the global real interest rate is low, capital outflows will reduce the domestic interest rate. At a low global world interest rate, the government does not earn high returns on outflows, so it does not seek to raise as much funds domestically and the domestic real interest rate is lower. When the domestic real interest rate is lower, the rental rate of capital is lower, and capital accumulation increases. As a result, output and wages increase.

[20]Note that r^* can be less than one but the world nominal interest rate I^* still be greater than one. This would hold iff $r^* \times \sigma^* > 1$.

[21]In our model, $\hat{b}^d$ represents the stock of foreign bonds held by the domestic government. However, the experiment is such that $\hat{b}^d$ changes at date 0 and stays at that level forever (a change in the steady state asset holdings of corporate securities in the advanced country by the government in the low income country). In this case, we are studying the long-run effects of a transitory increase in capital flows which is never reversed. So, to assess the effects of an increase in public/official capital outflows, we look at the effects of a higher stock of foreign debt holdings, $\hat{b}^d$.

However, when the world interest rate is high, an increase in capital outflows will lead to an increase in the domestic real interest rate. As the government increases outflows, it will seek to raise more funds domestically to lend abroad. As such, local demand for funding increases, driving up the domestic real interest rate. Higher interest rates domestically crowd out capital formation, reduce wages, and lower output. Thus, the implications of lending abroad for domestic economic activity critically depend on conditions in the global capital market.

This result separates our work from others in the following way. Firstly, the impact of uphill public capital flows on economic activity in a small open economy depends on the level of global interest rates, *ceterus paribus*. Von Hagen and Zhang (2014) argue that full capital mobility can raise output in the poor country as well as globally, despite "uphill" net capital flows. However, this result does not separate the effect of private flows versus public flows. Plus, it does not consider the possibility of restrictions being placed on the cross-border flow of private capital. We show that even with restrictions on private capital flows, public flows can increase output if global rates are low. Further, Alfaro et al. (2014) show that public debt financed by private creditors raises economic activity, but says nothing about how private debt financed by public flows impact the lender economy. Finally, Prasad et al. (2006) state that uphill capital flows may not necessarily lead to output growth. However, their result was not conditioned on global rates as ours is.

We now look at how an increase in the money growth rate will impact the equilibrium real interest rate in the low-income country. We get the following result:

Proposition 2 *(Effect of an increase in the money growth rate in the low income country on the domestic real interest rate and domestic economic activity.) In a SSE, a higher rate of money growth in the low income country leads to an increase in the equilibrium real interest rate on bonds issued domestically by the low income country's government* $\left(\frac{\partial R}{\partial \sigma} > 0\right)$ *and a decrease in domestic economic activity.*

Proposition 2 suggests that an increase in the money growth rate would lead to an increase in the equilibrium real interest rate in the bond market for the low income country. Intuitively, a higher rate of money growth means that the government generates more seigniorage revenue. With more revenue, the government's demand for funds increases, since it has greater revenue with which it can pay off its debt. To continue to support the same amount of capital outflows, the interest rate on local debt will be higher so that the government can raise the same amount of income domestically. Higher interest rates domestically crowd out capital formation, reduce wages, and lowers output. In this manner, monetary policy produces a reverse-Tobin effect in these economies. Though a similar mechanism emerges in the standard Schreft and Smith (1997, 1998) framework, the novelty here is that the government borrows funds domestically *while lending internationally.*

We proceed to examine how the relationship between the domestic real interest rate and the money growth rate depends on the level of capital outflows. In essence, we want to see the result of $\left(\frac{\partial R}{\partial \sigma \partial \tilde{b}^d}\right)$. However, due to the complexity of the exercise, we were unable to proceed

analytically. Hence, we use a numerical example. In particular, consider the parameters studied in Table 1.[22] The impact of monetary policy is illustrated by an increase in the growth rate of money from 6% to 15% in increments of 1%. Our findings from the table show that in the low income country, the effects of monetary policy are weakened when capital outflows are relatively higher. At higher capital outflows, more of the government's revenues derive from lending to the rest of the world – thus, seigniorage plays a smaller role. Consequently, the effects of monetary policy on interest rates would be weaker.

Table 1: The effects of monetary policy in the low income country and capital outflows from its government.

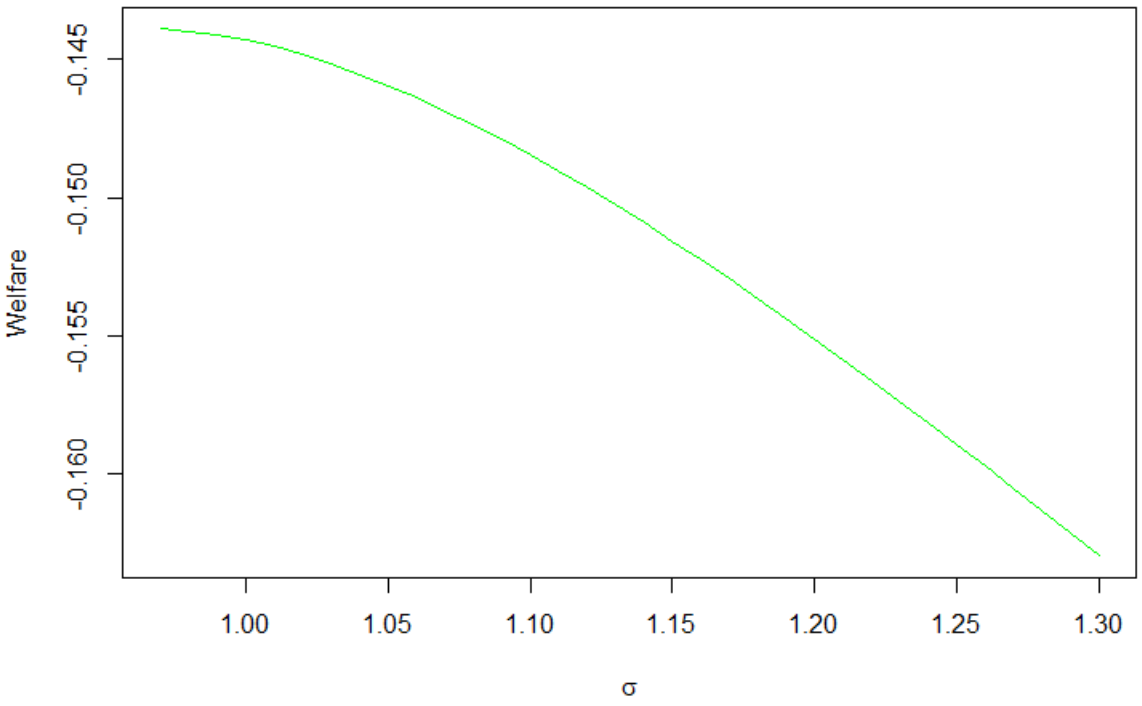
$\hat{b}^d$	0.16	0.19	0.22	0.25	0.28
σ	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$
1.06					
1.07	1.0428	1.0363	1.0300	1.0237	1.0175
1.08	1.0187	1.0125	1.0064	1.0004	0.9945
1.09	0.9958	0.9899	0.9840	0.9783	0.9726
1.10	0.9740	0.9683	0.9627	0.9572	0.9517
1.11	0.9532	0.9477	0.9423	0.9370	0.9318
1.12	0.9333	0.9281	0.9229	0.9178	0.9127
1.13	0.9144	0.9093	0.9043	0.8993	0.8945
1.14	0.8962	0.8913	0.8865	0.8817	0.8770
1.15	0.8788	0.8740	0.8694	0.8648	0.8602

3.6 Optimal monetary policy

All of our previous analysis considers that the money growth rate is exogenous. Now, we study the optimal monetary policy that will maximize total welfare in the small open low income economy. In the low income country, total welfare is defined by (8). Finding the optimal monetary policy is akin to solving the Ramsey problem in which the planner (central bank/government) optimally selects the money growth rate to maximize household's lifetime expected utility as measured by (8), subject to all private first order conditions and budget constraints. Due to the complexity of the exercise, however, we are unable to proceed analytically. As a result, we use numerical examples.

[22]Prior to this point, we have provided conditions in which a SSE exists if $\alpha = 0.5$ – implying that the labor share of income and the capital share of income are essentially the same. As shown in the recent work by Jones et al. (2003), this is close to recent estimates for the capital share across many advanced economies. However, it seems fairly obvious that the labor income shares in the developing world would be much higher than the advanced world. While the equal share case allows us to make many points analytically, we would like to show that a SSE exists for a wider set of parameters than economies where $\alpha = 0.5$. In particular, by using $\alpha = 0.1$, we have expanded the analysis to look at more general parameter spaces so the analytical results are shown to be quite robust. Further, Guerriero (2019) provides evidence that the labor share can be this high in developing economies though the number has trended down in recent years. In addition to using $\alpha = 0.1$, we use $A = 1$, $\pi = 0.5$, and $r^* = 1.048$ as parameter values to solve for the real interest rate from the government budget constraint (15). From there, calculate the percent change in R with respect to a one percent change in the rate of money growth at different levels of public outflows.

Figure 2: The optimal monetary policy in the small open low income country



For a given level of public portfolio debt flows, a higher rate of money growth lowers the returns promised to relocated agents, which reduces insurance against liquidity shocks. By comparison, a higher rate of money growth also increases the real interest rate in the low income country and will increase the returns promised to agents who are not relocated.[23] However, the demand for capital will fall, and wages will decrease, reducing the bank's deposit base. Overall, the welfare losses of relocated agents dominates that of the welfare gains by non-relocated agents as the rate of money growth increases. As a result, the economy's total welfare falls.

The mechanism described above ensures that welfare is strictly decreasing in the rate of money growth. Thus, it would be optimal for the monetary authority to keep the rate of money growth as low as possible. However, the money growth rate cannot be too low, to the point where the real returns to government bonds are less than that of real money balances. Based upon the parameter set we study, the optimal growth policy is -3% as shown in Figure 2.[24]

[23]Because money growth has a reverse-Tobin effect, it raises the return to capital which raises the returns to agents who are not relocated.

[24]Again, to show that a SSE exists across a wide range of parameters, we present results from an alternative set of parameters. For example, we use $A = 1$, $\pi = 0.5$, $\alpha = 0.1$, $\hat{b}^d = 0.19$, and $r^* = 1.112$ as parameter values.

3.7 Optimal capital outflows

As with our previous work, we have also treated capital outflows by the government of the low income country as exogenous. In this section, we seek to study how public portfolio debt flow to the corporate sector of the advanced economy will affect welfare in the low income country. In this case, the Ramsey planner (central bank/government) optimally selects the amount of foreign external asset holdings (for a given money growth rate) to maximize household's lifetime expected utility.

For a fixed rate of money growth, an increase in capital outflows will lead to an increase in the domestic real interest rate. As the government increases capital outflows, it will seek to raise more funds domestically to lend abroad. Since the money growth rate is fixed, the government is limited in its ability to generate additional revenue from seigniorage required to satisfy the budget constraint. As such, local demand for funding increases, driving up the domestic real interest rate. When the domestic interest rate increases, the returns to holding government bonds are now higher such that banks can now promise higher returns to agents who are not relocated. However, as banks devote more resources to holding interest bearing assets, less resources are made available for holding reserves. Therefore, relocated agents are less insured against liquidity shocks, and this lowers their welfare as public capital outflows increase. Overall, the welfare losses of relocated agents is dominated by the welfare gains of non-relocated agents as public holdings of foreign assets increase.[25] Consequently, our analysis at this point indicates that the recent pattern in which developing countries lend to the advanced world is welfare-improving as shown in Figure 3. However, in the small open economy case, the global interest rate is constant and independent of the level of capital outflows from the small country. We turn to address this issue in Section 5.

4 High Income Country

We now move to consider a high income economy. The environment here largely mirrors the structure of the small open low income economy so we will only emphasize the elements of the framework which differ from the previous section. In particular, in contrast to the previous setting, there is an additional group of agents: entrepreneurs who represent the corporate sector of the advanced economy.

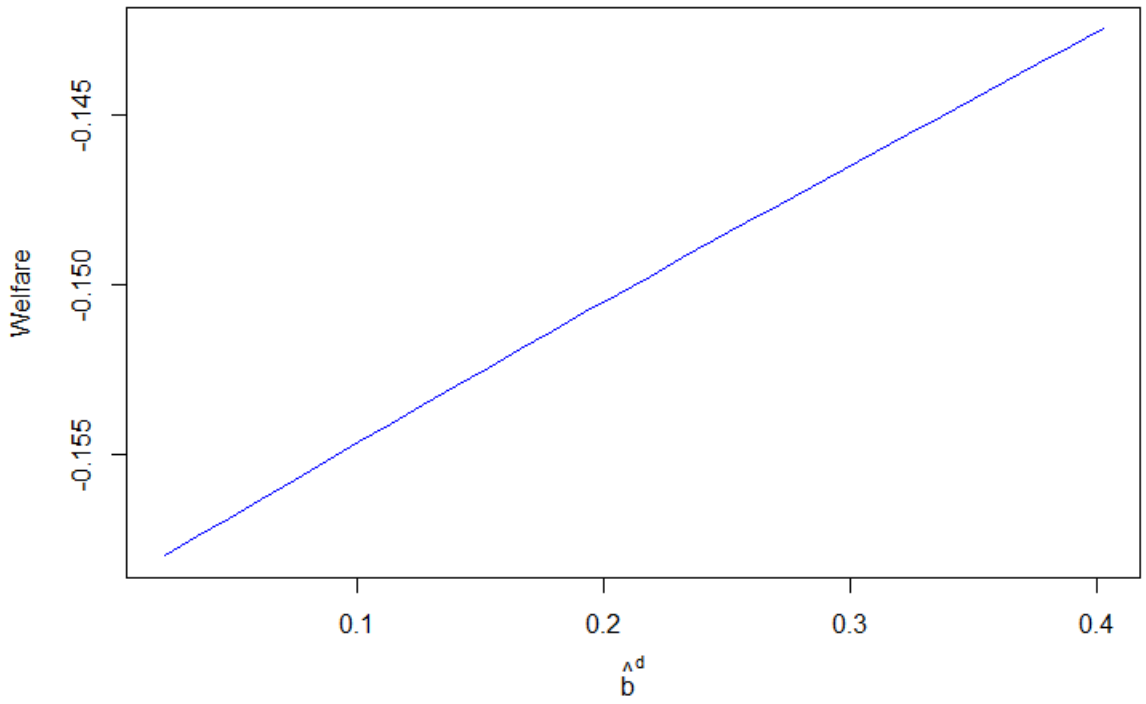
Thus, there are now five different groups of agents: workers (depositors), firms, entrepreneurs, financial intermediaries (banks), and the government. As in the case of the small open economy, workers only value consumption in old-age with preferences given by

$$U(\tilde{c}_t) = ln(\tilde{c}_t).$$

Factor markets consist of firms who produce a single final good using a Cobb-Douglas production function where $\tilde{y}_t = \tilde{A}\tilde{k}_t^{\tilde{\alpha}}$ and $\tilde{k}_t \equiv \frac{\tilde{K}_t}{\tilde{L}_t}$. In addition to wage income, young agents receive a transfer of currency, τ_t, from the government and they deposit all their income in the bank.

[25] For this exercise, we use $A = 1$, $\pi = 0.5$, $\alpha = 0.1$, $\sigma = 1.11$, and $r^* = 1.08$ as parameter values.

Figure 3: The optimal public capital outflows from the small open low income country



As in the case of workers, entrepreneurs work when young, receiving income $\tilde{w}_t$, and a transfer of currency from the government, τ_t. Yet, it is not enough to undertake the investment project they wish to pursue. As such, they need to borrow in the credit market against the future return of their investment project. Therefore, entrepreneurs will seek funding from local banks.[26]

There are three assets in this country: a homogeneous consumption-investment good, locally-issued fiat currency, and bonds issued by entrepreneurs seeking external funding. We denote the per worker monetary base as $\tilde{m}_t$ and bonds issued by entrepreneurs as $\tilde{b}_t^s$. Since the total population equals one, the per capita values of variables also represent the aggregate values. Furthermore, one unit of investment by a young agent at time t becomes one unit of capital at time $t+1$. So, $\tilde{i}_t$ units of goods invested become $\tilde{k}_{t+1}$ units of capital the following period.

At the end of the first period of life, each young worker faces a probability $\tilde{\pi}$ of being relocated to the other island. As in the previous section, banks arise endogenously to provide workers with insurance against liquidity risk. Thus, a bank's portfolio will consist of domestic currency, local bonds issued by entrepreneurs, and capital. In contrast to workers, entrepreneurs are not subject to relocation shocks.

[26] In Huybens and Smith (1999), entrepreneurs would need to obtain funding by announcing loan contract terms which financial intermediaries either accept or reject. For simplicity, we abstain from this and assume that all entrepreneurs who seek funding will receive it from their local bank. When we turn to the open economy case, entrepreneurs will also look to the international capital market to secure remaining funds.

The monetary authority of the high income country also follows a simple money growth rule by setting the gross money growth rate $\tilde{\sigma} > 1$ so that

$$\tilde{M}_{t+1} = \tilde{\sigma}\tilde{M}_t. \tag{17}$$

However, in contrast to the small open low income country, the monetary authority of the high income country does not issue debt. Instead, seigniorage revenues are redistributed to young agents as lump-sum transfers in the amount:[27]

$$\tau_t = \frac{\tilde{M}_t - \tilde{M}_{t-1}}{\tilde{P}_t} = \left(\frac{\tilde{\sigma} - 1}{\tilde{\sigma}}\right)\tilde{m}_t \tag{18}$$

4.1 Benchmark: Closed-Economy Environment

As a benchmark to understanding the effects of capital flows from the developing world to an advanced economy, we will begin by studying activity in the high income country as a closed economy.

4.1.1 Trade

4.1.2 Factor Markets

Analogous to the low income economy, factor inputs also earn their marginal products such that the rental rate and wage rate are

$$\tilde{r}_t = \tilde{\alpha}\tilde{A}\tilde{k}_t^{\tilde{\alpha}-1} \tag{19}$$

and

$$\tilde{w}_t = (1 - \tilde{\alpha})\tilde{A}\tilde{k}_t^{\tilde{\alpha}} \tag{20}$$

respectively.

4.1.3 The Entrepreneur's Problem

Let $\tilde{b}_t^s$ denote the amount borrowed by an entrepreneur (in real terms) in period t. Each potential entrepreneur has one investment project which can only be operated at the scale $q > 1$. By assumption, $q > \tilde{w}_t + \tau_t$. Clearly then, $\tilde{b}_t^s = q - \tilde{w}_t - \tau_t$. Entrepreneurs are the only ones with access to this type of scale-based technology. The productivity level on their project is scaled by a parameter z in which total output is $zlog(q_t)$, and from it they repay the amount borrowed at the prevailing interest rate, $\tilde{R}_t$.

An entrepreneur will seek to choose q to maximize his profits. In particular, a typical

[27] To reiterate, there is not the same level of reliance on seigniorage revenue in advanced countries as in developing countries. In particular, Aisen and Veiga (2008) find that on average, seigniorage was responsible for about 1.6% of government revenue in advanced economies.

entrepreneur solves the following problem:

$$\max_{q_t} zlog(q_t) - \tilde{R}_t \tilde{b}_t^s \tag{21}$$

subject to

$$\tilde{b}_t^s = q_t - \tilde{w}_t - \tau_t \tag{22}$$

and

$$zlog(q_t) \geq \tilde{R}_t \tilde{b}_t^s \tag{23}$$

The second constraint ensures that the costs of borrowing do not exceed the real value of the investment project. Because the scale of production has to be large, entrepreneurs will not be able to finance the first-best investment unless they borrow at a cost that is lower than the value of their project.

An entrepreneur's profit maximization occurs where the marginal income from the investment project is equal to the marginal cost of the project. From here, it follows that the scale of production that maximizes the entrepreneur's profit is equal to the return on the investment project per unit of the return on the amount borrowed. Specifically,

$$q_t = \frac{z}{\tilde{R}_t}. \tag{24}$$

Since q_t must be larger than one in order for positive production to take place, it follows that $z > \tilde{R}_t$. Substituting the level of investment into the expression for borrowing by the entrepreneurs yields:

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - \tilde{w}_t - \tau_t. \tag{25}$$

4.1.4 The Bank's Problem in the High Income Country

The details about the bank's problem in the high income country are analogous to the low income country. For example, since banks provide insurance against liquidity shocks, each young depositor at time t will put all of his income in the bank rather than holding assets directly. In addition, the banking sector in the high income country is also perfectly competitive. The bank uses the funds to acquire a real amount of money balances, $\tilde{m}_t$, a real amount of bond balances issued by entrepreneurs, $\tilde{b}^d$, and makes an investment in capital, $\tilde{i}_t$. The bank balance sheet constraint is:

$$\tilde{w}_t + \tau_t \geq \tilde{m}_t + \tilde{b}_t^d + \tilde{i}_t \tag{26}$$

Since currency is the only asset that can be transported across locations, relocated agents will choose to liquidate their asset holdings into currency. This is accomplished using the real amount of money balances that the bank holds. Also, each unit of real money balances paid to relocated agents at time t has a return $\dfrac{\tilde{P}_t}{\tilde{P}_{t+1}}$ between t and $t+1$. Depending on the bank's

money holdings and the inflation rate, the return to relocated agents satisfies:

$$\tilde{\pi}\tilde{r}_t^m(\tilde{w}_t + \tau_t) \leq \tilde{m}_t \left(\frac{\tilde{P}_t}{\tilde{P}_{t+1}} \right) \tag{27}$$

Those depositors who are not relocated will therefore be paid from the return on the bank's bond holdings of entrepreneurs and capital investments. Bonds held from entrepreneurs earn a real return $\tilde{R}_t$ between periods t and $t+1$. Also, since capital is rented at $t+1$ and then depreciates completely, capital investments have a return of $\tilde{r}_{t+1}$. Thus, the return paid to agents who are not relocated must satisfy:

$$(1 - \tilde{\pi})\tilde{r}_t^n(\tilde{w}_t + \tau_t) \leq \tilde{R}_t \tilde{b}_t^d + \tilde{r}_{t+1}\tilde{i}_t \tag{28}$$

The bank's problem is now:

$$\max_{\tilde{r}_t^m, \tilde{r}_t^n, \tilde{m}_t, \tilde{b}_t^d, \tilde{i}_t} \tilde{\pi} ln\left(\tilde{r}_t^m[\tilde{w}_t + \tau_t]\right) + (1 - \tilde{\pi}) ln\left(\tilde{r}_t^n[\tilde{w}_t + \tau_t]\right) \tag{29}$$

subject to (26), (27), and (28).

The bank's optimal demand for money is:

$$\tilde{m}_t = \tilde{\pi}[\tilde{w}_t + \tau_t] \tag{30}$$

The money market clears when the demand by banks for cash reserves is equal to the supply of money at each date, as expressed by (30). Similarly, the period $t+1$ per capita capital stock, k_{t+1}, must equal i_t. Hence, equilibrium capital holdings are:

$$\tilde{k}_{t+1} = \tilde{w}_t + \tau_t - \tilde{m}_t - \tilde{b}_t^d \tag{31}$$

Additionally, an absence of arbitrage opportunities requires that the interest rate on bonds issued by entrepreneurs, $\tilde{R}_t$, is equal to the rate of return on capital investments, $\tilde{r}_{t+1}$. Specifically,

$$\tilde{R}_t = \tilde{r}_{t+1}. \tag{32}$$

4.1.5 General Equilibrium

Equations (19), (20), (17), (18), (25), (30), (26), and (32) now characterize the complete set of equilibrium conditions for the high income country. By combining these equations, the issuance of bonds by entrepreneurs is:

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - \frac{(1 - \tilde{\alpha})\tilde{A}}{1 - \left(\frac{\tilde{\sigma} - 1}{\tilde{\sigma}}\right)\tilde{\pi}} \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}_t}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} \tag{33}$$

4.1.6 Steady State Analysis

We proceed to characterize the equilibrium conditions in the steady state. To begin, steady state bond supply is:

$$\tilde{b}^s = \frac{z}{\tilde{R}} - \frac{(1-\tilde{\alpha})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} \tag{34}$$

In the bank balance sheet of the high income country, we substitute the expression for capital derived from (19), and rewrite the bank balance sheet constraint in terms of total demand for bonds by the bank, while also taking note of the no arbitrage condition (32), as:

$$\tilde{b}^d = \frac{(1-\tilde{\alpha})(1-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-\tilde{\alpha}}} \tag{35}$$

It is clear that the domestic bond market is non-linear in the real interest rate. Thus, the interconnection between the bond market, the money market and economic activity may not be apparent. Nonetheless, we can find analytical solutions in the special case where the factor market is equally capital and labor-intensive. In order to find an analytical solution to the real interest rate that clears the bond market under autarky, we set (34) and (35) equal to each other and solve for the special case where the factor market is equally capital and labor-intensive ($\tilde{\alpha} = 0.5$).[28]

Lemma 2. *Suppose that* $\widetilde{\alpha} = 0.5$.[29] *Under this condition, the real interest rate in the high income country under autarky is:*

$$\tilde{R}^{aut*} = \tilde{R}(z, \tilde{\pi}, \tilde{A}, \tilde{\sigma}) \tag{36}$$

Because the economy is in autarky, entrepreneurs are limited in their ability to raise funds on the global stage. In particular, they will have only one source of funding – their local banks. However, certain economic shocks can hurt their ability to raise adequate funding for their projects. For example, if there is an increase in the probability of a relocation shock ($\tilde{\pi}$), banks adjust their portfolios to hold more cash and less bonds. As such, entrepreneurs seeking to borrow more scarce funds will be required to pay a higher rate of interest. This would lower their ability to complete their investment projects, since the cost of borrowing is now higher. Similarly, firms seeking to rent capital will now have to pay higher rates for capital.[30]

A monetary tightening by the government can also lower the ability of entrepreneurs to raise

[28] In numerical examples, we choose different values for $\tilde{\alpha}$ to show that our analytical results are robust.

[29] Again, as reported by Jones et al. (2003), the capital share for many advanced economies is relatively close to one-half.

[30] Even though this is a lager issue for developing countries, Ghossoub and Reed (2010) show that in a closed economy setting, lower levels of financial development can coerce individuals into holding more liquid assets. This is turn, can have significantly adverse effects on economic activity.

funds for their projects under autarky. In particular, if there is a decrease in the money growth rate in the high income country ($\tilde{\sigma}$), this will result in a shrinkage of the banks' balance sheet since there is less seigniorage revenue being redistributed to workers who are also depositors. With a smaller deposit base, banks become limited in their asset holdings, including their debt holdings. On the demand side,the redistribution of seigniorage revenue to entrepreneurs also fall. Because of this, they have less income to put towards their investment project, and now have greater reliance on their local banks for additional funding. With higher demand and limited resources, the interest rate will rise which makes it harder for entrepreneurs to fund their projects. This of course, would reduce production. In a similar manner, higher interest rates raise the cost of capital for firms which has adverse consequences for output.[31]

4.2 The high income country as a small open economy

We proceed to look at the high income country as a small open economy. In this setting, entrepreneurs now have the ability to obtain funding from the rest of the world, in addition to raising funds domestically. For this international demand for funds to be plausible, the world real interest rate (r^*) must be lower than the autarky real interest rate ($\tilde{R}^{aut*} > r^*$). Otherwise, entrepreneurs would seek all their funding domestically, despite the ability to raise funds internationally. Also, seeing that the high income country is a small open economy, the no-arbitrage condition is now augmented so that the interest rate in (32) will be equal to the world interest rate. Thus, all endogenous variables that depend on (32) are now pinned down by r^*. Moreover, since the country is an advanced economy, we consider that the supply curve from the rest of the world is perfectly elastic at r^*. In this manner, entrepreneurs turn to the global capital market after obtaining funding domestically.

The amount of debt that entrepreneurs will issue to the rest of the world is equal to the difference between the amount of debt they issue to their local banksand the amount debt those banks are willing to hold. Denote this difference as $\hat{b}^s$.

Lemma 3. *The amount of bonds issued by entrepreneurs to the rest of the world is:*

$$\hat{b}^s = \tilde{b}^s - \tilde{b}^d \tag{37}$$

$$= \frac{z}{r^*} - \frac{(1-\tilde{\alpha})(2-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} + \left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{1}{1-\tilde{\alpha}}} \tag{38}$$

In comparison to autarky, we can now see the importance of the international capital market to entrepreneurs in their attempt to raise funds for their investment projects. In particular, access to the global bond market can alleviate the adverse effects of idiosyncratic economic shocks on their ability to raise funds. For example, if there is an increase in the probability

[31] Ahmed and Rogers (2000) find evidence of a long-run Tobin effect for the United States. As is shown in the appendix, higher rates of inflation stimulate investment activity.

of a relocation shock ($\tilde{\pi}$), entrepreneurs will make greater use of the global bond market by borrowing more internationally. Also, with a higher probability of a relocation shock, there will be greater demand for cash so that they devote more resources to real money balances, and less to fund entrepreneurs. Therefore, entrepreneurs will demand more funds from the rest of the world at the prevailing world real interest rate by issuing more debt internationally.

Moreover, entrepreneurs can make use of global markets in the presence of a monetary tightening domestically. To illustrate, if there is a fall in the money growth rate in the high income country ($\tilde{\sigma}$) such that the bank's balance sheet shrinks (as a result of a lower transfer of seigniorage revenue to workers/depositors), entrepreneurs will respond by issuing more debt internationally. In particular, this will raise the amount of capital inflows to high income country. Because there is now lower seigniorage revenue being transferred to entrepreneurs, they now require a larger portion of their funding internationally at the prevailing world real interest rate.

4.2.1 Steady-State Equilibrium

In this section, we focus on the *steady-state equilibrium* in the context of the environment where both the rest of the world and banks in the high income country lend to the corporate sector in the high income country.

Definition 2 *(Steady-state equilibrium).* A *non-trivial, steady-state equilibrium* (SSE) is satisfied under the following conditions:
(H-1) banks maximize the expected utility of each depositor (29);
(H-2) entrepreneurs have positive profits and borrow at the prevailing world real interest rate;
(H-3) the rest of the world holds international debt issued by entrepreneurs in the high income country at the prevailing world real interest rate;
(H-4) the nominal interest rate at (r^*) is greater than 1.

Theorem 2 *(Existence) Suppose that Lemmas 2 & 3 hold and there exists z such that $z > \underline{z}$ and r^* such that $r^* < \tilde{R}^{aut*}$. Then, there exists a unique steady-state equilibrium interest rate in the international bond market.*

Recall that the entrepreneur's borrowing costs cannot exceed the real value of the investment project, as stated in (23). As we show in the appendix, this is viable if the productivity level on their project is above some minimum value, $\underline{z}$. Finally, we show in the appendix that for capital to flow into the high income country while entrepreneurs have simultaneous positive production and profits, $r^* < \tilde{R}^{aut*}$.

4.2.2 Comparative Statics

We next consider the effect of an increase in the world interest rate on the financial market

in the high income economy:[32]

Proposition 3 *(Effect of a decrease in the world interest rate on the domestic real interest rate and domestic economic activity in the high income country.) In a SSE, a decrease in the world interest rate leads to an increase in the amount of bonds issued by entrepreneurs domestically* $\left(\frac{\partial \tilde{b}^s}{\partial r^*} < 0\right)$ *and a decrease in the amount of debt demanded by banks* $\left(\frac{\partial \tilde{b}^d}{\partial r^*} > 0\right)$ *in the high income country, with more capital accumulation and larger output across all sectors of the high income economy.*

The implications of Proposition 3 are as follows. At lower interest rates in the global market for funding, it will become less costly for entrepreneurs to borrow abroad. Also, due to the no arbitrage condition (32), a lower real interest rate reduces the cost of using capital and therefore promotes capital formation. In turn, the increase in capital formation raises wages and increases the deposit base of banks in the high income country. Thus, a lower global interest rate leads to more economic activity across all sectors of the high income country.

We next consider the effect of an increase in the money growth rate in the high income economy on its financial market:

Proposition 4 *(Effect of an increase in the money growth rate in the high income country on domestic economic activity.) In a SSE, an increase in the money growth rate in the high income country leads to a decrease in the amount of bonds issued by entrepreneurs domestically* $\left(\frac{\partial \tilde{b}^s}{\partial \sigma} < 0\right)$ *and an increase in the amount of debt demanded by banks* $\left(\frac{\partial \tilde{b}^d}{\partial \sigma} > 0\right)$ *so that there is no change in output in any sector of the economy. Thus, the only change is that entrepreneurs will borrow less from the global capital market* $\left(\frac{\partial \hat{b}^s}{\partial \sigma} < 0\right)$*. However, the impact of the money growth rate on the amount of bonds issued by entrepreneurs globally is weaker if global interest rates are lower* $\left(\frac{\partial \hat{b}^s}{\partial \sigma \partial r^*} > 0\right)$*.*

The implications of Proposition 4 are as follows. A higher money growth rate increases the amount of deposit funding that banks obtain. In addition, entrepreneurs have more internal funding because the seigniorage revenue generated from the higher money growth rate is transferred to both workers and entrepreneurs. Consequently, the higher rate of money growth leads to a decrease in the amount of bond supply issued by entrepreneurs. The higher rate of money growth also leads to an increase in bonds demanded by banks so that there is no change in interest rates.[33]

Therefore, in a small open economy, monetary policy primarily affects how much funding the corporate sector seeks from the global capital market. Yet, the ability of a central bank to influence the impact on credit sought domestically by entrepreneurs will be weaker if global interest rates are lower. This takes place because of the no-arbitrage condition between bonds issued domestically and bond funding provided in the global capital market. Similarly, if the

[32] A decrease in the world interest rate can occur because of a global savings glut (see Bernanke, 2005).

[33] In this manner, the "balance sheet" channel (via an increase in the size of the bank balance sheet) of monetary policy is offset by the rise in internal finding for the entrepreneur (See Rey (2016) for a discussion).

central bank lowers the money growth rate, it would have less of an impact on debt funding sought by the corporate sector of the economy in the presence of lower global interest rates (due possibly to a savings glut). In this manner, our framework helps to begin to explain how active global capital markets may render domestic monetary policy actions in advanced economies to be less effective – as policy has less of an impact on funding demanded internally. We turn to this issue in more detail in Section 5 where we look at activity across two large open economies.

4.2.3 Welfare and Monetary Policy

It is trivial to show that an expansionary monetary policy is welfare improving in the small open high income economy. In particular, if the monetary authority has entirely egalitarian objectives and seeks to maximize the expected life utility of depositors and the profits of entrepreneurs, it will pursue a high inflation target. Since seigniorage revenue is redistributed to workers and entrepreneurs, a higher rate of money growth increases the welfare of both sets of agents, and thus, total welfare (which is the sum of 21 and 29). For entrepreneurs, a higher rate of money growth reduces their dependence on funding, which reduces their debt obligations, increases their profits, and thus, increasing welfare. Also, an increased money growth rate gives them more income to put towards their investment projects, which means that they do not need to borrow as much. This translates to lower production costs and increased welfare.

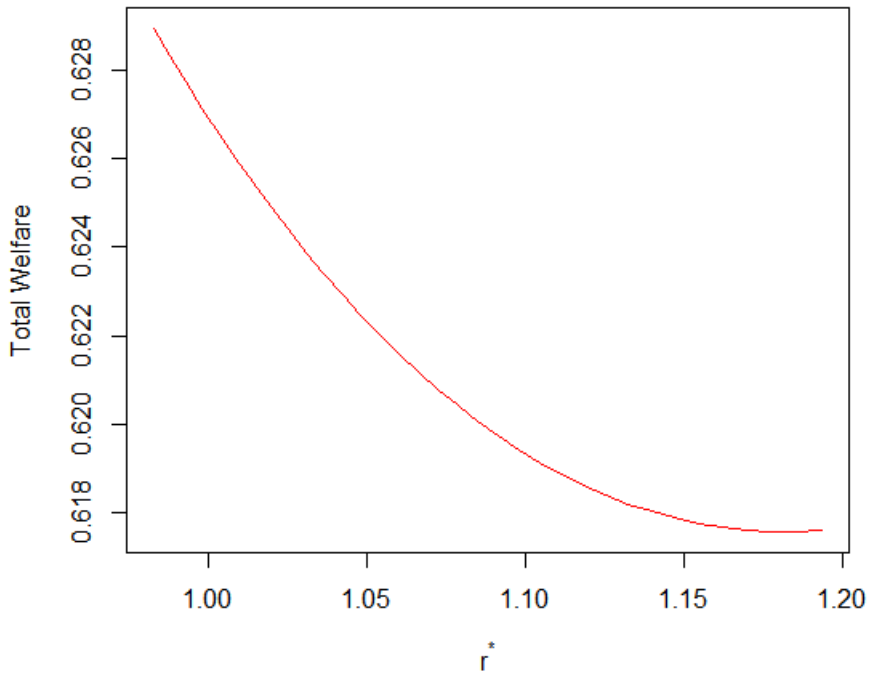
For workers, a higher rate of money growth means that their income increases through the transfer of seigniorage revenue. When this increased income is deposited in the bank, it is greater able to provide insurance against the risk of liquidity shocks. This increases the welfare of each depositor. These coupled together ensure that welfare is strictly increasing in the money growth rate.

4.2.4 Welfare and varying global market conditions

In this section, we consider how welfare in a small open advanced country depends on global capital market conditions. In particular, we examine the impact of global interest rates in the financial system. As in the welfare analysis for a small open low income country, we use numerical examples to illustrate these considerations. Due to a no-arbitrage condition, the higher world interest rate would lead to a higher interest rate on bonds issued domestically by entrepreneurs. In turn, higher interest rates lead to lower profits for entrepreneurs. Moreover, workers experience lower wages when the rental rate on capital is higher. However, non-movers earn higher returns on deposits.

Figure 4 shows total welfare in the high income country. From initially low interest rates, total welfare falls as the world interest rate increases. Thus, the welfare of entrepreneurs dominates here. However, there is an inflection point where eventually welfare rises – this occurs from the increase in welfare of depositors who do not experience liquidity shocks and earn returns on long-maturing assets: bonds and capital.

Figure 4: Welfare in the small open high income country and the world interest rate



5 Two Large Open Economies: Capital Flows from the Low Income Country to the High Income Economy

Having developed a basic framework with small open economies, we now arrive at the main part of our paper. In particular, we look at two large open economies. In the previous sections, both countries took the world interest rate as given. By extending the analysis to look at global economic activity, we can study the factors that affect the steady state world interest rate. That is, both economies are large in the sense that the world interest rate is determined by the international demand for funds from entrepreneurs in the high income country and the supply of funds internationally by the government of the low income country. The environment from both countries are applicable here. Specifically, the agents, assets, and timing of events are the same here as in the small open economy cases.

At this point, the global bond market clearing condition ensures that the supply of bonds coming from entrepreneurs in the high income country (34) is equal to the total demand for bonds, coming from both the banks in the high income country (35) and the government of the low income country ($\hat{b}^d$). Assuming that the amount of bonds demanded by the government of the low income country in the international bond market is fixed at some $\hat{b}_0^d$, then equilibrium in the global bond market now satisfies:

$$\hat{b}^s = \tilde{b}^s - \tilde{b}^d = \hat{b_0}^d \tag{39}$$

$$= \frac{z}{\tilde{R}} - \frac{(1-\tilde{\alpha})(2-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} + \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-\tilde{\alpha}}} = \hat{b_0^d} \tag{40}$$

We see that (40) is nonlinear in the endogenous world interest rate. However, as in the previous analyses, we consider the special case where the factor market in the advanced economy is equally capital and labor intensive in order to obtain analytic solutions.[34] With this, we can find the government's steady state inverse demand for bonds issued internationally by entrepreneurs:

Lemma 4. Let $\tilde{\alpha} = 0.5$ *Also, suppose that* $z < \bar{z}$. *Under this condition, the real interest rate for bonds issued by entrepreneurs in the high income country is:*[35]

$$\tilde{R}^* = \tilde{R}(z, \tilde{\pi}, \tilde{A}, \tilde{\sigma}, \hat{b_0}^d) \tag{41}$$

As long as the productivity level on entrepreneurs' investment project (z) does not exceed some upper bound, then the cost of borrowing in global bond markets will not be negative. Thus, under the conditions in the lemma, there is only one positive solution for the global real interest rate as shown in Figure 5. Also, in contrast to the small open economy case, it is obvious that exogenous shifts in the steady state holdings of domestic debt by foreigners can impact the world interest rate.

Notably, when the low income economy lends more abroad, the global interest rate falls which does not occur in the small open economy case. A lower world interest rate makes it less expensive for entrepreneurs to borrow internationally. By the no-arbitrage condition between bonds and capital in the advanced country, the costs of using capital will also fall. Consequently, there will be increased production across both sectors of the high income country. This result is in line with, for example, Prasad et al. (2006) who find via empirical evidence that uphill capital flows are positively related to output growth in high income countries.

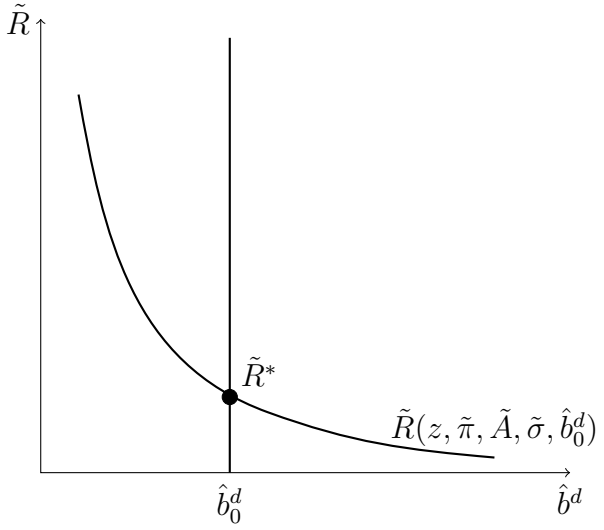
5.1 Implications of the Endogenous World Interest Rate for the Domestic Bond Market in the Low Income Country

We proceed to study how global credit market activity feeds back into the market for domestically issued sovereign debt in the low income country. To begin, we substitute the market-clearing global interest rate from (41) into the government budget constraint of the developing

[34] As before, analyses involving numerical examples use differing values for capital and labor intensity to show that our results are robust.

[35] See appendix for details.

Figure 5: The real interest rate that clears the international bond market as a function of bonds held by the government of the low income country that is issued by entrepreneurs



country, while utilizing the assumption of equal capital and labor intensity to obtain an analytical solution for the real interest rate (R) that clears the domestic bond market::

Lemma 5. *Let $\alpha = 0.5$. Suppose that $\sigma > \bar{\sigma}_1$ and $\pi < \sigma[2 - \sigma]$. In addition, let $\tilde{A} < \tilde{A}_0$. Under these conditions, there is only one equilibrium real interest rate on bonds issued by the government of the low income country:*

$$R^* = R(A, \pi, \sigma, \hat{b}_0^d, z, \tilde{\pi}, \tilde{A}, \tilde{\sigma}) \tag{42}$$

Interestingly, the conditions in Lemma 5 depend on activity in both countries. The first set of conditions is required so that the low income country has sufficient seigniorage revenues to support the domestic bond market to the point where money in the developing country is dominated in rate of return. It also helps establish that the discriminant in the solution for the equilibrium interest rate on bonds is positive. As in the case of the small open low income economy, the second condition is such that banks are able to allocate scarce resources to assets with the highest returns. By comparison, if productivity is too high in the advanced country, wages would also be very high and entrepreneurs would have no need to borrow. Thus, the international bond market would be obsolete.

5.2 Steady-State Equilibrium

In this section, we focus on the *steady-state equilibrium* in the current economic environment. This allows us to highlight the influence of capital flows from the low income country on interest rates in the global capital market and worldwide economic activity.

Definition 3 *(Steady-state equilibrium).* A *non-trivial, steady-state equilibrium* (SSE) is

satisfied under the following conditions:

(J-1) banks in both countries maximize the expected utility of each depositor (8), (26);
(J-2) entrepreneurs have positive profits and borrow at the prevailing real interest rate;
(J-3) the government of the low income country borrows domestically and lends internationally at the prevailing interest rates;
(J-4) returns to entrepreneurial production must be positive and larger than the prevailing real interest rate at which they borrow;
(J-5) in the international bond market, there is a unique real interest rate to satisfy (J-2) and (J-3);
(J-6) in the bond market of the low income country, there is only one valid real interest rate that clear the market.
(J-7) in both bond markets, the nominal interest rate is greater than 1.

Theorem 3 *(Existence) Suppose that Lemmas 4 & 5 hold and there exists z such that $z > \underline{z}$ and $\hat{b}^d$ such that $\hat{b}^d > \underline{\hat{b_0}}^d$. Then, there exists both a unique steady-state equilibrium interest rate in the international bond market and a unique equilibrium real interest rate in the domestic bond market.*

Similar to the small open high income economy, entrepreneurs will choose to borrow as long as $z > \underline{z}$. Yet, in order to ensure that entrepreneurs have positive production and profits simultaneously at $\tilde{R}$, the developing country must engage in a sufficiently large amount of lending to entrepreneurs in the advanced country – this supports activity by promoting sufficiently low interest rates in global capital markets.

5.3 Comparative Statics

We proceed to look at some comparative statics. We are particularly interested in learning about the global implications of monetary policy. To begin, we obtain the following:

Proposition 5 *(Effect of an increase in the money growth rate in the high income country on global interest rates and global economic activity.) In a SSE, a higher rate of money growth in the high income country leads to a decrease in real interest rates across the world $\left(\frac{\partial \tilde{R}}{\partial \tilde{\sigma}} < 0\right)$ and an increase in worldwide economic activity. By comparison, changes in the money growth rate in the low income country do not impact the equilibrium real interest rate on international debt $\left(\frac{\partial \tilde{R}}{\partial \sigma} = 0\right)$. As such, changes in the money growth rate in the low income country do not impact international economic activity.*

The implications of Proposition 5 are as follows. First, a higher money growth rate in the high income country will lead to a higher deposit base in the banking system. With a larger deposit

base, banks can hold more assets including debt. Moreover, some of the seigniorage revenue is also distributed to entrepreneurs. With this additional income, they do not need to borrow as much from either local banks or abroad. Consequently, interest rates in the global capital market will be lower.[36]

In turn, there are implications for the domestic real interest rate in the low income country. With lower interest rates in the global financial system, the government of the developing country will have less income. Thus, it cannot issue as much debt to banks in the poor country. Consequently, the real interest rate on domestically issued bonds will be lower – this weakens the crowding out effect from sovereign debt in the developing country. In this manner, the effects of monetary expansion in the advanced economy will be felt across the globe so that there will be an increase in worldwide economic activity.

However, changes in monetary policy in the poor country will be isolated to the domestic economy. Simply put, an increase in the money growth rate will provide the government with more seigniorage revenues. As a result, they borrow more at home and interest rates on sovereign debt will be higher – the increase in borrowing leads to a larger crowding out effect on capital accumulation which causes economic activity to fall.[37]

Next, we study how the relationship between the global real interest rate and the money growth rate in the high income country depends on the amount of lending by the developing country $\left(\frac{\partial \tilde{R}}{\partial \tilde{\sigma} \partial \tilde{b}_0^d}\right)$. As an example, consider the parameters studied in Table 2.[38] The impact of monetary policy is illustrated by an increase in the growth rate of money from 1% to 10% in increments of 1%. First, by looking at activity within each individual column, we see that an increase in the rate of money growth in the large advanced country will cause the interest rate paid by entrepreneurs to decline as explained in Proposition 5. However, as one moves across columns where the low income country lends more abroad, the impact of actions by the central bank in the advanced country is weaker. Alternatively, if the central bank were to adopt more restrictive policy by lowering the money growth rate, the impact of the policy on raising interest rates would also be weaker. This suggests that the effects of monetary policy

[36]Notably, the model produces a real balance effect because seigniorage taxes are not borne by all of the demographic groups in our overlapping generations setup -- seigniorage revenues are redistributed to all of the young, but the costs of the inflation tax only fall upon a segment of the old. Similarly, in the presence of zero nominal interest rates, Ireland (2005) shows that monetary policy can produce real balance effects in economies with population growth – under population growth, there are distributional effects from monetary policy much like there are in our overlapping generations setup.

[37]This result of course is because we have assumed a fixed level of external debt holdings by the government of the low income country. If instead, we assumed that the external debt holdings of the government was endogenous, then monetary policy in the low income country would have implications for the global financial market. In particular, the seigniorage revenue generated from an expansionary monetary policy would increase the government's demand for external assets, thus driving down global interest rates and raising global economic activity. Indeed, Miranda-Agrippino et al. (2020) find that through the Chinese experience, monetary policy has implications for the global economy. In particular, they note that global activity increases after an expansionary monetary policy.

[38]For this example, we use $z = 3.212$, $\tilde{\pi} = 0.15$, $\tilde{A} = 2.91$, and $\tilde{\alpha} = 0.3$ as parameter values to solve for the world real interest rate from the global bond market condition (40). From there, we calculate the percent change in $\tilde{R}$ with respect to a one percent change in the rate of money growth at different levels of public outflows. The interpretation here is based on the absolute value of the values in Table 2. The negative sign just signifies the relationship between $\tilde{R}$ and $\tilde{\sigma}$. In other words, when there is a one unit increase in the money growth rate in the high income country, the real interest rate falls.

in the advanced country depend on global capital market conditions. Again, we show with our framework how active global capital markets may render domestic monetary policy actions in advanced economies to be less effective – whether they be small or large.

Table 2: The effects of monetary policy in the high income country and international lending by the government of the low income country.

$\hat{b}^d$	0.16	0.19	0.22	0.25	0.28
$\tilde{\sigma}$	$\%\Delta\tilde{R}/\%\Delta\tilde{\sigma}$	$\%\Delta\tilde{R}/\%\Delta\tilde{\sigma}$	$\%\Delta\tilde{R}/\%\Delta\tilde{\sigma}$	$\%\Delta\tilde{R}/\%\Delta\tilde{\sigma}$	$\%\Delta\tilde{R}/\%\Delta\sigma$
1.01					
1.02	-0.2084	-0.2056	-0.2028	-0.2002	-0.1977
1.03	-0.2067	-0.2039	-0.2012	-0.1986	-0.1961
1.04	-0.2050	-0.2022	-0.1996	-0.1970	-0.1945
1.05	-0.2033	-0.2006	-0.1980	-0.1954	-0.1930
1.06	-0.2017	-0.1990	-0.1964	-0.1939	-0.1915
1.07	-0.2001	-0.1974	-0.1949	-0.1924	-0.1900
1.08	-0.1985	-0.1959	-0.1933	-0.1909	-0.1885
1.09	-0.1970	-0.1944	-0.1918	-0.1894	-0.1871
1.10	-0.1955	-0.1929	-0.1904	-0.1880	-0.1856

We proceed to look at how an increase in the demand for international bonds will impact the domestic real interest rate of the low income country.

Proposition 6. *(Effect of international lending by the government of the low income country on the domestic real interest rate and domestic economic activity.) Suppose that* $\hat{b_0}^d < \hat{b}_T^d$, where $\hat{b}_T^d$ is some threshold value.[39] *Under this condition, the equilibrium real interest rate on government debt in the low income country is increasing in* $\hat{b}^d$ $\left(\frac{\partial R}{\partial \hat{b}_0^d} > 0\right)$ *and domestic economic activity is decreasing in* $\hat{b}^d$. *However, suppose that* $\hat{b_0}^d > \hat{b}_T^d$. *Under this condition, the equilibrium real interest rate on government debt in the low income country is decreasing in* $\hat{b}^d$ $\left(\frac{\partial R}{\partial \hat{b}_0^d} < 0\right)$ *and domestic economic activity is increasing in* $\hat{b}^d$.

The implications of Proposition 6 are illustrated in Figure 6. An increase in the demand for international debt by the government of the low income country leads to an increase in the domestic real interest rate when $\hat{b}^d < \hat{b}_T^d$. Intuitively, at relatively low demand for bonds from the corporate sector in the advanced country, interest rates in the global capital market would be relatively high. With more revenue from lending abroad, the developing country government will have more income and will borrow more in the domestic bond market. The increased issuance of debt causes the equilibrium interest rate to be higher. This is depicted by all values of $\hat{b}^d < \hat{b}_T^d$ in Figure 6.

That is, at relatively low levels of lending to the advanced country, increased sovereign debt issuance will crowd out capital formation domestically and lower local economic activity. Thus,

[39] See appendix for details.

as argued by Bernanke (2005), the government acts as a financial intermediary, channeling domestic saving *away from local uses* and into international capital markets.

However, if $\hat{b}_0^d > \hat{b}_T^d$, then an increase in the demand for bonds by the government of the low income country leads to a lower equilibrium real interest rate in the domestic bond market. The decrease in demand for funds domestically causes the government to issue less debt to its local banks, which drives the real interest rate in the domestic bond market downwards in equilibrium. This is depicted by all values of $\hat{b}^d > \hat{b}_T^d$ in the figure.

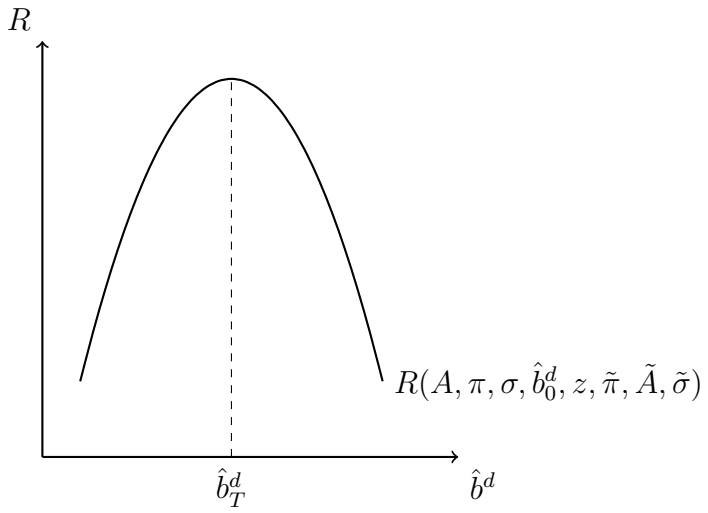


Figure 6: The impact of an increase in the amount of funds lent internationally by the government of the low income country on the domestic interest rate.

We next consider how the relationship between the real interest rate and monetary policy in the low income country depends on the government's level of capital outflows $\left(\frac{\partial R}{\partial \sigma \partial \hat{b}_0^d}\right)$. The impact of monetary policy is illustrated in Table 3 by an increase in the growth rate of money from 6% to 15% in increments of 1%. Interestingly, there are non-monotonicities that arise. Starting from a point with low levels of capital outflows, the effects of monetary policy are weaker if the government lends more – at higher levels of lending, the relative impact of seigniorage on the developing country's budget constraint will be lower because interest rates in the global capital market will be relatively high. However, once capital outflows eclipse a certain point, higher capital outflows lead to a stronger impact from monetary policy because global interest rates would be lower if the government aggressively lends to the advanced country. This does not occur in the small open economy case because the interest rate in the global capital market is unaffected by the individual country's lending activity to the global financial system.

5.4 The optimal monetary policy

All previous analysis assumes that in a SSE, the money growth rate is exogenous. Now, we study the optimal monetary policy that will maximize total welfare in both countries. In the high income country, total welfare is a summation of (21) and (29), while in the low income

Table 3: The effects of monetary policy in the large open low income country and capital outflows from its government.

$\hat{b}^d$	0.16	0.19	0.22	0.25	0.28
σ	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$	$\%\Delta R/\%\Delta\sigma$
1.06					
1.07	0.9903229	0.9858599	0.9843398	0.98560057	0.989545794
1.08	0.9684309	0.9641481	0.9626891	0.96389915	0.967685295
1.09	0.947561	0.9434454	0.9420432	0.9432062	0.946844572
1.10	0.9276391	0.9236792	0.9223299	0.92344903	0.926949827
1.11	0.9085984	0.9047838	0.9034838	0.90456207	0.907934495
1.12	0.8903786	0.8866998	0.885446	0.88648594	0.889738359
1.13	0.8729249	0.8693733	0.8681627	0.86916679	0.872306789
1.14	0.8561873	0.8527552	0.8515851	0.85255559	0.855590092
1.15	0.8401206	0.8368007	0.8356688	0.83660762	0.839542953

country, total welfare is defined by (8).[40] Due to the complexity of the exercise, we are unable to proceed analytically. As a result, we use a numerical example.

5.4.1 Optimal monetary policy in the low income country

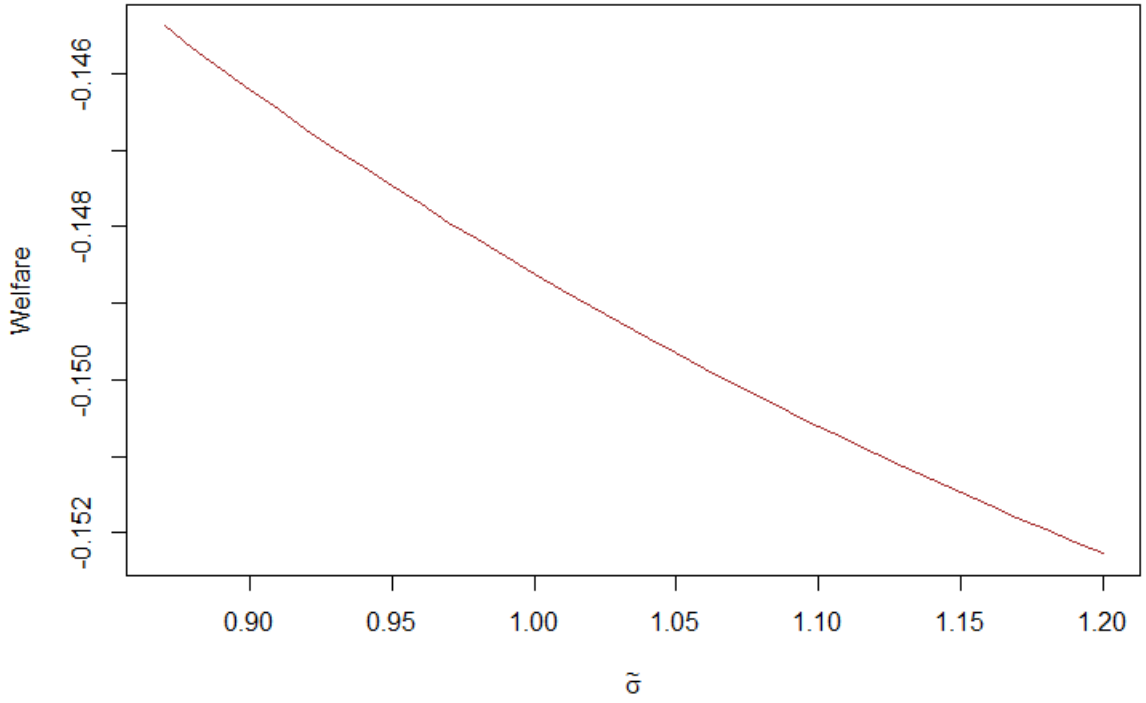
The effect that the rate of money growth in the low income country has on the welfare of its citizens follows from our discussion of the small open economy case. In particular, a higher rate of money growth lowers the returns promised to relocated agents, which reduces insurance against liquidity shocks. By comparison, a higher rate of money growth also increases the real interest rate in the low income country and will increase the returns promised to agents who are not relocated. However, the demand for capital will fall, and wages will decrease, reducing the bank's deposit base. Overall, the welfare losses of relocated agents dominates that of the welfare gains by non-relocated agents as the rate of money growth increases. As a result, the economy's total welfare falls.

5.4.2 Optimal monetary policy in the high income country

It is easy to see that welfare in the advanced country will be higher if the money growth rate is higher – the government redistributes seigniorage revenues across the population of workers and entrepreneurs. With a larger income base, both groups of individuals would obtain more utility. However, what is the impact on welfare in the developing world? We have previously shown that the effects of monetary policy in the high income country will be felt across the globe. At this point, we find it interesting to study how monetary policy in the advanced world affects the welfare of individuals in the developing country. As previously mentioned, an increase in the money growth rate in the high income country will lower the real interest

[40] As in the low income country case, finding the optimal monetary policy in both countries is akin to solving the Ramsey problem in which the planner (central bank/government) optimally selects the money growth rate to maximize household's lifetime expected utility, subject to all private first order conditions and budget constraints.

Figure 7: The optimal growth rate in the high income country for depositors in the low income country



rate in the low income country, which does not occur in the small open economy case. While lower real interest rates promote economic activity in the advanced country, they also lower the bank's ability to promise high returns to depositors in the developing country.

The mechanism described above is shown in Figure 7.[41] From the figure, welfare in the developing world would be higher if the rate of money growth is low. Thus, there is a great conflict that results from monetary policy in the advanced world.[42]

5.4.3 Optimal capital outflows

As with previous sections, we have also treated capital outflows by the government of the low income country as exogenous. In this section, however, we relax this view as we seek to find the amount of international lending that will maximize total welfare in both countries. This

[41] In this example, we use $z = 3.212$, $\tilde{\pi} = 0.15$, $\tilde{A} = 2.91$, $\tilde{\alpha} = 0.3$, $\hat{b}^d = 0.19$, $A = 1$ and $\pi = 0.5$, $\alpha = 0.1$, $\sigma = 1.11$ and begin with $\tilde{\sigma} = 0.87$ as parameter values.

[42] As is well-known, one of the key channels through which monetary policy in advanced economies (e.g., in the U.S.) impacts developing countries (subject to currency mismatches) is through movements in exchange rates and resulting balance sheet effects (see, for example, Bruno and Shin, 2015a; Banerjee et al., 2016; Rey, 2016; Akinci and Queralto, 2019; Miranda-Agrippino and Rey, 2020, among others). Though we do not consider the exchange rate channel in our framework, we argue that similar results can be seen through the *interest rate channel with government budget constraint effects* shown here. In particular, monetary policy in the advanced economy affects developing countries through interest rate differentials, which has effects on the government budget constraint, leading to bank balance sheet effects and thus, welfare implications.

proved too complex to move forward analytically, so we proceed with a numerical example.[43]

The top panel in Figure 8 shows total welfare in the high income country. As we can see, past a certain point, total welfare rises as capital flows from the low income country increase. An increase in international lending to entrepreneurs in the high income country reduces the real interest rate in the international bond market. The reduced demand for funds domestically drives down the domestic real interest rate. Lower interest rates have differing welfare effects for different sets of agents in the high income country. For depositors, welfare falls because lower interest rates reduce the bank's ability to promise high returns. However, for entrepreneurs, welfare rises through lower borrowing costs, higher production, and higher profits. The welfare gains to entrepreneurs outway the welfare losses to depositors as capital flows from the low income country increases. Thus, the net effect is driven by welfare of the entrepreneurs which results in higher social welfare.

By comparison, the lower panel shows total welfare in the low income country. In contrast to the small open economy case, there is a welfare-maximizing amount of outflows. When $\hat{b}^d$ is low, the government's revenue from lending internationally would be low. In order to increase its revenues on the international stage, the government has to raise more funds locally. As a result, its demand for funds from its domestic bond market increases, causing the domestic interest rate to be higher. A higher interest rate means that banks will promise higher expected returns to depositors, thus improving their welfare. However, once the flow of capital passes the welfare-maximizing point, increasing capital outflows causes global interest rates to fall which drives down expected returns in the banking system. Thus, the model suggests that it is indeed optimal for the government to act as an intermediary, following Bernanke's description.

5.5 Capital flows to advanced countries: Are they sustainable?

There is a large literature that considers differences in the level of financial development as a primary reason for capital to flow to advanced countries (see, for example, Caballero et al., 2008; Mendoza and Terrones, 2008; Mendoza et al., 2009; Bacchetta and Benhima, 2010; Forbes, 2010; Ju and Wei, 2011, among others).[44] We now consider the relationship between the level of financial development in the low income country and the domestic real interest rate. The results are presented in Figure 9.

At relatively high levels of financial development, banks will have more resources available to hold sovereign debt, thus lowering the domestic real interest rate. Consequently, at these low rates, the government of the low income country can borrow large amounts domestically to loan to entrepreneurs in the high income country.

However, as the level of financial development deteriorates, the ability of financial intermediaries to allocate credit will be lower such that their appetite for sovereign debt will be weak. As a result, the reduction in the supply of loanable funds puts upward pressure on the

[43] We use $\tilde{A} = 2.91$, $\tilde{\pi} = 0.15$, $\tilde{\alpha} = 0.3$, and $z = 3.212$, $\tilde{\sigma} = 1.02$, $\sigma = 1.11$, $\alpha = 0.1$, $A = 1$ and $\pi = 0.5$ as parameter values. We begin with $\hat{b}_0^d = 0.025$.

[44] See Forbes and Warnock (2012) for more discussion.

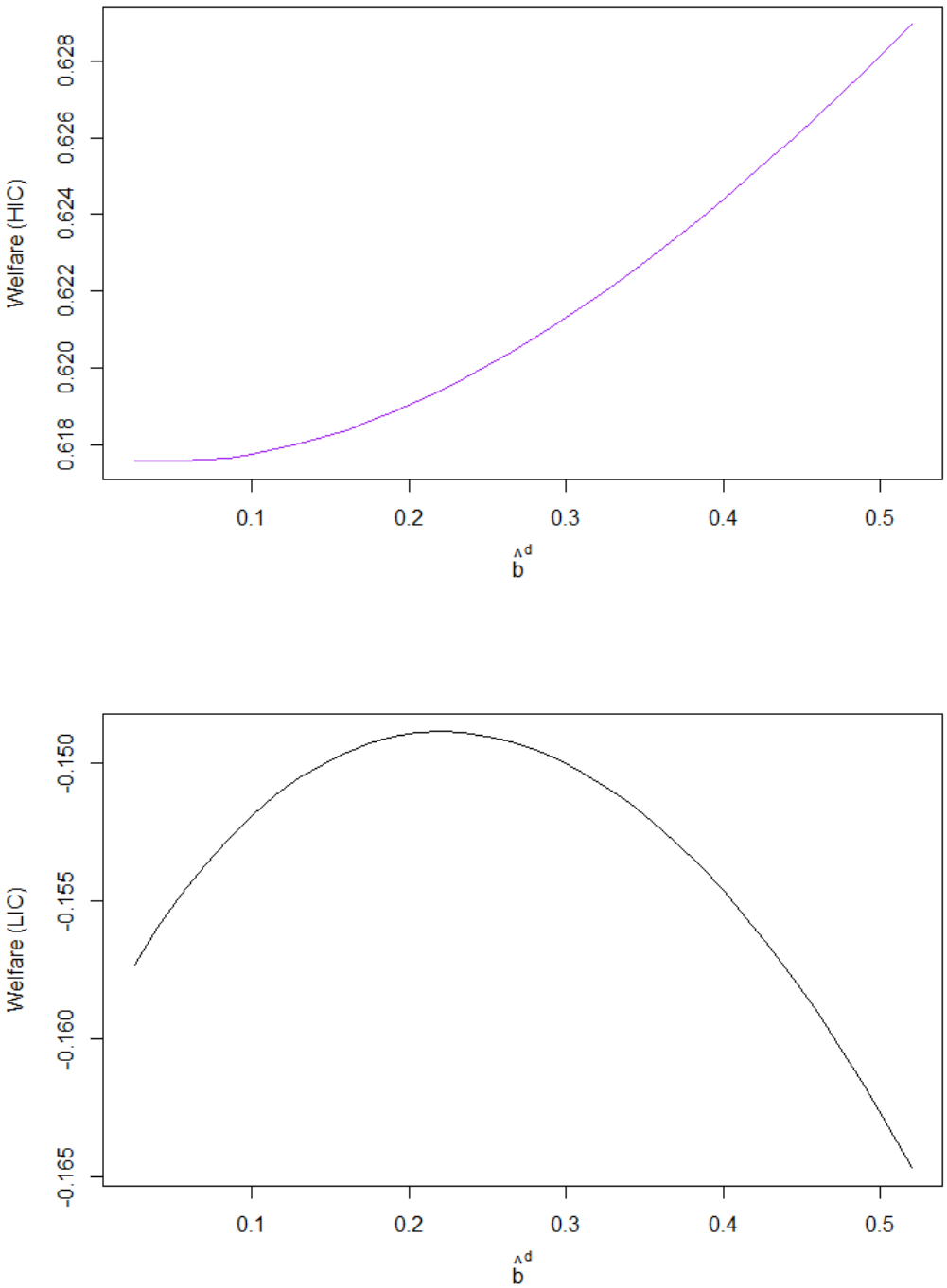


Figure 8: The optimal capital outflow from the low income country.

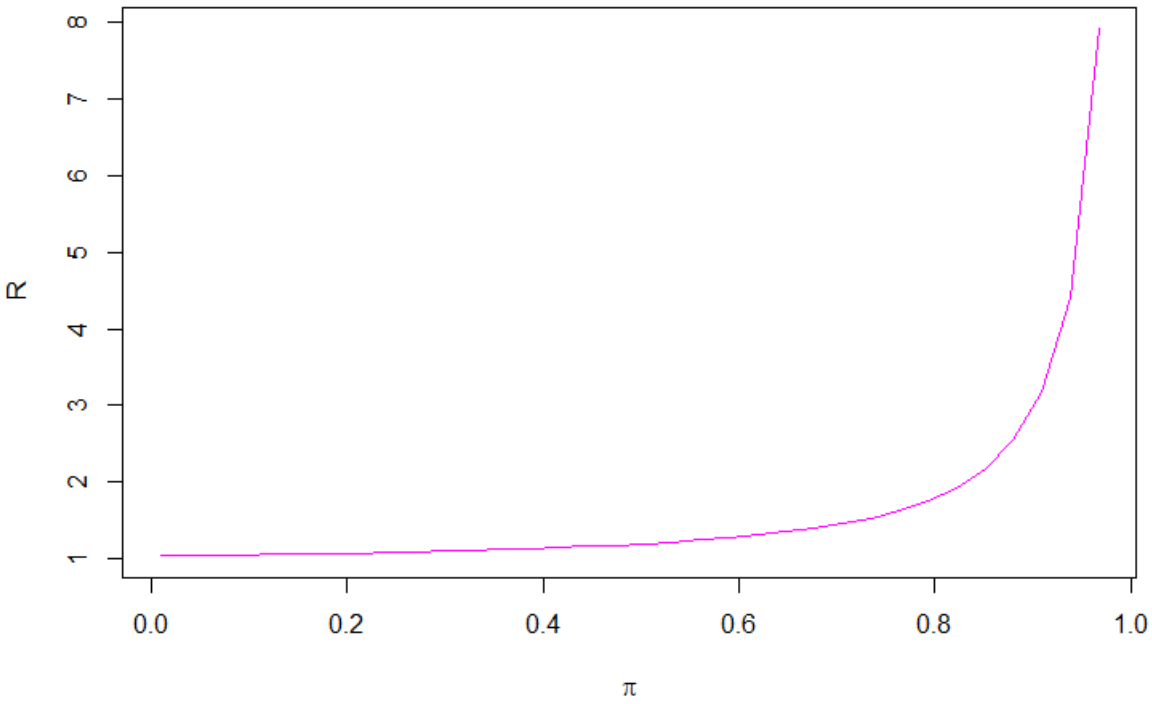


Figure 9: The relationship between the level of financial development and the real interest rate in the low income country

real interest rate. Hence, to maintain the same level of funding to the corporate sector of the advanced country (while also maintaining the inflation target), the government would have to borrow domestically at higher interest rates. This also raises the rental rate of capital in the consumption goods sector, and lowers the demand for domestic capital. Finally, wages will be lower which causes the deposit base in the banking system to contract. Thus, it is more difficult for the government of the low income country to maintain a position as a lender in the international capital market when there is more liquidity risk in the country's banking system. This result is in contrast to Wang et al. (2017) who argue that capital flows to advanced countries are sustainable even in the presence of heterogeneity in the level of financial development across both sets of countries. While this is true to some extent, we show that if the the level of financial development in the developing world is sufficiently low, then it may not be optimal to support capital flows to the advanced world.

6 Empirical results

We now seek to look to empirical results to test the main assumptions and predictions of the model. A key component of our framework is that the effect of changes in the cost of borrowing in developing countries depends on the level of capital outflows by its government. In particular, for a small open economy, if the world interest rate is initially low, then governments in these

developing countries will not seek to raise large amounts of funds in order to lend to the advanced world. In turn, the domestic real interest rate is lower.

However, when the world interest rate is high, an increase in capital outflows will lead to an increase in the domestic real interest rate. As the government increases outflows, it will seek to raise more funds domestically to lend abroad. As such, local demand for funding increases, driving up the domestic real interest rate.

Monetary policy is associated with a crowding out effect in developing countries. In particular, an increase in the rate of money growth in developing economies leads to an increase in the cost of borrowing as governments use seigniorage revenue as leverage to issue more debt. With intermediaries devoting more of their resources to hold government debt, less resources are on hand to hold capital and liquid assets. Thus, the lower level of resources drive up the interest rate, meaning that the effect of money growth on the cost of borrowing is positive.

The cost of borrowing we consider is the bank lending rate (LIR). It is the bank rate that usually meets the short and medium-term financing needs of the private sector. Summary statistics for each country are reported in the *Online Supplementary Materials*.

We next describe the primary control variables. The level of capital outflows is the main variable of interest. The data is obtained from the U.S. Department of Treasury and is measured in millions of dollars. Summary statistics are reported in the *Online Supplementary Materials*. In order to avoid problems with scale effects across countries, we consider the level of U.S long-term corporate debt holdings as a percentage of Gross Domestic Product (corpgdp). GDP data is obtained from the World Bank. Summary statistics on the holdings of U.S. long-term corporate debt as a percentage of GDP are reported in the *Online Supplementary Materials*.

It is important to note that corporate holdings of US securities by foreigners are not separated by private and public holdings. Thus, in order to isolate the effects of official holdings of US corporate debt from private holdings, we interact corporate holdings with an index that measures the degree of capital account openness. In particular, we use the Chinn-Ito index developed by Chinn and Ito (2006), normalized to range between zero and one (zero means the economy is financially closed such that no cross-border flow of private capital is allowed and 1 means that the economy has full capital mobility) which we denote as (ka_open). With this measure of financial openness, we are concerned with the correlation between foreign holdings of corporate debt and the cost of borrowing when these economies are financially closed. This implies that flows to the U.S corporate sector are public, not private.

In order to capture how the relationship between the cost of borrowing and the level of official flows to the corporate sector of the advanced world depends on the world interest rate as predicted by our model, we include an interaction term for the world interest rate. In particular, we use the 10-year treasury yield of the United States (tenyr) as the world interest rate. If the coefficient on the 10-year treasury yield is significant, then this ensures that the correlation between official gross outflows and the cost of borrowing depends on global interest rates.

We primarily study the growth rate of narrow money (m1g) as an indicator of monetary

policy. This variable is obtained from the first difference of the money supply in log levels. The descriptive statistics for (m1g) in each country are detailed in the *Online Supplementary Materials*.[45] Moreover, the level of income is likely to be correlated with the cost of borrowing. As a result, we examine GDP growth per capita (grrgdppc) as an important control variable.

We proceed to discuss the data used to investigate the implications of financial development for the domestic cost of borrowing in the low income country. Thus, we control for the level of development of the banking system. In particular, we utilize information on the amount of deposit bank assets relative to GDP (ba) as a measure of the extent of deposit-based financial intermediation.

Our model predicts differing effects of capital outflows on the cost of borrowing in small and large open developing economies. In addition, the efficacy of monetary policy differs across developing economies, depending on whether or not they can influence global interest rates. Thus, in order to see these differing effects in the data, we split developing countries into "small" and "large" groups. In this manner, countries that are "large" have the ability to influence global market conditions while countries that are "small" should not. To measure this numerically, a country is considered "large" if its holdings of long term corporate securities in the U.S. are more than the median U.S. corporate debt holdings of high income countries at all points in time between 2002 and 2018. In particular, developing countries that hold a consistently high level of US debt will more likely have the ability to influence global market rates. In a similar manner, a country is considered "small" if its holdings of U.S. corporate debt securities are less than or equal to the median holdings among high income countries over the same time frame.

We begin by examining the prediction that the relationship between the cost of borrowing in the small open low income country the level of capital outflows depends on the world interest rate. In this manner, we employ the following empirical specification:

$$LIR_{i,t} = \beta_1 \text{grrgdppc}_{i,t} + \beta_2 \text{m1g}_{i,t} + \beta_3 \text{tenyr}_{i,t} + \beta_4 \text{corpgdp}_{i,t} + \beta_5 \text{ka_open}_{i,t} + \beta_6 \text{ba}_{i,t} + \beta_7 \text{tenyr}_{i,t} * \text{corpgdp}_{i,t} + \beta_8 \text{tenyr}_{i,t} * \text{ka_open}_{i,t} + \beta_9 \text{corpgdp}_{i,t} * \text{ka_open}_{i,t} + \beta_{10} \text{tenyr}_{i,t} * \text{corpgdp}_{i,t} * \text{ka_open}_{i,t} + \varepsilon_{i,t}$$

In particular, we are interested in the marginal effect of corpgdp when the economy is financially closed (ka_open $= 0$), such that $\frac{\partial LIR_{i,t}}{\partial \text{corpgdp}_{i,t}} = \beta_4 + \beta_7 \times \text{tenyr}_{i,t}$. Thus, based our model predictions, the coefficient β_4 should be negative while the coefficient β_7 should be positive. This is because a low global world interest rate, the government does not earn high returns on outflows, so it does not seek to raise as much funds domestically when it increases its holdings of foreign debt. As such, demand for local funds fall, which lowers the domestic real interest rate. However, when the world interest rate is high, it will seek to raise more funds domestically to lend abroad. As such, local demand for funding increases, driving up the domestic real interest rate.

The results are presented in Table 4. As a simple benchmark, column (1) does not take into

[45] In other specifications, we use the growth rate of broad money (m2g) as an indicator of monetary policy. With this, we show that the main empirical results are robust to the conditioning information set. Because the results are largely unchanged, they are not reported.

Table 4: Proposition 1 – Small open economies

	Dependent variable:				
	Lending Interest Rate				
	(1)	(2)	(3)	(4)	(5)
grrgdppc	-0.076*	-0.063	-0.042*	-0.059*	-0.041
	(0.046)	(0.039)	(0.025)	(0.032)	(0.027)
m1g	18.940***	-2.939	1.440	11.956**	15.898***
	(5.696)	(3.581)	(3.546)	(5.655)	(5.869)
corpgdp	2.619***	-0.859***	-0.989***	-0.246	-0.646
	(0.642)	(0.289)	(0.313)	(0.546)	(0.567)
tenyr	3.811***	0.390	1.294	0.100	0.725
	(0.210)	(0.250)	(0.991)	(0.465)	(1.517)
ka_open	16.412***	-3.365	-2.798	-3.810	-4.204*
	(1.783)	(2.624)	(2.497)	(2.502)	(2.437)
ba	-0.025**	-0.080***	-0.044*	-0.037***	-0.033***
	(0.011)	(0.030)	(0.025)	(0.011)	(0.011)
corpgdp:tenyr	-0.805***	0.309***	0.467***	-0.129	0.064
	(0.195)	(0.099)	(0.104)	(0.202)	(0.203)
corpgdp:ka_open	-3.613**	2.016**	2.268**	0.373	1.250
	(1.754)	(0.797)	(0.922)	(1.854)	(1.906)
tenyr:ka_open	-3.511***	1.345***	1.445***	1.885**	2.096***
	(0.558)	(0.507)	(0.454)	(0.795)	(0.754)
corpgdp:tenyr:ka_open	0.888**	-0.617***	-0.878***	-0.015	-0.404
	(0.422)	(0.203)	(0.223)	(0.465)	(0.477)
Observations	738	738	738	738	738
R^2	0.766	0.947	0.953	0.801	0.811
Country Fixed-Effects	No	Yes	Yes	No	No
Region Fixed-Effects	No	No	No	Yes	Yes
Year Fixed-Effects	No	No	Yes	No	Yes

From the model specification we employ, we are interested in the marginal effect of (corpgdp). In particular, $\frac{\partial LIR}{\partial \text{corpgdp}} = \beta_4 + \beta_7 \times \text{tenyr} + \beta_9 \times \text{tenyr} \times \text{ka_open} + \beta_{10} \times \text{corpgdp} \times \text{ka_open}$ when ka_open = 0, such that the effect of public capital flows to the corporate sector of the U.S. depends on the world interest rate. Standard errors are robust to heteroskedasticity. *Note:* *p<0.1; **p<0.05; ***p<0.01.

account country-specific unobservables or year fixed-effects. Column (2) includes country fixed-effects while column (3) includes both country fixed-effects and year fixed-effects. Columns (4) and (5) follows the previous two columns but focus on region-fixed effects.

When controlling for country fixed effects, column (2) of Table 4 shows results are in line with the predictions of the model. Specifically, when the economy is financially closed such that there is no cross-border flow of private capital, the association between capital outflows of corporate debt and the cost of borrowing depends on the world interest rate. In particular, when the world interest rate is relatively high, there is a positive association between capital outflows and the cost of borrowing in financially closed developing economies. When including year fixed-effects along with country fixed-effects, column (3) shows results that confirm our findings.

We next examine the prediction that the relationship between the cost of borrowing in the large open low income country and the level of capital outflows depends on how much funds are initially being lent. In order to capture the non-linear relationship between the cost of borrowing and the level of official flows to the corporate sector of the advanced world as predicted by our model, we include the squared term (corpgdp2) in the regression and interact it with the degree of financial openness. Including this squared term is indicative of a quadratic relationship between the two variables. If the coefficient on this squared term is significant, this ensures that the relationship between the cost of borrowing in the large open low income country and the level of capital outflows depends on its initial value. Thus, we view this as an important control variable. In particular, the model specification is as follows:

$$LIR_{i,t} = \beta_1 \text{grrgdppc}_{i,t} + \beta_2 \text{m1g}_{i,t} + \beta_3 \text{corpgdp}_{i,t} + \beta_4 \text{corpgdp2}_{i,t} + \beta_5 \text{ka_open}_{i,t} + \beta_6 \text{ba}_{i,t} + \beta_7 \text{ka_open}_{i,t} * \text{corpgdp}_{i,t} + \varepsilon_{i,t}$$

Again, we are interested in the marginal effect of corpgdp when the economy is financially closed (ka_open $= 0$), such that $\frac{\partial LIR_{i,t}}{\partial \text{corpgdp}_{i,t}} = \beta_3 + 2\beta_4 \times \text{corpgdp}_{i,t}$. Thus, based on the predictions of our model, the coefficient β_3 should be positive while the coefficient β_4 should be negative. This is because an initial low demand for bonds of the corporate sector will mean low revenue for the government of the low income country. With more revenue, the government can give more funding to the high income country's corporate sector. As a result, its demand for funds from its domestic bond market increases, causing its debt issuance to increase. The increased issuance of debt raises demand for local funds, which causes the interest rate to be higher. However, for an initial high level of capital outflows, then the more funds supplied to the corporate sector of the high income country, the more interest payments it receives on those funds. Thus, it does not need to borrow much domestically which drives the real interest rate downwards. The results are presented in Table 5.

In the absence of introducing controls for country-specific unobservables, we see from column (1) of Table 5 that the level of capital outflows is robustly correlated to changes in the cost of borrowing when the economy is financially closed. Specifically, for an initial low level of capital outflows, the cost of borrowing is positively correlated with capital outflows. However,

Table 5: Proposition 6 – Large open economies

	Dependent variable:				
	Lending Interest Rate				
	(1)	(2)	(3)	(4)	(5)
grrgdppc	-0.105	-0.215	-0.335	0.286**	0.373
	(0.338)	(0.166)	(0.320)	(0.123)	(0.263)
m1g	17.583	-13.226	-14.900	-33.640**	-34.675
	(29.775)	(9.717)	(16.531)	(15.136)	(21.361)
corpgdp	14.526***	2.213*	-1.909	5.363**	1.344
	(3.817)	(1.238)	(2.497)	(2.378)	(2.517)
corpgdp2	-5.849***	-1.054*	0.398	-2.615**	-0.831
	(1.788)	(0.570)	(1.277)	(1.155)	(1.454)
ka_open	1.736	-1.660	1.180	-17.887***	-16.617***
	(3.213)	(5.047)	(7.314)	(3.316)	(4.918)
ba	0.009	-0.060**	-0.028	0.017	0.132
	(0.018)	(0.026)	(0.050)	(0.025)	(0.095)
corpgdp2:ka_open	1.571	0.984**	0.425	1.903**	0.186
	(1.601)	(0.482)	(1.042)	(0.764)	(1.651)
Observations	46	46	46	46	46
R^2	0.824	0.978	0.989	0.953	0.979
Country Fixed-Effects	No	Yes	Yes	No	No
Region Fixed-Effects	No	No	No	Yes	Yes
Year Fixed-Effects	No	No	Yes	No	Yes

From the model specification we employ, we are interested in the marginal effect of (corpgdp). In particular, $\frac{\partial LIR}{\partial \text{corpgdp}} = \beta_3 + 2\beta_4 \times \text{corpgdp} + 2\beta_7 \times \text{corpgdp} \times \text{ka_open}$ when ka_open = 0, such that the effect of public capital flows to the corporate sector of the U.S. depends on its initial value. Standard errors are robust to heteroskedasticity. *Note:* *p<0.1; **p<0.05; ***p<0.01.

as capital outflows become particularly high, the correlation between the cost of borrowing and capital outflows is negative. Columns (2) and (4) which control for country fixed effects and region fixed effects, respectively, are also consistent with our findings.[46]

Perhaps the most important prediction emerging from our modeling framework concerns how the relationship between the domestic real interest rate and the money growth rate depends on the level of capital outflows to the corporate sector in advanced countries. In particular, for small open low income economies, the effects of monetary policy on the domestic interest rates would be weaker as capital outflows increase. This is because at higher capital outflows, more of the government's revenues derive from lending to the rest of the world. Thus, seigniorage plays a smaller role in crowding out capital formation.

To test this prediction, we include an interactive term for the foreign corporate debt holdings and the rate of money growth with the degree of financial openness. Thus, the model specification is:

$$LIR_{i,t} = \beta_1 \text{grrgdppc}_{i,t} + \beta_2 \text{m1g}_{i,t} + \beta_3 \text{corpgdp}_{i,t} + \beta_4 \text{ka_open}_{i,t} + \beta_5 \text{ba}_{i,t} + \beta_6 \text{m1g}_{i,t} * \text{corpgdp}_{i,t} + \beta_7 \text{m1g}_{i,t} * \text{ka_open}_{i,t} + \beta_8 \text{corpgdp}_{i,t} * \text{ka_open}_{i,t} + \beta_9 \text{m1g}_{i,t} * \text{corpgdp}_{i,t} * \text{ka_open}_{i,t} + \varepsilon_{i,t}$$

We are interested in how the correlation between the cost of borrowing and the rate of money growth depend on foreign holdings of corporate debt when these economies are financially closed (ka_open $= 0$) such that $\frac{\partial LIR_{i,t}}{\partial \text{m1g}_{i,t}} = \beta_2 + \beta_6 \times \text{corpgdp}_{i,t}$. Thus, based upon the predictions of our model, the coefficient β_2 should be positive while the coefficient β_6 should be negative.

The results are in Table 6. In the absence of controlling for country and year fixed effects, column (1) of Table 6 supports our predictions about the efficacy of money policy in small open economies. In particular, the relationship between the cost of borrowing and monetary policy is weakened as capital flows to the corporate sector of advanced countries increase.

For large open low income economies, the effects of money growth should be weaker for low levels of capital outflows and stronger as the level of capital outflows become relatively large. This is because at low levels of capital outflows, the government will seek to raise more funds locally to lend internationally. By doing so, the government raises its debt issuance, thus driving up the cost of borrowing. As a result, monetary policy has a weaker impact on the cost of borrowing. On the other hand, the effect of monetary policy on the cost of borrowing will become stronger when capital outflows are relatively high. This occurs because as the government increases its capital outflows, it reduces its use of the bank lending channel to raise funds.

To test this prediction, our specification follows Table 6, but with the sample for large open economies. However, in this case, our model predicts that the coefficient β_2 should be negative

[46] It should be noted that the sample size here is relatively small, as there are only 3 countries in sample that held U.S. corporate debt securities above the median holdings of high income countries every year between 2002 and 2018 (China, Colombia, and Mexico). In the *Online Supplementary Materials*, we relax the specification of "large" developing countries to include any that held more than the median of high income countries at ***any*** point in time between 2002 and 2018. In this classification, we find that the relationship between the cost of borrowing in the large open low income country and the level of capital outflows depends on its initial value, particularly when controlling for country specific unobservables.

Table 6: Small open economies, public flows, and the efficacy of monetary policy.

	Dependent variable:				
	Lending Interest Rate				
	(1)	(2)	(3)	(4)	(5)
grrgdppc	0.050	-0.069*	-0.043	-0.053*	-0.047*
	(0.048)	(0.037)	(0.028)	(0.031)	(0.029)
m1g	114.508***	-6.396	-6.037	11.618	11.038
	(10.974)	(6.227)	(5.967)	(9.940)	(9.723)
corpgdp	2.473***	-0.121	0.327*	-0.594**	-0.414
	(0.481)	(0.181)	(0.191)	(0.264)	(0.281)
ka_open	18.009***	1.304	0.664	1.426	1.340
	(1.187)	(1.898)	(1.788)	(1.277)	(1.284)
ba	0.034***	-0.133***	-0.037	-0.040***	-0.032***
	(0.012)	(0.027)	(0.025)	(0.011)	(0.011)
m1g:corpgdp	-29.711***	0.775	2.215	-2.604	-0.409
	(8.031)	(2.616)	(2.351)	(4.146)	(4.100)
m1g:ka_open	-129.286***	15.541	19.097*	20.089	24.296
	(21.219)	(12.298)	(10.557)	(19.086)	(18.438)
corpgdp:ka_open	-4.850***	0.165	-0.524	0.470	0.071
	(0.925)	(0.314)	(0.319)	(0.454)	(0.482)
m1g:corpgdp:ka_open	51.381***	-2.341	-5.303	-1.078	-6.700
	(17.278)	(5.372)	(4.755)	(8.315)	(8.176)
Observations	738	738	738	738	738
R^2	0.705	0.944	0.953	0.798	0.811
Country Fixed-Effects	No	Yes	Yes	No	No
Region Fixed-Effects	No	No	No	Yes	Yes
Year Fixed-Effects	No	No	Yes	No	Yes

From the model specification we employ, we are interested in the marginal effect of (m1g). In particular, $\frac{\partial LIR}{\partial \text{m1g}} = \beta_2 + \beta_6 \times \text{corpgdp} + \beta_7 \times \text{ka_open} + \beta_9 \times \text{m1g} \times \text{ka_open}$ when ka_open = 0, such that the effect of money growth depends on public capital flows to the corporate sector of the U.S. Standard errors are robust to heteroskedasticity. *Note:* *p<0.1; **p<0.05; ***p<0.01.

while the coefficient β_6 should be positive. The results are in Table 7. Upon controlling for region and year fixed-effects, column (5) of Table 7 supports our predictions about the efficacy of money policy in large open economies. In particular, the relationship between the cost of borrowing and monetary policy is stronger as capital flows to advanced countries increase. Starting from a point with low levels of capital outflows, the effects of monetary policy are weaker if the government lends more. However, once capital outflows eclipse a certain point, higher capital outflows lead to a stronger impact from monetary policy because global interest rates would be lower if the government aggressively lends to the advanced country.

7 Conclusion

For developing countries, public sector lending to other countries has been higher than any other sector, while the corporate sector in advanced countries issued the largest share of portfolio debt to other countries of any other sector pre-crisis. What are the implications of these events for the global financial system and overall economic activity? Did it have an impact on the transmission channels of monetary policy across the different stages of development? We investigate these important issues using a micro-founded model of money and banking based on the level of financial development so that the effects of monetary policy across countries can be meaningfully studied. Notably, at high global interest rates, the capital drain away from the poor country adversely affects domestic investment. Moreover, the increase in capital inflows to the advanced economy renders monetary policy in advanced countries to be less effective. Yet, our results indicate that it is indeed optimal for lower income countries to lend to the advanced world. Nevertheless, we do find that the optimal amount of lending to advanced countries critically depends on the level of financial development – if it deteriorates sufficiently, then public sector lending to advanced economies is not warranted. Consequently, our results indicate that governments in developing countries should carefully consider how much capital they send abroad to foreign countries.

Table 7: Large open economies, public flows, and the effects of monetary policy.

	Dependent variable:				
	Lending Interest Rate				
	(1)	(2)	(3)	(4)	(5)
grrgdppc	-0.039	-0.163	-0.130	0.367**	0.477*
	(0.196)	(0.168)	(0.272)	(0.187)	(0.256)
m1g	276.258***	11.724	-35.269	37.910	-101.148*
	(37.694)	(22.954)	(41.359)	(26.208)	(57.086)
corpgdp	25.034**	-1.778	-10.033**	-1.963	-18.098***
	(9.969)	(2.870)	(4.112)	(3.631)	(5.633)
ka_open	43.407***	3.522	-4.480	-2.900	-19.932**
	(8.686)	(7.030)	(8.276)	(10.063)	(8.804)
ba	-0.061***	-0.066***	-0.019	-0.003	0.130*
	(0.015)	(0.024)	(0.051)	(0.019)	(0.078)
m1g:corpgdp	-417.975***	19.123	128.997*	17.465	275.191***
	(141.956)	(49.460)	(71.499)	(67.432)	(100.410)
m1g:ka_open	-1,028.333***	-143.511	-19.289	-328.802**	24.459
	(172.930)	(109.405)	(137.572)	(144.072)	(155.886)
corpgdp:ka_open	-62.630***	1.328	17.560*	0.331	28.533**
	(14.265)	(6.933)	(10.408)	(9.968)	(12.955)
m1g:corpgdp:ka_open	1,216.701***	37.429	-251.120	80.562	-480.746*
	(264.865)	(142.155)	(213.183)	(211.471)	(252.866)
Observations	46	46	46	46	46
R^2	0.908	0.980	0.991	0.958	0.985
Country Fixed-Effects	No	Yes	Yes	No	No
Region Fixed-Effects	No	No	No	Yes	Yes
Year Fixed-Effects	No	No	Yes	No	Yes

From the model specification we employ, we are interested in the marginal effect of (m1g). In particular, $\frac{\partial LIR}{\partial \text{m1g}} = \beta_2 + \beta_6 \times \text{corpgdp} + \beta_7 \times \text{ka_open} + \beta_9 \times \text{m1g} \times \text{ka_open}$ when ka_open = 0, such that the effect of money growth depends on public capital flows to the corporate sector of the U.S. Standard errors are robust to heteroskedasticity. *Note:* *p<0.1; **p<0.05; ***p<0.01.

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Appendix: Derivations (Not Intended for Publication)

1. Analysis of the low income, small open economy

We begin with the bank's problem:

$$\max_{r_t^m, r_t^n, m_t, b_t^d, i_t} \pi ln(r_t^m w_t) + (1-\pi) ln\left(r_t^n w_t\right)$$

subject to

$$m_t + b_t^d + i_t \leq w_t.$$

$$\pi r_t^m w_t \leq m_t \left(\frac{P_t}{P_{t+1}}\right)$$

$$(1-\pi) r_t^n w_t \leq R_t b_t^d + r_{t+1} i_t$$

It will prove convenient to transform the constraints as follows. Let $\gamma_t \equiv \frac{m_t}{w_t}$ be the reserve to deposit ratio of the bank at time t, and let $\beta_t \equiv \frac{b_t^d}{w_t}$ be the ratio of bonds to deposits. This implies that $1-\gamma_t-\beta_t$ is the ratio of capital investment to deposits. With these transformations, the bank's problem can be rewritten as:

$$\max_{\gamma_t,\beta_t} \pi ln\left\{\frac{\gamma_t}{\pi}\left(\frac{P_t}{P_{t+1}}\right)w_t\right\} + (1-\pi)ln\left\{\left(\frac{r_{t+1}[1-\gamma_t-\beta_t]+R_t\beta_t}{1-\pi}\right)w_t\right\}$$

The first-order conditions for each type of asset, respectively:

$$\frac{\pi}{\gamma_t} - \frac{(1-\pi)r_{t+1}}{r_{t+1}[1-\gamma_t-\beta_t]+R_t\beta_t} = 0$$

$$r_{t+1} = R_t$$

$$\pi = \gamma_t \implies m_t = \pi w_t$$

The money market clears when:

$$m = \pi w(k)$$

The rate of money growth is equal to the inflation rate in the steady-state:

$$\sigma = \frac{P_{t+1}}{P_t}$$

From the inverse demand for capital, (3), we get the following expression for k :

$$k = \left[\frac{\alpha A}{r}\right]^{\frac{1}{1-\alpha}}$$

From the bank's balance sheet condition:

$$b^d = (1-\alpha)(1-\pi)A\left[\frac{\alpha A}{r}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{\alpha A}{r}\right]^{\frac{1}{1-\alpha}}$$

The government budget constraint for the low income country is satisfied when:

$$\hat{b}^d + Rb^s = \frac{(\sigma-1)m}{\sigma} + r^*\hat{b}^d + b^s$$

We substitute b^d for b^s, which occurs when the low income country's bond market clears. Also, by the no-arbitrage condition:

$$(R-1)\left\{(1-\alpha)(1-\pi)A\left[\frac{\alpha A}{R}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{\alpha A}{R}\right]^{\frac{1}{1-\alpha}}\right\} + \hat{b}^d = \frac{(\sigma-1)\pi(1-\alpha)A}{\sigma}\left[\frac{\alpha A}{R}\right]^{\frac{\alpha}{1-\alpha}} + r^*\hat{b}^d$$

Let $\alpha = 0.5$, such that:

$$(R-1)\left\{(1-0.5)(1-\pi)A\left[\frac{0.5A}{R}\right]^{\frac{0.5}{1-0.5}} - \left[\frac{0.5A}{R}\right]^{\frac{1}{1-0.5}}\right\} + \hat{b}^d = \frac{(\sigma-1)\pi(1-0.5)A}{\sigma}\left[\frac{0.5A}{R}\right]^{\frac{0.5}{1-0.5}} + r^*\hat{b}^d$$

After a few lines of algebra:

$$\left[0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right]R^2 - \left\{\frac{0.25A^2[(2-\pi)\sigma + (\sigma-1)\pi]}{\sigma}\right\}R + 0.25A^2 = 0$$

The solutions to this quadratic equation are:

$$R^{-*}(r^*) = \frac{\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\} - \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\}}}{2\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\}}$$

and

$$R^{+*}(r^*) = \frac{\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\} + \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\}}}{2\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\}}$$

Proof. [**Proof of Lemma 1.**] First, we need to show that $\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\} > 0$.

$$\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\} > 0$$

iff:

$$1 + \frac{0.25A^2(1-\pi)}{\hat{b}^d} > r^*$$

Next, we need to show that the discriminant is positive:

$$\left\{-\frac{0.25A^2[(2-\pi)\sigma + (\sigma-1)\pi]}{\sigma}\right\}^2 > A^2\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\}$$

$$r^* > 1 + \frac{0.25A^2(1-\pi)}{\hat{b}^d} - \left\{\frac{0.25[(2-\pi)\sigma + (\sigma-1)]}{\sigma}\right\}^2\frac{A^2}{\hat{b}^d}$$

Now, to ensure that the two solutions are positive, we need to show that the numerator of R^{-*} is positive. iff:

$$\left\{\frac{0.25A^2[(2-\pi)\sigma + (\sigma-1)\pi]}{\sigma}\right\} > \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma + (\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{0.25A^2(1-\pi) - \hat{b}^d(r^*-1)\right\}}$$

which reduces to:

$$A^2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}>0$$

since $1+\frac{0.25A^2(1-\pi)}{\hat{b}^d}>r^*$. □

Proof. [Nominal Interest Rate] Though both real interest rates are positive, it is possible that the nominal interest rate (I) at R^- is less than 1. If that is the case, we can rule out R^- as an equilibrium interest rate:

Recall that:

$$I=\sigma R$$

At R^-, $I^-<1$ iff:

$$\sigma R^-<1$$

$$\frac{\sigma\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}-\sigma\sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2-A^2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}}}{2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}}<1$$

$$r^*>1+\frac{A^2}{4\hat{b}^s}[\sigma^2+1-2\sigma]$$

If we can show that $1+\frac{A^2}{4\hat{b}^d}[\sigma^2+1-2\sigma]<1+\frac{0.25A^2(1-\pi)}{\hat{b}^d}$, then we prove that $0<I^-<1$.

$$\cancel{1}+\frac{A^2}{4\hat{b}^d}[\sigma^2+1-2\sigma]<\cancel{1}+\frac{0.25A^2(1-\pi)}{\hat{b}^d}$$

$$\pi<\sigma[2-\sigma]$$

□

Proof. [**Proof of Proposition 1.**] This requires differentiation of the equilibrium real interest rate with respect to the demand for funds from overseas by the government of the low income country.

$$\frac{\partial R}{\partial \hat{b}^d} \implies \frac{\frac{1}{2}\left(\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}\right)^{-\frac{1}{2}}[A^2(r^*-1)]}{2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}} +$$

$$\frac{\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\} + \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}} \times [2(r^*-1)]}{\left[2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}\right]^2}$$

$$\lessgtr 0$$

$\frac{\partial R}{\partial \hat{b}^d} > 0$ iff $r^* - 1 > 0$ and $\frac{\partial R}{\partial \hat{b}^d} < 0$ iff $r^* - 1 < 0$. □

Proof. [**Proof of Proposition 2.**] This requires a differentiation of the equilibrium real interest rate with respect to the money growth rate in the low income country.

$$\frac{\partial R}{\partial \sigma} \implies \frac{0.25A^2\left(\frac{\pi}{\sigma^2}\right)}{2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}} +$$

$$\frac{\left(\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}\right)^{-\frac{1}{2}}\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}0.25A^2\left(\frac{\pi}{\sigma^2}\right)}{2\left\{0.25A^2(1-\pi)-\hat{b}^d(r^*-1)\right\}}$$

$$> 0.$$

□

2. Analysis of the High Income Closed Economy

First, we turn to the entrepreneur's problem:

$$\max_{q_t} zlog(q_t) - \tilde{R}_t\tilde{b}_t^s$$

subject to

$$\tilde{b}_t^s = q_t - \tilde{w}_t - \tau_t$$

$$\max_{q_t} zlog(q_t) - \tilde{R}_t\left[q_t - \tilde{w}_t - \tau_t\right]$$

Therefore,

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - \tilde{w}_t - \tau_t.$$

The bank's problem in the high income country is similar to the problem in the developing country:

$$\max_{\tilde{\gamma}_t,\tilde{\beta}_t,\hat{\beta}_t} \tilde{\pi} ln \left\{ \frac{\tilde{\gamma}_t}{\tilde{\pi}} \left(\frac{\tilde{P}_t}{\tilde{P}_{t+1}} \right) [\tilde{w}_t + \tau_t] \right\} + (1-\tilde{\pi}) ln \left\{ \left(\frac{\tilde{r}_{t+1}[1-\tilde{\gamma}_t - \tilde{\beta}_t] + \tilde{R}_t \tilde{\beta}_t}{1-\tilde{\pi}} \right) [\tilde{w}_t + \tau_t] \right\}$$

$$\tilde{\gamma}_t : \frac{\tilde{\pi}}{\tilde{\gamma}_t} - \frac{(1-\tilde{\pi})\tilde{r}_{t+1}}{\tilde{r}_{t+1}[1-\tilde{\gamma}_t - \tilde{\beta}_t] + \tilde{R}_t \tilde{\beta}_t} = 0$$

Optimal Risk-Sharing Condition:

$$\frac{\tilde{\pi}}{\tilde{\gamma}_t} = \frac{(1-\tilde{\pi})\tilde{r}_{t+1}}{\tilde{r}_{t+1}[1-\tilde{\gamma}_t - \tilde{\beta}_t] + \tilde{R}_t \tilde{\beta}_t}$$

$$\tilde{\beta}_t : \frac{(1-\tilde{\pi})\left[-\tilde{r}_{t+1} + \tilde{R}_t\right]}{\tilde{r}_{t+1}[1-\tilde{\gamma}_t - \tilde{\beta}_t] + \tilde{R}_t \tilde{\beta}_t} = 0$$

$$\tilde{r}_{t+1} = \tilde{R}_t$$

No-Arbitrage Condition between physical capital and debt issued by entrepreneurs:

$$\tilde{r}_{t+1} = \tilde{R}_t$$

So,

$$\tilde{\pi} = \tilde{\gamma}_t \implies \tilde{m}_t = \tilde{\pi}[\tilde{w}_t + \tau_t]$$

Let $\tilde{D}_t$ be the total per capita deposits in the high income country. Recall that the total deposits in the high income country are equal to the sum of wage income and seigniorage revenue:

$$\tilde{D}_t = \tilde{w}_t + \tau_t$$

$$\tilde{D}_t = \tilde{w}_t + \left(\frac{\tilde{\sigma} - 1}{\tilde{\sigma}} \right) \tilde{m}_t$$

$$\tilde{D}_t = \frac{(1-\tilde{\alpha})\tilde{A}\tilde{k}_t^{\tilde{\alpha}}}{1 - \left(\frac{\tilde{\sigma} - 1}{\tilde{\sigma}} \right) \tilde{\pi}}$$

From the inverse demand for capital, (19), we get the following expression for capital:

$$\tilde{k}_t = \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{r}_t} \right]^{\frac{1}{1-\tilde{\alpha}}}$$

$$\tilde{D}_t = \frac{(1-\tilde{\alpha})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{r}_t}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}$$

The issuance of bonds by entrepreneurs comes from their demand for investment net of funding from the banks in the high income country. This is expressed as:

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - \tilde{w}_t - \tau_t$$

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - [\tilde{w}_t + \tau_t]$$

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - \tilde{D}_t$$

$$\tilde{b}_t^s = \frac{z}{\tilde{R}_t} - \frac{(1-\tilde{\alpha})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{r}_t}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}$$

Imposing steady state in the high income country:

$$\tilde{r} = \tilde{R}$$

So,

$$\tilde{b}^s = \frac{z}{\tilde{R}} - \frac{(1-\tilde{\alpha})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}$$

Recall the bank balance sheet:

$$\tilde{w} + \tau = \tilde{m} + \tilde{b}^d + \tilde{i}$$

$$\tilde{b}^d = \frac{(1-\tilde{\alpha})(1-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-\tilde{\alpha}}}$$

Proof. [**Proof of Lemma 2.**] In autarky, the bond market clearing condition ensures that the supply of bonds coming from both the entrepreneurs in the high income country is equal to the demand for bonds, which comes from banks in the high income country.

$$\tilde{b}^s = \tilde{b}^d$$

$$\frac{z}{\tilde{R}} - \frac{(1-\tilde{\alpha})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} = \frac{(1-\tilde{\alpha})(1-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-\tilde{\alpha}}}$$

Now, let $\tilde{\alpha} = 0.5$:

$$\frac{z}{\tilde{R}} = \frac{(1-0.5)(2-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{0.5\tilde{A}}{\tilde{R}}\right]^{\frac{0.5}{1-0.5}} - \left[\frac{0.5\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-0.5}}$$

$$\tilde{R}^{aut*} = \frac{0.25\tilde{A}^2\left[1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right]}{0.25(2-\tilde{\pi})\tilde{A}^2 - z\left[1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right]}$$

□

3. Analysis of the High Income, Small Open Economy

Proof. [**Proof of Lemma 3.**] The global bond market clearing condition ensures that the supply of bonds coming from entrepreneurs in the high income country is equal to the total demand for bonds, coming from both the banks in the high income country and the government of the low income country at the world interest rate r^*.

$$\hat{b}^s = \tilde{b}^s - \tilde{b}^d = \hat{b}^d$$

$$\hat{b}^s = \frac{z}{r^*} - \frac{(1-\tilde{\alpha})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \left\{\frac{(1-\tilde{\alpha})(1-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{1}{1-\tilde{\alpha}}}\right\}$$

$$\hat{b}^s = \frac{z}{r^*} - \frac{(1-\tilde{\alpha})(2-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} + \left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{1}{1-\tilde{\alpha}}}$$

□

Proof. [**Proof of Proposition 3.**] This requires a differentiation of the amount of locally issued bonds by entrepreneurs and the amount of debt demanded by banks with respect to the world real interest rate.

$$\frac{\partial\tilde{b}^s}{\partial r^*} \implies -\frac{z}{r^{*2}} + \frac{\tilde{\alpha}}{\cancel{1-\tilde{\alpha}}}\frac{\cancel{(1-\tilde{\alpha})}\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}-1}\left(\frac{\tilde{\alpha}\tilde{A}}{r^{*2}}\right)$$

$$\implies \frac{1}{r^{*2}}\left[\frac{\tilde{\alpha}\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{2\tilde{\alpha}-1}{1-\tilde{\alpha}}}-z\right]$$

Evaluated at $\tilde{\alpha}=0.5$:

$$\implies \frac{1}{r^{*2}}\left[\frac{0.25\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-z\right]<0.$$

since $z>\dfrac{0.25\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}$.

Thus, if the world real interest rate increases, entrepreneurs issue less debt.

$$\frac{\partial\tilde{b}^d}{\partial r^*}\implies -\frac{\tilde{\alpha}}{\cancel{1-\tilde{\alpha}}}\frac{\cancel{(1-\tilde{\alpha})}(1-\tilde{\pi})\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}-1}\left(\frac{\tilde{\alpha}\tilde{A}}{r^{*2}}\right)+\frac{1}{1-\tilde{\alpha}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}\left(\frac{\tilde{\alpha}\tilde{A}}{r^{*2}}\right)$$

$$\implies \frac{1}{r^{*2}}\left[-\frac{(\tilde{\alpha}\tilde{A})^2(1-\tilde{\pi})}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{2\tilde{\alpha}-1}{1-\tilde{\alpha}}}+\frac{\tilde{\alpha}\tilde{A}}{1-\tilde{\alpha}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}\right]$$

Evaluated at $\tilde{\alpha}=0.5$:

$$\implies \frac{1}{r^{*2}}\left[\frac{0.5\tilde{A}^2}{r^*}-\frac{0.25(1-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right]>0$$

since $\dfrac{0.5\tilde{A}^2}{r^*}>\dfrac{0.25\tilde{A}^2}{r^*}>\dfrac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-z \quad \forall\ z.$

Therefore, banks will demand more debt from entrepreneurs at a higher world real interest rate.

□

Proof. [**Proof of Proposition 4.**] This requires differentiation of the amount of locally issued bonds by entrepreneurs and the amount of debt demanded by banks with respect to the rate of money growth in the high income country.

$$\frac{\partial\tilde{b}^s}{\partial\tilde{\sigma}}\implies -\frac{(1-\tilde{\alpha})\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}^2}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}\left(\frac{\tilde{\pi}}{\tilde{\sigma}}\right)<0$$

Entrepreneurs borrow less from their local banks as they receive an increase in the transfer of seigniorage revenue when the money growth rate increases.

$$\frac{\partial \tilde{b}^d}{\partial \tilde{\sigma}} \implies \frac{(1-\tilde{\alpha})(1-\tilde{\pi})\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}^2}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}\left(\frac{\tilde{\pi}}{\tilde{\sigma}}\right) > 0.$$

Banks will have larger deposit base as workers deposit their increased transfer of seigniorage revenue in the bank. Thus, banks will demand more bonds from entrepreneurs. Entrepreneurs will borrow less internationally $\left(\frac{\partial \hat{b}^s}{\partial \tilde{\sigma}} < 0\right)$. □

$$\frac{\partial^2 \tilde{b}^s}{\partial \tilde{\sigma} \partial r^*} \implies \frac{\tilde{\alpha}}{\cancel{1-\tilde{\alpha}}}\frac{\cancel{(1-\tilde{\alpha})}(2-\tilde{\pi})\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}-1}\left(\frac{\tilde{\alpha}\tilde{A}}{r^{*2}}\right)\left(\frac{\tilde{\pi}}{\tilde{\sigma}}\right) > 0.$$

$$\frac{\partial^2 \tilde{b}^d}{\partial \tilde{\sigma} \partial r^*} \implies -\frac{\tilde{\alpha}}{\cancel{1-\tilde{\alpha}}}\frac{\cancel{(1-\tilde{\alpha})}(2-\tilde{\pi})\tilde{A}}{\left\{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}\right\}}\left[\frac{\tilde{\alpha}\tilde{A}}{r^*}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}-1}\left(\frac{\tilde{\alpha}\tilde{A}}{r^{*2}}\right)\left(\frac{\tilde{\pi}}{\tilde{\sigma}}\right) < 0.$$

The impact of the money growth rate on the amount of bonds issued by entrepreneurs is stronger if global interest rates are higher but its impact is weaker on debt sought by banks as global interest rates rise.

4. Analysis of Two Large Open Economies

Proof. [**Proof of Lemma 4.**] Following from the proof of Lemma 2, replace r^* with $\tilde{R}$ since we are no longer in the small open economy setting. Additionally, we replace $\hat{b}^s$ with $\hat{b}^d$ since both are equal in equilibrium.

$$\hat{b}^d = \frac{z}{\tilde{R}} - \frac{(1-\tilde{\alpha})(2-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} + \left[\frac{\tilde{\alpha}\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-\tilde{\alpha}}}$$

Now, let $\tilde{\alpha} = 0.5$ such that:

$$\hat{b}^d = \frac{z}{\tilde{R}} - \frac{(1-0.5)(2-\tilde{\pi})\tilde{A}}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\left[\frac{0.5\tilde{A}}{\tilde{R}}\right]^{\frac{0.5}{1-0.5}} + \left[\frac{0.5\tilde{A}}{\tilde{R}}\right]^{\frac{1}{1-0.5}}$$

$$\tilde{R}^2\hat{b}^d - \tilde{R}\left[z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right] - 0.25\tilde{A}^2 = 0$$

First, we show that $z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} < 0.$

$$z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} < 0$$

iff:

$$z < \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}$$

Then the solution to this quadratic equation is:

$$\tilde{R} = \frac{\left\{z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right\} + \sqrt{\left\{-\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}^d}}{2\hat{b}^d}$$

Now, to ensure that only one solution is positive, we need to show that the second term in the numerator is larger than the first in absolute terms.

$$-\left\{z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right\} < \sqrt{\left\{-\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}^d}$$

$$0 < \tilde{A}^2\hat{b}^d$$

□

Now, we show the condition necessary for entrepreneurs to borrow. With $\tilde{\alpha} = 0.5$,

$$\tilde{b}^s = \frac{z}{\tilde{R}} - \frac{0.25\tilde{A}^2}{\tilde{R}}\left[\frac{\tilde{\sigma}}{\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}}\right]$$

$$\frac{z}{\tilde{R}} - \frac{0.25\tilde{A}^2}{\tilde{R}}\left[\frac{\tilde{\sigma}}{\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}}\right] > 0$$

iff:

$$z > \frac{0.25\tilde{A}^2\tilde{\sigma}}{\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}}$$

It is easy to see that Lemma 4 simultaneously holds with the entrepreneurial borrowing condition, since $\exists\ z$ such that:

$$\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} > \frac{0.25\tilde{A}^2\tilde{\sigma}}{\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}}$$

We next show the condition necessary for entrepreneurs to have positive profits.

$$z \log\left(\frac{z}{\tilde{R}}\right) > \tilde{R}\left[\frac{z}{\tilde{R}} - \frac{0.25\tilde{A}^2}{\tilde{R}}\left[\frac{\tilde{\sigma}}{\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}}\right]\right]$$

Let $\phi \equiv e^{1-\frac{0.25\tilde{A}^2\tilde{\sigma}}{z[\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}]}}$ such that:

$$z > \tilde{R}\phi$$

Also, let $\chi \equiv \frac{0.25\tilde{A}^2\tilde{\sigma}(2-\tilde{\pi})}{\tilde{\sigma}(1-\tilde{\pi})+\tilde{\pi}} = \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}$.

$$z > \frac{\phi\{z-\chi\} + \phi\sqrt{\{-(z-\chi)\}^2 + \tilde{A}^2\hat{b}^d}}{2\hat{b}^d}$$

$$\hat{b}^d > \frac{\phi^2\tilde{A}^2}{4z^2} - \frac{\phi\chi}{z} + 1$$

Entrepreneurs will produce as long as $z > \tilde{R}$. This holds since $z > \tilde{R}\phi$ and $\phi > 1$. $\phi > 1$ because it is the exponential of the entrepreneurial borrowing condition normalized by z. Since the borrowing condition is positive, then $\phi > 1$. Therefore, entrepreneurs will produce, borrow, and have positive profits at $\tilde{R}$ as long as the above condition holds.

Proof. [**Proof of Lemma 5.**] Here, we solve for the real interest rate in the low income country and find conditions to ensure it holds.

$$R^2\left[0.25A^2(1-\pi) - \left(\frac{z - 2\hat{b}_0^d - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} + \sqrt{\left\{-\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)\right]$$
$$- R\left\{\frac{0.25A^2[(2-\pi)\sigma + (\sigma-1)\pi]}{\sigma}\right\} + 0.25A^2 = 0$$

The solutions are:

$$R^{-*} = \frac{\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\} - \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)\right\}}}{2\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)}$$

and

$$R^{+*} = \frac{\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\} + \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)\right\}}}{2\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)}$$

First, we show that the discriminant is positive.

$$\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 > A^2\left\{\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)\right\}$$

$$\sigma > \frac{\pi}{2 - \frac{4}{A}\sqrt{\left\{\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right)\right\}}}$$

Now, to ensure that the two solutions are positive, we just need to show that the numerator of R^- is positive.

$$\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\} > \sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z - \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right\}}$$

Square both sides:

$$\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 >$$

$$\left(\sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2 - A^2\left\{\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right\}}\right)^2$$

$$A^2\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d - z + \frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}}{2}\right) > 0$$

since $z < 0.5A^2(1-\pi) + 2\hat{b}_0^d + \dfrac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{\left\{\left(z-\dfrac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}$.

It is easy to show the above expression holds since it does not violate Lemma 4. For it to not violate Lemma 4, conditions on z are such that:

$$\left(\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2 + \tilde{A}^2\hat{b}_0^d}\right)^2 < \left(0.5A^2(1-\pi)+2\hat{b}_0^d\right)^2$$

$$\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}} - \sqrt{A^2(1-\pi)\left\{0.25A^2(1-\pi)+2\hat{b}_0^d\right\} + \hat{b}_0^d\left\{4\hat{b}_0^d - \tilde{A}^2\right\}} < z$$

The second term on the left hand side is greater than zero iff: $\tilde{A} < (1-\pi)\left\{\frac{0.25A^2}{\hat{b}_0^d}A^2(1-\pi)+2\right\} + 4\hat{b}_0^d$. □

Proof. [Nominal Interest Rate] Similar to the case in Lemma 1, we can show that though both real interest rates are positive, the nominal interest rate (I) at R^- is less than 1. If that is the case, we can rule out R^- as an equilibrium interest rate. Recall that:

$$I = \sigma R$$

At R^-, $I < 1$ iff:

$$\sigma R^- < 1$$

$$\frac{\sigma\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}-\sigma\sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2-A^2\left\{\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d-z+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}}{2}\right)\right\}}}{2\left(\frac{0.5A^2(1-\pi)+2\hat{b}_0^d-z+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}}{2}\right)}<$$

Square both sides:

$$\left[\sigma\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}-\left(0.5A^2(1-\pi)+2\hat{b}_0^d-z+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}\right)\right]^2<$$
$$\left[\sigma\sqrt{\left\{-\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}^2-\frac{A^2}{2}\left\{\left(0.5A^2(1-\pi)+2\hat{b}_0^d-z+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}\right)\right\}}\right]^2$$

$$0.5A^2(1-\pi)+2\hat{b}_0^d+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}-2\sigma\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}+\frac{\sigma^2A^2}{2}<z$$

If we can show that

$$0.5A^2(1-\pi)+2\hat{b}_0^d+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}-2\sigma\left\{\frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}\right\}+\frac{\sigma^2A^2}{2}<$$
$$0.5A^2(1-\pi)+2\hat{b}_0^d+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d},$$

Then, $I<1$ at R^-if:

$$\pi<\sigma[2-\sigma]$$

□

Proof. [**Proof of Proposition 5**.] This requires a differentiation of the world equilibrium real

interest rate with respect to the money growth rate in the high income country.

$$\frac{\partial \tilde{R}}{\partial \tilde{\sigma}} \implies -\left(\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\left(\frac{\tilde{\pi}}{\tilde{\sigma}^2}\right) -$$
$$\frac{1}{2}\left(\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}^d\right)^{-\frac{1}{2}} 2\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}$$
$$\times\left(\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\left(\frac{\tilde{\pi}}{\tilde{\sigma}^2}\right)<0$$

□

Proof. [**Proof of Proposition 6.**] This requires a differentiation of the equilibrium real interest rate in the low income country with respect to the demand for international debt by the government of the low income country. First, let $\Omega \equiv \frac{0.25A^2[(2-\pi)\sigma+(\sigma-1)\pi]}{\sigma}$, and

$$\Xi \equiv 0.5A^2(1-\pi)+2\hat{b}_0^d-z+\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}-\sqrt{\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d}$$

$$\frac{\partial R}{\partial \hat{b}_0^d} \implies \frac{\Omega}{\Xi^2}\left(\frac{\tilde{A}^2}{2}\left[\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d\right]^{-\frac{1}{2}}-2\right)+$$
$$\frac{\frac{1}{2}\left(\Omega^2-\frac{A^2}{2}\Xi\right)^{-\frac{1}{2}}}{\Xi}\left(\frac{A^2}{2}\left[\frac{\tilde{A}^2}{2}\left[\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d\right]^{-\frac{1}{2}}-2\right]\right)+$$
$$\frac{\sqrt{\Omega^2-\frac{A^2}{2}\Xi}}{\Xi^2}\left(\frac{\tilde{A}^2}{2}\left[\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d\right]^{-\frac{1}{2}}-2\right)\leqslant 0$$

$$\frac{\partial R}{\partial \hat{b}_0^d}>0$$

iff:

$$\frac{\tilde{A}^2}{2}\left[\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d\right]^{-\frac{1}{2}}-2>0$$

$$\frac{\tilde{A}^2}{2}\left[\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d\right]^{-\frac{1}{2}}>2$$

$$\left(\frac{\tilde{A}^2}{4}\right)^2>\left(\left[\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d\right]^{\frac{1}{2}}\right)^2$$

$$\frac{\tilde{A}^4}{16}>\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2+\tilde{A}^2\hat{b}_0^d$$

$$\frac{\tilde{A}^4}{16}-\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2>\tilde{A}^2\hat{b}_0^d$$

$$\frac{\tilde{A}^2}{16}-\frac{1}{\tilde{A}^2}\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2>\hat{b}_0^d$$

And, iff $\frac{\tilde{A}^2}{16}-\frac{1}{\tilde{A}^2}\left\{-\left(z-\frac{0.25(2-\tilde{\pi})\tilde{A}^2}{1-\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)\tilde{\pi}}\right)\right\}^2<\hat{b}_0^d$, $\frac{\partial R}{\partial \hat{b}_0^d}<0$. □